PRECISION Q0 MEASUREMENT OF AN SRF CAVITY WITH A DIGITAL RF TECHNIQUES*

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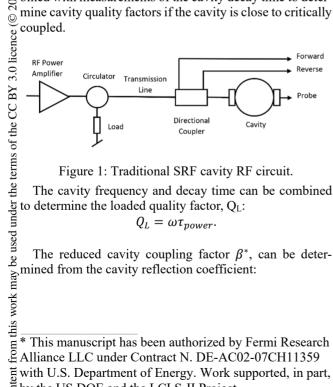Fermi National Accelerator Laboratory, Batavia, IL 60510, USA

Abstract

Direct measurement of the quality factor of SRF cavity using traditional RF techniques is essential for cavity production and development. Systematic effects of the measurement can contribute significant amounts of error to these measurements if not accounted for. This paper will present measurements taken at Fermilab using a digital RF system to characterize and correct for these systematic effects and directly measure the quality factor versus gradient curve for a single spoke resonator in the Spoke Test Cryostat at Fermilab. These measurements will be compared to traditional calorimetric measurements, and a discussion of improving/extending these techniques to other testing situations will be included.

INTRODUCTION

Cavity quality factors are typically measured using a circuit like that shown in Figure 1 [1]. The cavity is driven by an amplifier through a transmission line. A directional coupler separates the forward and reflected waves inside this transmission line. A probe monitors the cavity field.

The power of the forward, reverse and transmitted waves during steady state operation on resonance in can be combined with measurements of the cavity decay time to determine cavity quality factors if the cavity is close to critically coupled.

Figure 1: Traditional SRF cavity RF circuit.

The cavity frequency and decay time can be combined to determine the loaded quality factor, Q_L:

$$Q_L = \omega \tau_{power}.$$

The reduced cavity coupling factor β^*, can be determined from the cavity reflection coefficient:

$$\beta^* = Q_{Ext}^{-1}(Q_0^{-1} + Q_T^{-1})^{-1} = \frac{\sqrt{P_{Forward}^{SS}} \pm \sqrt{P_{Reverse}^{SS}}}{\sqrt{P_{Forward}^{SS}} \mp \sqrt{P_{Reverse}^{SS}}}$$

where SS represents Steady State, Q_0 is the cavity intrinsic quality factor, and Q_{Ext} is the input coupler quality factor. The top signs are for an over-coupled cavity, and the bottom signs are for an under-coupled cavity.

The cavity field probe coupling can then be determined from Q_L, β^*, and the steady state forward and probe power measurements as follows:

$$Q_T = \frac{P_{Reactive}}{P_{Probe}} = \frac{4Q_L}{1 + \beta^{*-1}} \frac{P_{Forward}^{SS}}{P_{Probe}^{SS}}.$$

Finally, the intrinsic quality factor, Q_0, can be calculated from the above quantities as follows:

$$Q_0 = \frac{Q_T Q_L (1 + \beta^*)}{Q_T - Q_L (1 + \beta^*)}.$$

Several systematic effects can bias such measurements [2]. Impedance mismatches between the circulator and the transmission line can reflect reverse energy back into the forward wave. During the decay this can lead to a non-zero forward wave which can interfere constructively or destructively with the cavity field, lengthening or shorting the cavity decay time. Additionally, cross-talk (directivity) in the directional coupler used to separate the forward and reflected waves can cross-contaminate the forward and reflected signals leading to systematic biases in the cavity coupling measurement.

ERROR CHARACTERIZATION

Circulator mismatches and imperfect directivity can be measured by inserting a variable length transmission line (trombone) in the circuit between the directional coupler and the cavity and recording the complex cavity baseband waveforms for a range of cavity detuning values as the length of the trombone is systematically varied over a wavelength at the cavity frequency.

Changing the length of the trombone will change the phase with respect to the cavity field of energy reflected by the circulator from the reverse wave back into the forward wave. As the trombone length is swept over a wavelength, the phase of this energy should sweep through 4π. This phase change should modulate the measured decay time of the cavity/waveguide system sinusoidally through 2 complete cycles. The magnitude and phase of the modulation can be used to determine the complex circulator reflection coefficient which can be used in turn to correct the measured decay time for circulator impedance mismatches.

Changing the trombone length will also change the relative phases of the forward, reverse with respect to the cavity field. As the length of the trombone is swept over a wavelength, the forward/probe and reverse/probe phase will sweep from 0 through 2π in opposite directions. If the phase of the RF signals in addition to the magnitude is recorded during the trombone sweep, the complex directivity of the coupler can be determined and used to suppress cross-contamination of the two signals offline.

The cavity signals can then be corrected for these two systematic effects, improving the accuracy of the decay time and cavity coupling measurements.

This technique is illustrated using data recorded from a 325 MHz single spoke resonator [3, 4] installed in the Fermilab STC [5] operating at 2K with and a nominal gradient of 5 MV/m. The RF circuit was modified by installing a trombone between the directional coupler and the cavity. The length of the trombone was systematically varied over one wavelength (c/325MHz) in 10 steps. At each step of the trombone, the phase of the phase lock loop locking the drive frequency to the cavity resonance frequency was varied in 7 steps between $-45°$ and $45°$ while the complex baseband RF signals were recorded using a digital RF control system provided by the Fermilab AD/LLRF group [6] for an interval of 10 seconds. During each recording the generator power was shut off after approximately 7 seconds, allowing the cavity to decay.

For comparison, a control sample was recorded without the trombone in the circuit while the lock phase was varied over the same range.

CAVITY DECAY MEASUREMENTS

The blue and red points in Figure 2 respectively compare the loaded cavity quality factor, Q_L, as a function of trombone position before and after correction for impedance mismatches at the circulator. The uncorrected measurements were multiplied by a factor to account for non-zero forward power during the decay to obtain the corrected measurements. Remaining linear trend in the data is likely from additional attenuation in the trombone.
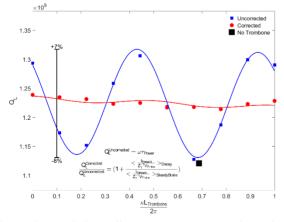

Figure 2: Loaded Quality Factor Q_L vs trombone length (Blue Squares). Q_L measured without the trombone circuit (Black Diamond). Using directivity-corrected powers, the measured Q_L can be corrected (Red Circles).

The uncorrected measurements vary sinusoidally by up to 8% over two cycles as the length of the trombone is varied over a wavelength while the corrected measurements are much less sensitive to trombone length.

The black diamond shows the equivalent measurement with no trombone in the circuit. The value is consistent with the value expected from the trombone measurements when the phase length of the waveguide is the same.

CORRECTING DIRECTIVITY

Cross-contamination of the forward and reverse signals extracted by the directional coupler can lead to significant systematic biases in the determination of the cavity coupling if the cavity is not close to critically coupled.

The measurement of the forward and reverse waves by the directional coupler can be modelled by the product of a trombone dependent phase delay and a linear mixing matrix representing cross-talk (directivity) in the coupler:

$$C.$$

The derivatives of the measured forward/probe signal ratio and measured reflected/probe signal ratio with respect to changes in detuning can be used to determine the relative complex gains of the forward and reverse waves and cross-contamination coefficients. These components are separated in the frequency domain and normalized by the inverse transfer functions.

Figure 3 compares the transformed derivative ratio measured for the 325 MHz cavity under test before and after directivity determination and correction. As would be expected from the equations, the uncorrected ratio shows a large peak at -2 corresponding to the coefficient ratio G_R/G_F and smaller peaks at -4 and 0 corresponding to the coefficient ratios ϵ_F/G_F and ϵ_R/G_F. Before correction the directivity is 21 dB. Following offline correction, the directivity improves to 50 dB giving much better separation and relative calibration of the forward and reflected waves.
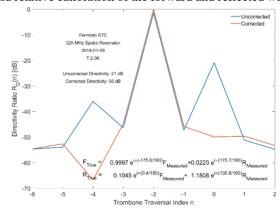

Figure 3: Directivity determination before and after correction factor applied.

INVERSE TRANSFER FUNCTIONS

The complex base-band envelopes of the forward and reverse waves at the cavity and the cavity voltage are related by the following inverse transfer functions:

$$T_{P/F}^{-1} = \frac{1}{2}\left(1 + \beta^{*-1} + i\left(\frac{\omega - \delta}{\omega_T}\right)\right)$$

$$T_{P/R}^{-1} = \frac{1}{2}\left(1 - \beta^{*-1} - i\left(\frac{\omega - \delta}{\omega_T}\right)\right).$$

The sum of the inverse transfer functions adds to unity while the derivatives with respect to detuning are equal and opposite. These constraints can be used to determine the relative complex gain of the three cavity RF signals from the data itself, as seen in Figure 4.
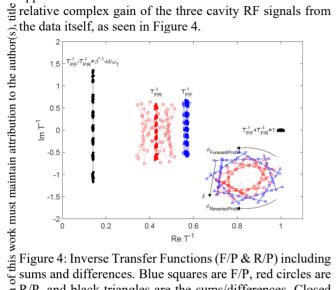

Figure 4: Inverse Transfer Functions (F/P & R/P) including sums and differences. Blue squares are F/P, red circles are R/P, and black triangles are the sums/differences. Closed symbols use directivity-corrected signals, and open symbols are uncorrected. Inset data (lower right) shows data before calibration/directivity correction.

This allows the inverse coupling to be determined directly from the calibrated inverse transfer functions with no ambiguity whether the cavity is over or under-coupled.

Following correction for directivity and calibration, the cavity inverse transfer functions can be determined from the complex steady state ratios of the forward/probe and reverse/probe signals.
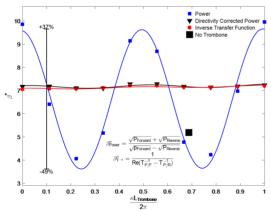

Figure 5: Measured β^* via power measurements versus trombone angle with uncorrected signals (Blue Squares) and directivity-corrected signals (Black Triangles) compared to directivity-corrected Transfer Functions (Red Circles)..

The inverse transfer functions measured for the 325MHz cavity under test operating at 2K are plotted in the complex plane in Figure 5, with and without directivity correction.

Measurement of Q_T and thus gradient and Q_0 include both directivity and circulator reflection errors, and thus the traditional technique error is reduced by using directivity corrected signals, but still significant. The measured error in Q_T drops from +8% to -12% to ±9% when using the directivity corrected powers. The inverse transfer function technique has an error of ±1%. The equivalent errors for gradient measurement are down by a factor of 2 from Q_T.

QUALITY FACTOR VS GRADIENT

Figure 6 shows the variation of the intrinsic quality factor with accelerating gradient measured using the power technique with no trombone, power technique with the trombone set to zero (which happens to gives the highest Q_0), and the inverse transfer function technique. The coupling calculated from the power in the cavity RF signals depends strongly on the phase length of the waveguide and cross-talk in the directional coupler.

The cavity quality factor was also measured calorimetrically, using the outlet mass flow meter in the STC cryogenic circuit and in-cryostat heaters. These measurements are limited by noise in the cryogenic system to above 1-2 Watts, so only high field points were taken, shown on Figure 6 as black diamonds. This measurement agrees very well with the directivity-corrected inverse transfer-function data.
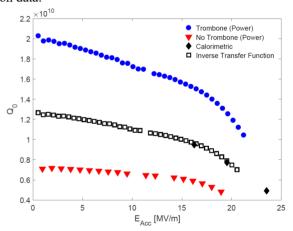

Figure 6: Q_0 vs E measured by power with no trombone (Red Triangles), at the zero trombone position (gives 'maximum Q_0') (Blue Circles), and as measured with Transfer Functions (Black Squares) compared with calorimetric measurements (Black Diamonds).

CONCLUSION

Systematic effects associated with impedance mismatches at the circulator and imperfect directivity limit the accuracy of cavity quality factor measurements. Consistency constraints can be used to improve the calibration of the RF signals if the complex base-band signals are recorded in conjunction with a trombone in the circuit. The improved calibration allows accurate measurements to be made over a wider range of couplings.

REFERENCES

[1] T. Powers, "Theory and Practice of Cavity RF Test Systems", IAEA-INIS 28.01, 2006.

[2] J. Holzbauer *et al.*, "Systematic Uncertainties in RF-based Measurement of Superconducting Quality Factors", Nuclear Instruments and Methods A, 830, 11 September 2016, pp. 22-29, 2016

[3] M.A. Hassan *et al.*, "Development of Low Single-Spoke Resonators for the Front End of the Proton Improvement Plan-II at Fermilab", *IEEE Transaction on Nuclear Science*, vol. 64, Issue 9, September 2017.

[4] A. Sukhanov *et al.*, "Results of Cold Testing of the Fermilab SSR1 Cavities", *in Proc. LINAC'16*, Geneva, CH, paper THPP057, 2014.

[5] M. Chen *et al.*, "Project X Superconducting Spoke Resonator Test Cryostat 2K Conversion", *AIP Conference Proceedings*, 1573, p. 790, 2014.

[6] J. Branland *et al.*, "LLRF Design for the HINS-SRF Test Facility at Fermilab", *in Proc. LINAC'10*, Tsukuba, Japan, paper MOP083, 2010.