9th International Particle Accelerator Conference IPAC2018, Vancouver, BC, Canada JACoW Publishing
ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-MOPMF010

# MEASUREMENT AND SIMULATION OF BETATRON COUPLING BEAM TRANSFER FUNCTION IN RHIC*

Y. Luo†, W. Fischer, A. Marusic, M. Minty
Brookhaven National Laboratory, Upton, NY 11973, USA

*Abstract*

Transfer function measurements are important for characterizing betatron tunes, betatron coupling, and beam spectrum in the routine operation of the Relativistic Heavy Ion Collider (RHIC). To counteract the linear betatron coupling, we developed a technique to continuously measure the betatron coupling coefficient with a base band phase lock loop tune meter in 2006. Based on this technique, we demonstrated and built a robust tune/coupling feedback in RHIC. In this article, we revisit the BTF measurement with betatron coupling to benchmark our BTF simulation code. We also compared the values of eigenmode projection ratios from BTF with those calculated with the single particle model.

## INTRODUCTION

Linear betatron coupling couples the horizontal and vertical particle's betatron motion in circular accelerators. In the Relativistic Heavy Ion Collider (RHIC), it pushes the horizontal and vertical tunes away toward the betatron resonances on the acceleration which may cause bad beam lifetime and even beam dump. In polarized proton operation, linear betatron coupling also reduces the proton polarization transmission efficiency during the acceleration. To counteract the linear betatron coupling, we developed a technique to continuously measure the betatron coupling coefficient with a base band phase lock loop tune meter (BBQ) [1]. Based on this technique, we built a robust tune/coupling feedback for the first time in RHIC in 2006 [2]. Since then, tune/coupling feedback has become an important tool for RHIC machine development and routine physics operation.

Our theory to measure the betatron coupling is based on single particle motion [1]. However, in reality, the coupling coefficient is measured with the help of BBQ which continuously kicks the beam with a chosen frequency. This process is actually the measurement of beam transfer function (BTF). To continuously measure the betatron coupling, the BBQ kicking frequency is fixed to betatron tunes. While in the BTF measurement, the kicking frequency sweeps across the whole beam tune distribution.

In the following we first briefly review the single particle model based perturbation theory to extract the betatron coupling coefficient. Then we presented the BTF measurements from the beam experiments performed in the 2018 RHIC ion run. The goal of the beam experiments is to benchmark our BTF simulation tool and to compare the eigenmode projection ratios from BTF measurement and those predicted from single particle model.

* Work supported by U.S. DOE under contract No DE-AC02-98CH10886
† yluo.bnl.gov

## PERTURBATION THEORY

To measure the linear difference coupling coefficient

$$C^- = |C^-|e^{i\chi} = \frac{1}{2\pi}\int_0^L \sqrt{\beta_x\beta_y}k_s e^{i[\phi_x-\phi_y-2\pi\Delta\cdot s/L]}dl, \quad (1)$$

we define the eigenmode projection ratios

$$\begin{cases} r_I &= \dfrac{A_{I,y}}{A_{I,x}} \\ r_{II} &= \dfrac{A_{II,x}}{A_{II,y}} \end{cases}, \quad (2)$$

and the eigenmode phase differences

$$\begin{cases} \Delta\phi_I &= \phi_{I,y}-\phi_{I,x} \\ \Delta\phi_{II} &= \phi_{II,x}-\phi_{II,y} \end{cases} \quad (3)$$

Here $A_{i,z}$ is the amplitude projection of eigenmode $i$ onto the $z$ plane, where $i = I, II, z = x, y$. $\phi_{i,z}$ is the phase difference of the eigenmode projections.

Based on the perturbation theory of single particle motion with linear betatron coupling [1], if we can measure the above eigenmode projections and the phases, the linear difference coupling coefficient $C^-$ can be determined,

$$|C^-| = \frac{2\sqrt{r_I r_{II}}}{1+r_I r_{II}}\,|Q_I - Q_{II}| \quad (4)$$

and $\Delta\phi_I$ is the phase of $C^-$. Knowing $C^-$, we can correct the betatron coupling with the existing skew quadrupole correctors in the accelerator.

## COUPLING BTF MEASUREMENT

BTFs are routinely measured and archived during the RHIC operation. In the 2018 100 GeV ion run, we measured BTFs with different skew quadruple settings in two beam experiment sessions. In the first session, we used a rebucketed Ruthenium (Ru) ion beam in the Blue ring. In the second session, we used non-rebucketed Ru ion beam in the Yellow ring. Rebucketing from 28 MHz to 197 MHz is used in the RHIC ion operation to shorten the ion bunch length and produce more collisions in the central area of the detector. Table 1 lists the key machine and beam parameters during these beam experiments.

During the experiments, we first corrected the betatron coupling with the online coupling monitor. Then we scanned the strengths of skew quadrupole families. There are three skew quadrupole families in RHIC. Normally we paired Families 1 and 2 together to produce a new family F13 which is orthogonal to Family 2. At each step, we recorded the

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-MOPMF010



Table 1: Key Parameters During the Two Beam Experiments

| Parameter | Session I | Session II |
|---|---|---|
| Fill number | 21537 | 21586 |
| RHIC ring used | Blue | Yellow |
| betatron tunes | ( 0.234, 0.227) | (0.235, 0.231) |
| linear chromaticity | unknown | (4,4) |
| rebucketing | Yes | No |
| rms trans. emittance (μm) | 1.3 | 1.7 |
| rms bunch length (m) | 0.25 | 1.0 |
| rms momentum spread | $4.6 \times 10^{-4}$ | $2.0 \times 10^{-4}$ |

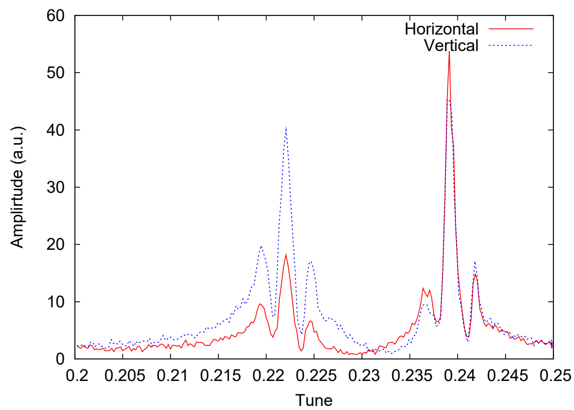


Figure 1: Measured BTF with a rebucketed beam.

eigenmode tunes, eigenmode projection ratios $r_I$ and $r_{II}$ from the eigenmode monitor, and took BTF measurements.

Figure 1 shows an example of coupling BTF measurement with a rebucketed beam in the first session. In the plot, both horizontal and vertical peaks appeared when one plane BTF was taken, which means that there was betatron coupling in the machine. There are side peaks on both sides of the betatron tune peaks which were caused by the rebucketing. Therefore, in the second section, we did not rebucket the beam.

Figure 2 shows an example of coupling BTF measurement with a non-rebucketed beam in the second session. There was no side peak in the BTF measurements. Figure 3 shows the longitudinal bunch profiles with and without rebucketing. Without rebucketing, the bunch profile was like Gaussian distribution. With rebucketing, there were particles outside the central 197 MHz RF bucket. Those particles generated the side peaks as shown in Figure 1.

Table 2 lists the measured eigenmode amplitude projection ratios in the two experiments. From Eq. (4), $r_I \times r_{II}$ can be used to determine the amplitude of coupling coefficient. During the experiment, the measured phases $\Delta\phi_{I,II}$ were very noisy and need averaging before use.

## COUPLING BTF SIMULATION

To understand the BTF with betatron coupling, we carried out numeric simulation to reproduce the measured BTFs. An element-by-element particle tracking code has been used [3].

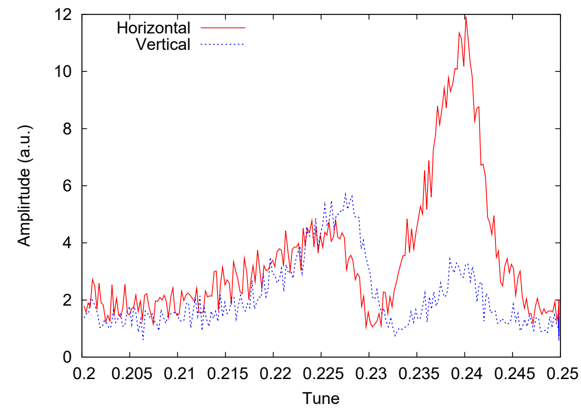


Figure 2: Measured BTF with a non-rebucketed beam.

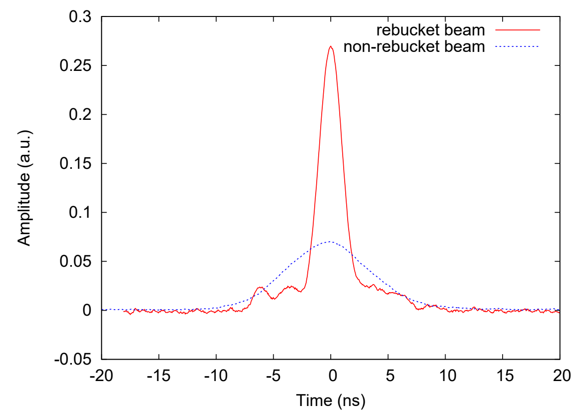


Figure 3: Measured bunch profiles w/o rebucketing.

The macro-particles are kicked horizontally or vertically at IP2 with 64 uniformly distributed frequencies between 0.21 to 0.25. The beam response includes in-phase and out-of-phase parts of the kick. The BTF is the ratio of beam response and the BBQ kick as function of the excitation frequency which covers the whole range of betatron ferquency. To save computing time, we only track 2000 macro-particles up to 4096 turns. Although there is a significant numeric

Table 2: Measured Eigenmode Projection Amplitude Ratios

| SkewQ Family | $\Delta(K_1 L)$ ( $10^{-3}$ ) | $r_I$ | $r_{II}$ | $r_I r_{II}$ |
|---|---|---|---|---|
| Session I : | | | | |
| F2 | + 0.1 | 1.0 | 0.6 | 0.06 |
| F2 | + 0.2 | 1.25 | 0.3 | 0.375 |
| F2 | + 0.3 | 1.4 | 0.4 | 0.56 |
| F13 | +0.1 | 0.8 | 0.3 | 0.24 |
| F13 | +0.2 | 1.4 | 0.5 | 0.70 |
| F13 | +0.3 | 1.6 | 0.6 | 0.96 |
| Session II : | | | | |
| F2 | +0.1 | 0.3 | 1.2 | 0.36 |
| F2 | +0.2 | 0.1 | 2.0 | 0.02 |
| F2 | -0.1 | 0.1 | 1.5 | 0.15 |
| F2 | -0.2 | 0.2 | 2.1 | 0.42 |

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-MOPMF010

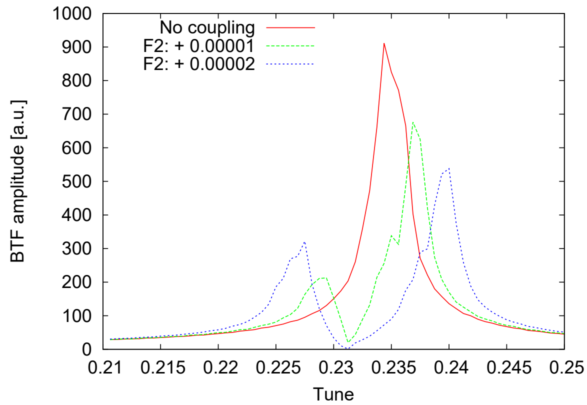


Figure 4: Simulated horizontal coupling BTFs with different skew quadrupole family settings.

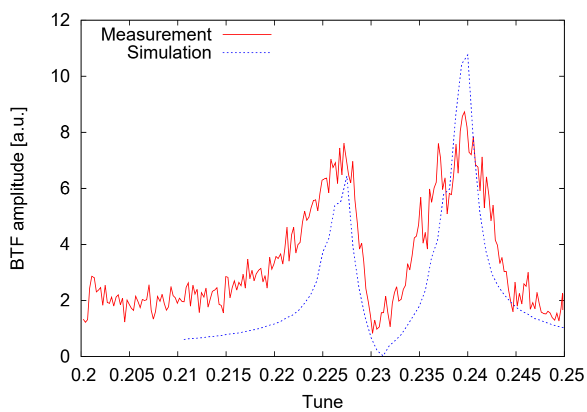


Figure 5: Comparison of hrizontal BTFs between measurement and simulation.

noise in the results, it is sufficient for comparison with the BTF measurements.

In the following, we simulate the BTFs taken from the second session with a non-rebucketed beam. At this point, we are not able to generate a realistic longitudinal beam profile with rebucketed beams. We assumed 3-d Gaussian particle distribution for non-rebucketed beam.

Figure 4 shows the calculated horizontal BTFs with different strengths of skew quadrupole Family 2. The distance between the two eigenmode tune peaks increases when more coupling is introduced in the machine by increasing the skew quadrupole family's strength.

Figure 5 shows the comparison of the horizontal BTFs from measurement and simulation with Family 2's strength at 0.0002 $m^{-1}$. The simulation largely reproduced the meaured BTF, although the widths of measured BTF are wider than the simulation. It may be caused by a smaller tune spread used in the simulation. We did not include other nonlinear fields other than the arc sextupoles in the simulation.

Table 3: Calculated Eigenmode Projection Amplitude Ratios

| SkewQ Family | $\Delta(K_1L)$ ( $10^{-3}$ ) | $r_I$ | $r_{II}$ | $r_I r_{II}$ |
|---|---|---|---|---|
| Session I: | | | | |
| F2 | +0.1 | 0.60 | 0.20 | 0.12 |
| F2 | +0.2 | 0.95 | 0.34 | 0.32 |
| F2 | +0.3 | 1.15 | 0.40 | 0.46 |
| F13 | +0.1 | 0.89 | 0.31 | 0.27 |
| F13 | +0.2 | 1.20 | 0.41 | 0.50 |
| F13 | +0.3 | 1.35 | 0.46 | 0.62 |
| Session II: | | | | |
| F2 | +0.1 | 0.85 | 0.31 | 0.27 |
| F2 | +0.2 | 1.16 | 0.45 | 0.52 |
| F2 | -0.1 | 0.85 | 0.33 | 0.28 |
| F2 | -0.2 | 1.16 | 0.43 | 0.50 |

Table 3 lists the calculated eigenmode projection ratios based on the single particle theory. The measured projection ratios during the first experiment session agreed better with their predictions from the single particle model. The differences are bigger in the second experiment session. During the second session, the eigenmode projection and phase measurements with BBQ were quite noisy. Also based on the single particle perturbation theory, the projection ratios vary with local $\beta$ functions although $r_I r_{II}$ does not.

## CONCLUSION

In this article, we presented the BTF measurements from two experiments performed in the 2018 RHIC ion run. The BTF measurements are used to benchmark our offline BTF simulation code. Side peaks were observed on both sides of the tune peaks when a rebucketed beam was used. We compared the eigenmode projection amplitude ratios from BTF and that predicted from the single particle model. The differences between them are smaller in the first beam experiment than in the second beam experiment. Next we will continue carrying out numeric simulation to reproduce every BTF measurement and try to understand the difference in the eigenmode projection amplitude ratios between the BTF measurement and that predicted from the single particle model.

## REFERENCES

[1] Y. Luo, et al, Phys. Rev. ST Accel. Beams 9, 124001 (2006).

[2] P. Cameron, et al., Phys. Rev. ST Accel. Beams 9, 122801 (2006).

[3] Y. Luo, Nucl.Instrum.Meth. A801 (2015) 95-103.