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MCMILLAN LENS IN A SYSTEM WITH SPACE CHARGE

I. Lobach*, The University of Chicago, Chicago, IL, 60637, USA
S. Nagaitsev[1], E. Stern, T. Zolkin, Fermilab, Batavia, IL, 60510, USA
[1]also at The University of Chicago

Abstract

Space charge (SC) in a circulating beam in a ring produces both betatron tune shift and betatron tune spread. These effects make some particles move on to a machine resonance and become unstable. Linear elements of beam optics cannot reduce the tune spread induced by SC because of its intrinsic nonlinear nature. We investigate the possibility to mitigate it by a thin McMillan lens providing a nonlinear axially symmetric kick, which is qualitatively opposite to the accumulated kick by SC. Experimentally, the proposed concept can be tested in Fermilab's IOTA ring. A thin McMillan lens can be implemented by a short (70 cm) insertion of an electron beam with specifically chosen density distribution in transverse directions. In this article, to see if McMillan lenses reduce the tune spread induced by SC, we make several simulations with particle tracking code Synergia. We choose such beam and lattice parameters that tune spread is roughly 0.5 and a beam instability due to the half-integer resonance 0.5 is observed. Then, we try to reduce emittance growth by shifting betatron tunes by adjusting quadrupoles and reducing the tune spread by McMillan lenses.

INTRODUCTION

Tune spread itself is not significantly detrimental to beams in rings. Emittance growth happens when particles in tune diagram cross some resonances, see Fig. 1 a for example. If tune spread is not large (<0.1), then it is usually enough to adjust quadrupoles and shift particles from the resonance region. However, if one deals with very dense bunches, tune spread may become large ($0.3-0.5$ and greater, see Fig. 1 a), bigger than the distance between two successive resonances. In this case, it is impossible to prevent crossing resonances by solely adjusting quadrupoles. This is the kind of situation considered in this contribution. It will be shown that big tune spread can be reduced by McMillan lenses. The motivation to use specifically McMillan lenses and not other types of nonlinear elements, is the fact that single particle motion in a ring with McMillan lens is integrable, granted specific choice of lattice. Whereas other possible lenses, e.g. Gaussian, Lorentz', do not possess this quality and may themselves lead to emittance growth.

The structure of the paper is the following. In section McMillan lens, integrability of single particle motion in a ring with McMillan lens is considered. The constraints on linear part of one turn transformation are given. In section Numerical simulations, a strategy for suppressing effects of large space charge is outlined and exemplified by particular simulations with Synergia.

* ilobach@uchicago.edu

MCMILLAN LENS

It was shown in [1] that a specific linear transformation followed by a kick by a round 2D McMillan lens results in two integrals of motion, one of which is the longitudinal component of particle's angular momentum. We have found that the result of [1] can be further generalized and applied to the following transformation:

$$\begin{pmatrix}\bar{x}\\ \bar{x}'\\ \bar{y}\\ \bar{y}'\end{pmatrix}=\begin{pmatrix}a_x & b & 0 & 0\\ -c_x & -a_x & 0 & 0\\ 0 & 0 & a_y & b\\ 0 & 0 & -c_y & -a_y\end{pmatrix}\begin{pmatrix}x\\ x'\\ y\\ y'\end{pmatrix}+\begin{pmatrix}0\\ K_m\bar{x}\\ 0\\ K_m\bar{y}\end{pmatrix}, \quad (1)$$

where $c_x=\left(1+a_x^2\right)/b$, $c_y=\left(1+a_y^2\right)/b$,

$$K_m=-\frac{k_m}{1+\left(\frac{\bar{r}}{r_m}\right)^2}, \quad (2)$$

with $\bar{r}=\sqrt{\bar{x}^2+\bar{y}^2}$. The first term in Eq. (1) represents linear part of the transformation, whereas the second term corresponds to nonlinear kick of a thin McMillan lens.

It turns out that for this transformation longitudinal part of angular momentum $(xy'-yx')$ is not conserved in general. Instead, the following quantity becomes invariant

$$L=\left(a_y-a_x\right)xy+b\left(xy'-yx'\right), \quad (3)$$

Second integral of motion, specific to a system with McMillan lens, is

$$T=\left(\frac{1}{r_m^2}+\frac{1}{r^2}\right)\left(a_xx^2+a_yy^2+brr'\right)^2 + bk_m\left(a_xx^2+a_yy^2+brr'\right)+\left(r^2+\frac{L^2}{r^2}\right). \quad (4)$$

The more general transformation with two invariants (see Eq. (1)) provides us with more freedom in terms of lattice structure. However, the betatron tunes for linear part of the transformation still have to be either 0.25 ($b>0$) or 0.75 ($b<0$), as in [1].

McMillan lens can be realized on experiment as a short insertion of electron beam (if circulating particles are protons) in the ring. This electron lens must have the following current density profile:

$$j_e(r)=\frac{I_e}{\pi r_m^2\left(1+\left(\frac{r}{r_m}\right)^2\right)^2}, \quad (5)$$

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where I_e is the total current in the electron beam. Then, in the thin lens approximation, integrated lens' strength (see Eq. (2)) is given by

$$k_m = \frac{2eI_eL_m\left(1 - v_e v_p/c^2\right)}{v_e r_m^2 \gamma_p m_p v_p^2}, \quad (6)$$

where e is elementary charge, L_m is the length of the insertion of electron beam, v_e and v_p are the speeds of electrons and protons respectively, $\gamma_p = \sqrt{1 - v_p^2/c^2}$, m_p is proton mass. In this formula it is assumed that electrons and protons move in the same direction, if their velocities are antiparallel, then one must change the sign in the numerator from minus to plus. To see if thin lens approximation works well for some set of parameters, one has to estimate beta function in the vicinity of McMillan lens (one cannot calculate it rigorously in a system with nonlinear elements) and compare it with the length of McMillan lens. If the latter is much smaller than the value of beta function, then thin lens approximation is reliable.

NUMERICAL SIMULATIONS

In this section, results of several simulations are presented which show that resonances do cause beam instabilities, and that a conjunction of quadrupoles adjustment and McMillan lenses can substantially reduce emittance growth in case of large initial tune spread. The simulations are done with a particle tracking code taking into account collective effects Synergia on Accelerator Simulations Wilson cluster at Fermilab, 3D space charge solver is used. We focus on a specific one turn lattice:

$$5 \times \text{ODOOFOODOMOFO} + \text{S}, \quad (7)$$

where O is a drift space, F and D are focusing and defocusing quadrupoles respectively, M represents a McMillan lens, and S corresponds to a thin axially symmetric linear defocusing kick (akin solenoid) with integrated strength k_s. Note that we do not use any bending magnets here for the sake of simplicity. Each cell also has an RF cavity to constrain longitudinal size of the bunch (dropped in the above lattice structure). The element S is introduced as a source of perturbation in the system with a period of one turn. It will induce 0.5 resonance. It may be thought of as an error in the lattice. However, we will be using high value of integrated strength in S for its effect on emittance growth to be profound and explicit. In all of the following simulations, protons' energy is 800 MeV, the number of protons in the bunch is $N_p = 9 \times 10^{10}$, length of the drift O is 5.5 m, length of quadrupoles F and D is 1.0 m, longitudinal rms size of the bunch is $\sigma_z = 0.5$ m, initial transverse rms emittances are $\epsilon_x = \epsilon_y = \epsilon = 0.86$ mm mrad, the bunch is always matched to the bare (no space charge, no McMillan lenses) lattice, Gaussian bunch is used, number of simulated turns is 1000.

In the first simulation, see Fig. 1 a, the strength of element S is zero ($k_s = 0$), and McMillan lenses are turned off too ($k_m = 0$). Hereinafter, we present 1D tune diagrams for x-axis, because in the lattice under consideration 2D tune diagrams are symmetric along the diagonal. The quantity along vertical axis, denoted by p, is particle density with respect to tune ν_x, p is normalized to 1. One can see that in this case the beam distribution in the tune diagram has a size about 0.5 and crosses the 0.5 resonance. However, there is no significant emittance growth because S is absent and there is nothing to induce the corresponding beam instability. Here, the strength of the quadrupoles $k_f = |k_d| = 0.1585\,\text{m}^{-2}$ are chosen in such a way that each cell ODOOFOODOOFO has bare betatron tunes $\nu_x = \nu_y = 0.75$, so that the requirement for integrability of motion with McMillan lenses inserted would be satisfied. Because of space charge, element S and quadrupoles adjustments in further simulations this requirement will not be rigorously satisfied, but we will try to keep betatron tunes per one cell close to 0.75 so that we are not too far from the integrability conditions. One turn has bare betatron tunes $\nu_x = \nu_y = 3.75$ (shown by a dashed blue vertical line in the tune diagram).

Analytically, one can estimate tune shift of an rms particle in Gaussian beam by the following formula (see [2])

$$\Delta\nu = -\frac{N_p r_p}{4\pi\epsilon_n\beta_p\gamma_p^2 B_f}, \quad (8)$$

where $r_p = e^2/m_p c^2$ is the classical proton radius, $\epsilon_n = \gamma_p\beta_p\epsilon$ is the normalized emittance of the proton bunch (the bunch is assumed round), $B_f = \sqrt{2\pi}\sigma_z/C$ is the bunching factor, $C = 240$ m is the ring circumference. For the beam under consideration, $\Delta\nu \approx -0.5$, which is about two times bigger than what wee see in Fig. 1 a. However, this is alright, because Eq. (8) was derived in assumption of very small tune shift (< 0.1).

In the second simulation, see Fig. 1 b, we set the integrated strength of element S to $k_s = -0.07923\,\text{m}^{-1}$ (defocusing) to introduce the perturbation inducing 0.5 resonance. You can see a dramatic increase of emittance due to the instability. In the tune diagram, the tune spread is reduced, because qualitatively it is inversely proportional to beam emittance (see Eq. (8)) and the beam has ballooned.

In the third simulation, see Fig. 1 c, we keep the same k_s, but we also adjust quadrupole strengths $k_f = |k_d| = 0.1623\,\text{m}^{-2}$ to shift the beam distribution from the 0.5 resonance. As you can see, the tune spread did not change much. However, emittance increase is reduced significantly. Emittance growth is still bigger than in Fig. 1 a, presumably, because element S also induces higher order resonances, although they are much weaker than the 0.5 resonance.

In the fourth simulation, see Fig. 1 d, we keep the same integrated strength of element S, adjust quadrupole strengths so that $k_f = |k_d| = 0.1606\,\text{m}^{-2}$, in addition we set McMillan lenses strengths to $k_m = 0.01\,\text{m}^{-1}$ and $r_m = 4.14$ mm (about rms beam size). One can see that the tune spread is reduced (roughly by 45%). However, there is also a moderate emittance increase with respect to the third simulation (roughly 20%). The difference in relative changes of tune spread and emittance is very important, because it means

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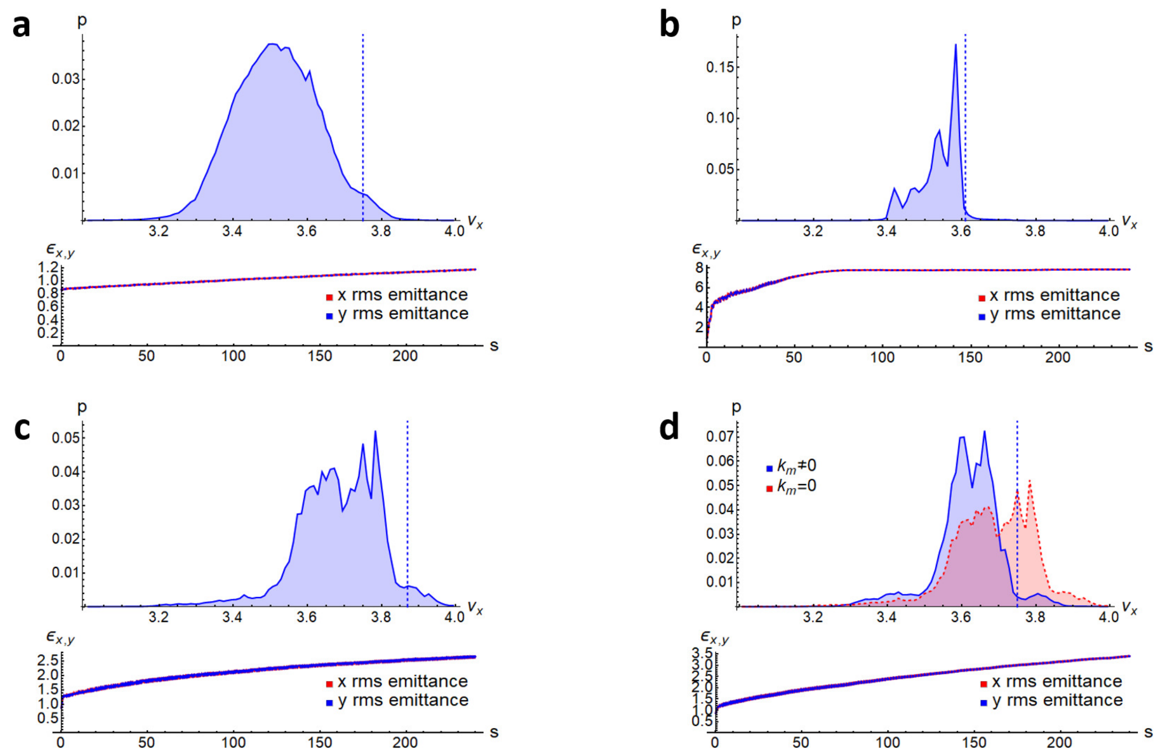

Figure 1: 1D tune diagrams and transverse rms emittance plots for the four simulations. ν_x is betatron tune, p is particle density with respect to ν_x, unit of rms emittance here is mm mrad, s is the distance traveled by the bunch in km, vertical dashed blue lines represent bare tunes. a) $k_f = |k_d| = 0.1585\,\mathrm{m}^{-2}$, $k_s = k_m = 0$, b) $k_f = |k_d| = 0.1585\,\mathrm{m}^{-2}$, $k_s = -0.07923\,\mathrm{m}^{-1}$, $k_m = 0$, c) $k_f = |k_d| = 0.1623\,\mathrm{m}^{-2}$, $k_s = -0.07923\,\mathrm{m}^{-1}$, $k_m = 0$, d) $k_f = |k_d| = 0.1606\,\mathrm{m}^{-2}$, $k_s = -0.07923\,\mathrm{m}^{-1}$, $k_m = 0.01\,\mathrm{m}^{-1}$, the red dashed plot is the case c for comparison.

that the tune spread reduction is not barely a consequence of emittance growth, as in Eq. (8). Let us qualitatively clarify here why McMillan lenses work. With space charge present, particles with small amplitudes encounter bigger tune shift, whereas particles with large amplitudes spend most of time far from the core of the beam and do not experience the effect of Coulomb force much, therefore their tune shift is smaller, see [3]. The kick of McMillan lens (see Eq. (2)) is such that particles with smaller amplitudes encounter stronger kick (toward beam axis), and particles with larger amplitudes encounter weaker kick. Because tune shift due to space charge is negative, and tune shift due to McMillan lenses is positive, at some choice of parameters they might even out and we might observe a net tune spread reduction.

CONCLUSIONS

In considered simulations we had only one strong resonance, namely 0.5, induced by axially symmetric defocusing kick by a thin element S. This is why it was possible to reduce emittance growth solely by adjustment of quadrupoles (see Fig. 1 c), even though tune spread was huge (~ 0.5). A real lattice would have more errors than this idealized model and in actuality emittance growth would be bigger in Fig. 1 c. One would have to reduce tune spread to prevent crossing resonances. In Fig. 1 d we show that it is possible to do with McMillan lenses. They do introduce some emittance growth (compare to Fig. 1 c), but it is much smaller than what one would have with resonances in real lattice (compare for example with Fig. 1 b), therefore a net effect of introduction of McMillan lenses would be positive.

The results of reducing large tune spread (~ 0.5) by McMillan lenses (compare Figs. 1 d and c) are not perfect. There still is room for further investigation. The simulations performed so far indicate that McMillan lenses can cope with tune spread of ~ 0.1 per one lens (we had total tune spread ~ 0.5 per 5 cells).

REFERENCES

[1] V.V. Danilov and E.A. Perevedentsev, "Two Examples of Integrable Systems with Round Colliding Beams", in *Proc. 17th IEEE Particle Accelerator Conference (PAC'97)*, Vancouver, B.C., Canada, May 1997, paper 8V001, pp. 1759–1761.

[2] arXiv, https://arxiv.org/pdf/1405.2352

[3] M. Reiser, "Space-Charge Tune Shift and Current Limits in Circular Accelerators," in *Theory and Design of Charged Particle Beams*, New York, NY, USA: John Wiley&Sons, 1994, pp. 260–273.