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ENERGY SPREAD COMPENSATION IN ARBITRARY FORMAT MULTI-BUNCH ACCELERATION WITH STANDING WAVE AND TRAVELING WAVE ACCELERATORS

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Abstract

In the E-driven ILC (International Linear Collider) positron source, the beam is generated and accelerated in a multi-bunch format with mini-trains. The macro-pulse contains 2 to 8 mini-trains with several train gaps, because the pulse format is a copy of a part of the bunch storage pattern in DR (Damping Ring). This pulse format causes a variation of the accelerator field in the pulse due to the transient beam loading and an intensity fluctuation of captured positron. In this article, we discuss the compensation of the energy spread of such beam in standing wave and traveling wave accelerators. For standing wave accelerator, it can be compensated by switching input RF at appropriate timings. For traveling wave accelerator, it can be compensated by amplitude modulation of the input RF.

INTRODUCTION

ILC (International Linear Collider) [1] is an e+e- linear collider based on super-conducting accelerator with CME from 250 to 1000 GeV. It would be constructed in Iwate, Japan, as the main project of High energy physics. In the E-driven ILC positron source, the positron is generated in a macro-pulse with 6.15 ns bunch spacing [2]. Because the macro pulse is a copy of a part of DR (Damping Ring) fill pattern, there is a gap as shown in Fig. 1. This is an example and number of gaps depends on DR fill pattern. Seimiya performed a start-to-end simulation of the E-Driven ILC positron source without the beam loading effect [3]. Kuriki perform the simulation with the beam loading effect, but it is only for the injector part (up to 250 MeV) [4]. To accelerate the beam in such format, the transient beam loading effect which varies the accelerating field at the macro pulse and every mini-train heads, has to be compensated. In this article,

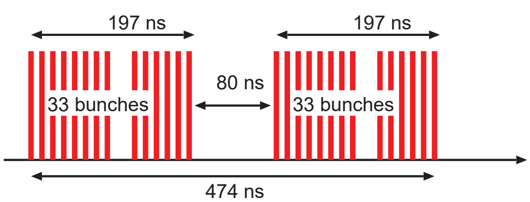

Figure 1: An example of macro-pulse structure in E-Driven ILC positron source. The macro pulse is composed from two mini-trains of 33 bunches with 6.15 ns bunch spacing. There is a gap of 80 ns.

we show that AM (Amplitude Modulation) of the input RF solve these issues. The method is effective for standing wave (SW) and traveling wave (TW) accelerators, respectively.

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SW ACCELERATOR

Standing wave accelerator is an pi-mode RF cavity with zero group velocity. It is composed from several cells. If we assume the cavity as a big cell (single cell model), the voltage evolution $V(t)$ with a constant beam loading I is [5],

$$V(t) = \frac{2\sqrt{\beta P_0 r L}}{1+\beta}\left(1-e^{-\frac{t}{T_0}}\right) - \frac{rIL}{1+\beta}\left(1-e^{-\frac{t-t_b}{T_0}}\right), \quad (1)$$

where P_0 is the input RF power, r is shunt impedance, L is length of the tube, β is the coupling beta, t is time, t_b is time to start acceleration, T_0 is defined as $T_0 = 2Q/[\omega(1+\beta)]$, where Q is Q-value and ω is angular frequency of RF. Because the time constant of the input RF and beam loading terms are same, the voltage can be flat if we choose an appropriate t_b. A real SW accelerator is composed from multi-cells. For example, SW accelerator designed for positron acceleration in ILC positron source [6] has 11 cells. Instead of equivalent circuit models (e.g. Ref. [7]), we account only the power flow in our multi-cell model [8]. It is enough to evaluate the time constants of the input RF and beam loading modes to examine the beam loading compensation accuracy. W_i, V_i, and P_i are stored energy, voltage, and dissipated power loss for i-th cell, respectively. We assume each cell is identical, i.e. $Q_0 = \omega W_i/P_i$, where Q_0 is the unloaded quality factor. We assume 11 cells SW accelerator. The time differential of W_6 (stored energy of the center cell) is

$$\frac{dW_6}{dt} = -P_6 - P_{5,6} - P_{6,5} + P_{7,6} + P_{6,7} + P_{in} - P_r - IV_6, \quad (2)$$

where P_{in} and P_r are input power and reflected power from and to the wave guide. I is beam loading current and IV_6 is the beam loading term. $P_{5,6} = kQP_5$ and $P_{6,5} = kQP_6$ are power flow from 5th cell to 6th cell and 6th cell to 5th cell, respectively. k is cell coupling. With $W_6 = \frac{Q}{R\omega}V_6^2$, $P_{in} = \beta/(RN)V_{in}^2$, $P_r = \beta/(RN)(V_{in} - NV_6)^2$, the equation can be converted to voltage as

$$\begin{aligned}\frac{dV_6}{dt} &= -\left[\frac{(1+N\beta)\omega}{2Q} + k\omega\right]V_6 \quad (3)\\ &+\frac{1}{2}k\omega\frac{V_7^2}{V_6} + \frac{1}{2}k\omega\frac{V_5^2}{V_6} + \frac{\omega\beta}{Q}V_{in} - \frac{\omega RI}{2Q},\end{aligned}$$

where V_i is voltage of i-th cell, V_{in} is voltage of input wave guide, R is shunt impedance of a cell, and N is number of cells of the structure (11). Because $1/k$ (typically 100) is much larger than Q (10000), time constant of cell coupling is much faster than that of RF power fill and decay and we expect $V_i(t) \sim V_{i\pm1}(t)$. Then, these terms can be approximated as $V_7^2/V_6 \sim V_7$ and $V_5^2/V_6 \sim V_5$ leading

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$$\frac{dV_6}{dt} = -\left[\frac{(1+N\beta)\omega}{2Q} + k\omega\right]V_6 + \frac{1}{2}k\omega V_7 + \frac{1}{2}k\omega V_5 + \frac{\omega\beta}{Q}V_{in} - \frac{\omega RI}{2Q}. \quad (4)$$

Similar equations are derived for other cells. These equations are simultaneous linear differential equations which can be solved with the matrix formalism. The equation in matrix form is

$$\frac{d\mathbf{V}}{dt} = \mathbf{AV} + \mathbf{C}, \quad (5)$$

$\mathbf{V}(V_i)$ is voltage vector and $\mathbf{A}$ is 11×11 matrix. The components are

$$a_{i,i} = -\frac{\omega}{2Q} - k\omega \quad (i \neq 1, 6, 11) \quad (6)$$

$$a_{6,6} = -\frac{(1+N\beta)\omega}{2Q} - k\omega \quad (7)$$

$$a_{1,1}, a_{11,11} = -\frac{\omega}{2Q} - \frac{1}{2}k\omega \quad (8)$$

$$a_{i,i+1}, a_{i+1,i} = \frac{1}{2}k\omega, \quad (9)$$

other components are zero. $\mathbf{C}(c_i)$ is a constant vector and the components are $-\omega RI/Q$ for $i \neq 6$ and $-\omega RI/Q + \omega\beta V_{in}/Q$ for i = 6. Because $\mathbf{A}$ is a symmetric real matrix which can be diagonalized with with a orthogonal matrix $\mathbf{R}$ as $\mathbf{B} = \mathbf{R^T AR}$, where $\mathbf{B}$ is a diagonal matrix. With this matrix $\mathbf{R}$, Eq. (5) can be transformed as

$$\frac{d\mathbf{R^T V}}{dt} = \mathbf{R^T ARR^T V} + \mathbf{R^T C}, \quad \frac{d\mathbf{V}'}{dt} = \mathbf{BV}' + \mathbf{C}', \quad (10)$$

where $\mathbf{V}' \equiv \mathbf{R^T V}$ and $\mathbf{C}' \equiv \mathbf{R^T C}$. Because $\mathbf{B}$ is a diagonal matrix, the mixing term among cells are disappeared and the differential equation is independent for each line as

$$\frac{dV_i'}{dt} = \lambda_i V_i' + C_i', \quad (11)$$

which can be solved as

$$V_i'(t) = \tau_i C_i' \left(1 - e^{-\frac{t}{\tau_i}}\right), \quad (12)$$

with $V'(t=0) = 0$. $\tau_i = -1/\lambda_i$. The solution for V_i is obtained as linear combination of V' as

$$\mathbf{V} = \mathbf{RV}'. \quad (13)$$

Instead of a single time constant in the single cell model, there are 11 modes with 11 time constants. The amplitude of each mode are different for input RF and beam loading, because the input RF mode is driven only by the center cell, cell 6. On the other hand, the beam loading mode is driven by all cells.

Figure 2 shows the result of evolution of acceleration voltage calculated by the multi-cell mode for 11 cells SW cavity [6] with 2.0 A beam loading. Input power is 22.5 MW with $\beta = 6.0$. The transient beam loading is compensated nicely, the energy spread in the macro pulse is less than 0.1%. Even the cavity has many modes with different τ_i, the effective time constants of the RF mode and beam loading mode are very close, resulting the good beam loading compensation. In Fig.2, the thick dotted and dashed horizontal

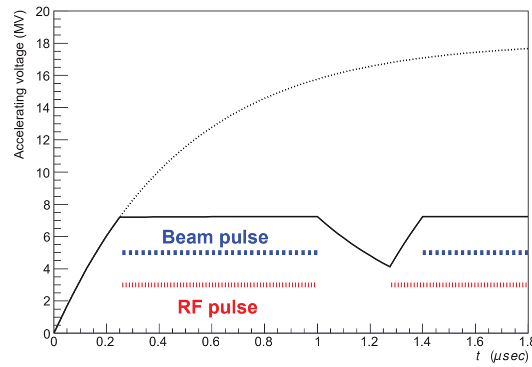

Figure 2: Voltage evolution with 2.0 A beam loading giving 7.3 MV acceleration.

lines show the periods when the beam current is on (2.0 A) and RF power is on, respectively. At the end of the first train (t=1.0 μs in this figure), the input RF is turned off. By turning on the input RF again prior to the second mini-train, voltage is recovered. By adjusting the timing, voltage at the beginning of the second train can be same as that at the first train. Voltage is kept for the second train because the situation is completely same as that at the fist train. We can repeat this process for every gaps. The accuracy of the beam loading compensation strongly depends on the band width of input RF source, i.e. speed of RF switching. The speed should be compared to the effective time constant of SW accelerator. It is mainly determined by Q, ω, k, and β. In this case, we set $\beta = 6$ for the heavy beam loading, the time constant is $\sim$ 0.5μs.

RF AM method with a phase-modulation is proposed by Urakawa [9]. In this method, two RF sources are combined by controlling the phase of each input RF. The amplitude of combined RF V_{com} is

$$V_{com} = V_1 \cos\phi + V_2 \cos\phi, \quad (14)$$

where V_1 and V_2 are input RF amplitude, and ϕ is the control phase. Because a typical speed of the phase modulation is $\sim$ 25 ns for $\pi/2$, the speed of AM is much faster than the time constant of the accelerator and a good compensation is expected with this method.

TW ACCELERATOR

The accelerating voltage of TW accelerator at t is determined by integral over the preceding time window of t_f which is the filling time defined as $t_f = \int_0^L 1/v_g dz$, where L is accelerator length and v_g is the group velocity. That causes the transient beam loading at the beam pulse head. It can be compensated by AM initially proposed by Satoh [10] with a matrix formalism. Here, we derive the analytic formula. The accelerating field by a TW accelerator in Laplace transformation is [11]

$$E(s) = \frac{\omega}{Q(1-e^{-2\tau})}\frac{V(s)}{L(s+\frac{\omega}{Q})}\left[1 - e^{-(s+\frac{\omega}{Q})t_f}\right] - \frac{\omega r_0}{2Q(1-e^{-2\tau})}\frac{I(s)}{s}\left[1 - e^{-\frac{\omega}{Q}t_f} - \frac{\omega(1-e^{-st_f - 2\tau})}{Q(s+\frac{\omega}{Q})}\right], \quad (15)$$

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where τ is attenuation constant, s is variable of Laplace transformation, $V(s)$ is input RF in voltage, $I(s)$ is the beam current. If we make an AM in the input RF, $V(s)$ is also a function of s in Eq. (16). Here, we consider AM as

$$V(t) = V_0U(t) + V_1U(t-t_f) + V_2(t-t_f)U(t-t_f) + V_2(t-2t_f)U(t-2t_f). \quad (16)$$

where V_0 is the accelerating voltage corresponding to the initial input RF power, V_1 is a DC component, and V_2 is a linear component, $U(t)$ is a step function, 0 for $t < 0$ and 1 for $t >= 0$. RF input is started at $t = 0$, and the beam acceleration is started at $t = t_f$. Please note that the dimension of V_2 is V/sec. Acceleration field $E_a(t)$ can be obtained by the inverse Laplace transformation as

$$E_a(t) = V_0 + \frac{\left[1-e^{-\frac{\omega}{Q}(t-t_f)}\right]}{1-e^{-2\tau}}\left(V_1 - \frac{Q}{\omega}V_2\right) + \frac{V_2(t-t_f)}{1-e^{-2\tau}} - \frac{r_0 I_0}{2(1-e^{-2\tau})}\left[-\frac{\omega}{Q}e^{-2\tau}(t-t_f) + 1 - e^{-\frac{\omega}{Q}(t-t_f)}\right], \quad (17)$$

where we assume the beam current $I(t)$ as $I(t) = I_0U(t-t_f)$. Here we set V_1 and V_2 as

$$V_1 = \frac{r_0LI_0}{2}(1-e^{-2\tau}), \quad V_2 = -\frac{r_0LI_0}{2}\frac{\omega}{Q}e^{-2\tau}, \quad (18)$$

resulting $E_a(t)$ to be a constant as

$$E_a(t) = \frac{V_0}{L} + \frac{\left(V_1 - \frac{Q}{\omega}V_2\right)}{L(1-e^{-2\tau})} - \frac{r_0I_0}{2(1-e^{-2\tau})} = \frac{V_0}{L}. \quad (19)$$

Figure 3 shows the evolution of accelerating voltage with (solid line) and without (dashed line) AM. Here, we assume L=3.0 m, r = 5.7×10^7 Ohm/m, 50 MW input power, f=3.0 GHz, Q=10000, and $\tau = 0.63$. For $t > 2t_f$, $E_a(t)$ is varied (decreased), but this variation will be compensated another AM pulse starting from $t = 2t_f$. The amplitude of the second modulation is less than the first one. For a bunch gap, the input RF power should be V_0, i.e. the power giving $E_a = V_0/L$ without beam loading, to suppress an overshoot of V. We start AM when we start the beam acceleration again at the second train head and the amplitude depends on the length of the gap period.

When the beam current is described as $I(t) = \sum I_o U(t-t_i)(-1)^i$, the number of terms are increased as number of the gaps are increased for the compensation, but the equation form is same, i.e. linear terms and exponential terms, as shown in Eq. (19). That means any transient effect on E_a can be compensated by the two components AM, i.e. constant and linear terms in principle.

Kim [12] presented the beam loading compensation of TW accelerator with AM. In his method, the beam acceleration timing and RF amplitude was adjusted empirically and there is still ~ 10% energy spread. In our method, the compensation is perfect and the accuracy is limited by the speed of AM. It is expected to be less than 25 ns which is much faster than a typical t_f of TW accelerator, in order of µs. A good compensation is expected.

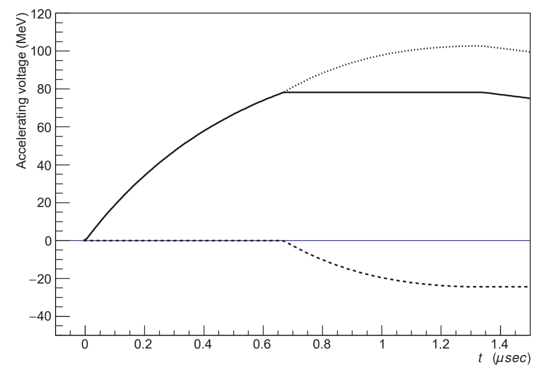

Figure 3: Voltage evolution with a constant beam loading. The solid line shows the acceleration voltage, the dashed line shows the beam loading voltage, and the dotted line shows the accelerating voltage without beam loading.

SUMMARY

We consider the transient beam loading compensation of SW and TW accelerators for a macro pulse acceleration with gaps. For SW, RF switching at an appropriate timing gives an uniform acceleration. For TW accelerator, the two components AM is effective to compensate the beam loading effect. This is a first solution giving a perfect compensation for the transient beam loading effect of TW accelerator. The technique is effective for a macro pulse with any number of gaps. The speed of AM determines the accuracy of the compensation for both cases. AM by the phase-modulation technique gives a fast AM which is enough for our purpose.

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