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A COMPUTATIONAL METHOD FOR MORE ACCURATE MEASUREMENTS OF THE SURFACE RESISTANCE IN SRF CAVITIES *

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Abstract

The principal loss mechanism for superconducting RF cavities in normal operation is Ohmic heating due to the microwave surface resistance in the superconducting surface. The typical method for calculating this field-dependent surface resistance $R_s(H)$ from RF measurements of quality factor Q_0 implicitly returns a weighted average of R_s over the surface as a function of peak surface magnetic field H, not the true value of R_s as a function of the local magnitude of H. In this work we present a computational method to convert a measured Q_0 vs. H_{peak} to a more accurate R_s vs. H_{local}, given knowledge about cavity geometry and field distribution.

INTRODUCTION

In superconducting radio-frequency accelerator physics (SRF), the signature figure of merit of accelerating cavities is the intrinsic quality factor Q_0. As for any harmonic resonator, this quality factor indicates the amount of power P needed to sustain an energy U stored in the cavity's electromagnetic field, shown here with resonant frequency ω:

$$Q_0 = \frac{\omega U}{P} = \frac{\omega \frac{1}{2} \int |\vec{H}|^2 \, dV}{P} \quad (1)$$

Thus a cavity with a higher quality factor needs less power to maintain a given field magnitude. For state-of-the-art SRF cavities in typical operation, losses arise almost exclusively from Ohmic power dissipation on the RF surface:

$$P = \frac{1}{2} \int |\vec{H}|^2 R_s(H) \, dS \quad (2)$$

Understanding and improving this surface resistance is a key factor in improving SRF technology and studying new SRF surfaces and surface treatments. Further, this resistance can depend strongly on the magnitude of the surface magnetic field, as it does for many materials currently under investigation (see for example impurity-doped niobium [1,2], Nb_3Sn [3], and thin film niobium [4]). As such, it is highly desirable to measure R_s as a function of H for SRF cavities.

For simplicity, in order to extract R_s from experimental measurements of Q_0, researchers calculate a "geometry factor"[1] G from Eqs. 1 and 2:

$$G = \frac{\omega \int |\vec{H}|^2 \, dV}{\int |\vec{H}|^2 \, dS} \quad (3)$$

This allows the following approximate form of Q_0:

$$Q_0(H_{pk}) = \frac{G}{R_s(H_{pk})} \quad (4)$$

Here, H_{pk} is the peak surface magnetic field.

The definition in Eq. 4 implicitly assumes that the surface resistance and magnetic field are constant across the surface of the cavity, allowing $R_s(H)$ to be pulled out of the integral in Eq. 3. To be more accurate, it is better to rewrite Eq. 4 noting that the surface resistance is actually a weighted average:

$$Q_0(H_{pk}) = \frac{G}{R_{av}(H_{pk})} \quad (5)$$

Rearranging Eqs. 1 to 5 demonstrates this averaging:

$$R_{av}(H_{pk}) = \frac{G}{Q_0(H_{pk})} = \frac{\int |\vec{H}|^2 R_s(H) \, dS}{\int |\vec{H}|^2 \, dS} \quad (6)$$

Given knowledge about the field distribution and the geometry of the cavity, it is possible to derive the observed $R_{av}(H_{pk})$ from a theoretical model of $R_s(H)$. On the other hand, "undoing" the averaging is not easy to do analytically without at least a parameterized functional form for $R_s(H)$. However, it is possible to approximate this reversal process (*i.e.* to calculate $R_s(H)$ from an observed $R_{av}(H_{pk})$) numerically, given the aforementioned knowledge about the cavity geometry and field distribution. In this work, we describe such a method using linear algebra.

MATHEMATICAL PROCESS

The right-hand side of Eq. 6 can be approached as an averaging operator **A** acting on a function $R_s(H)$ that transforms the actual surface resistance into the observed $R_{av}(H_{pk})$. To calculate the fundamental, local[2] resistance $R_s(H)$ from the observed $R_{av}(H_{pk})$, we need to find the inverse of the averaging function:

$$R_s(H) = \mathbf{A}^{-1}(\mathbf{A}(R_s(H))) \quad (7)$$

One way to calculate this inverse function is by discretizing the problem. We can find a good approximate solution by representing $R_s(H)$ as a vector **R**, with each entry of **R** denoting the surface resistance at a given field value H_i; the index i goes from 1 to N, **R** has N entries, and **A** has dimensions $N \times N$. In our case we will set the H_i values to be

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[1] For a full treatment of standard practices, see H. Padamsee's textbook [5].

[2] *N.B.*: "local" here does not imply that this method can find localized areas of heating; instead, it assumes a defect-free surface where the function $R_s(H)$ is the same everywhere.

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spaced evenly between H_{pk}/N and H_{pk}. Then the operation looks as follows:

$$\mathbf{R_{av}} = \mathbf{AR} \tag{8}$$

The averaging operator $\mathbf{A}$ is a positive lower-triangular matrix. To calculate the jth entry of the ith row (with $1 \leq j \leq i$) of $\mathbf{A}$, one should first separate the cavity surface into i sections $S_{j/i}$, each the union of the areas of the surface where the field H is approximately equal to $H_j = j\, H_{pk}{}^1/i$. Then the integral in Eq. 6 can be split into a sum of integrals over the sections $S_{j/i}$:

$$R_{av,i} = \frac{1}{\int |\vec{H}|^2\, dS} \sum_{j=1}^{i} \int_{S_{j/i}} |\vec{H}|^2 R_s(H)\, dS \tag{9}$$

Because the sections $S_{j/i}$ are split so that the local field (and thus the local surface resistance) is approximately uniform, we can approximate Eq. 9 as follows:

$$R_{av,i} = \frac{R_s(H_i)}{\int |\vec{H}|^2\, dS} \sum_{j=1}^{i} \int_{S_{j/i}} |\vec{H}|^2 dS \tag{10}$$

$\mathbf{R}$ defined as above contains these $R_s(H_i)$ values, so the rest of the right hand side of Eq. 10 must define $\mathbf{A}$. Indeed, the entries of $\mathbf{A}$ can be defined as follows:

$$A_{i,j} = \frac{\int_{S_{j/i}} |\vec{H}|^2 dS}{\int |\vec{H}|^2\, dS} \tag{11}$$

These integrals can be calculated easily from computer models of cavities.

Armed with $\mathbf{A}$ for a given cavity, it is now easy to calculate the local resistance $\mathbf{R}$ (*i.e.* $R_s(H)$) from a measurement of the average resistance $\mathbf{R_{av}}$ (*i.e.* $R_{av}(H)$) by inverting $\mathbf{A}$:

$$\mathbf{R} = \mathbf{A}^{-1}\mathbf{R_{av}} = \mathbf{A}^{-1}\frac{G}{\mathbf{Q}} \tag{12}$$

Here, $\mathbf{Q}$ is just the vector of $Q(H_{pk} = H_i)$.

DISCUSSION

This method is approximative – computationally it is limited by N (in the mathematical limit of $N \to \infty$, this discrete method approaches the continuous operation in Eq. 6), and it requires interpolating N points on the measured $Q(H_{pk})$ curve – but it is a step more accurate from the approximation in the traditional method where $R_s(H)$ is simply said to be equal to $R_{av}(H_{pk})$ (thus with $\mathbf{A}$ equal to the identity matrix $\mathbf{I}$).

In order to use this method in a non-computationally-expensive way, we suggest increasing N and repeating the process in the section above until the result converges within some acceptable threshold. This mitigates errors due to the first aspect of approximation above.

One could remove the second aspect of this approximation by building $\mathbf{A}$ such that the indices i, j were not evenly spaced but instead corresponded to the field values $H_{pk,i}$ measured in the experimental $Q(H)$ curve; this would remove the need for interpolation, but it might severely limit accuracy for curves with few points measured or irregular spacing between points. On the other hand, if one had a theoretical model of $R_s(H)$, it would be possible to use $\mathbf{A}$ (without inverting) to account for the geometry of a cavity and thereby compare theoretical predictions with experimental results (*e.g.* for theoretical parameter fitting).

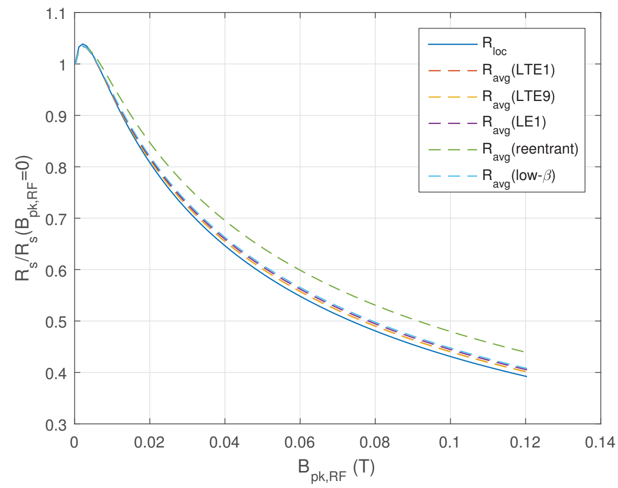

Figure 1: Local (intrinsic) surface resistance (solid line), generated here from theory [1, 7] at 1.3 GHz, and calculations of the implicitly-averaged measurements that would be made in various elliptical cavities (dashed lines). Material parameters used here were $\ell = 10$ nm, $\Delta/k_B T_c = 1.97$, $T_c = 9.2$ K, and $T = 2$ K.

This approach to determine the local surface resistance will have different impacts depending on cavity geometry. In particular, the ILC/TESLA [6] single-cell cavities common in today's fundamental SRF research [1–3] have very high field uniformity over the RF surface; as a result, for these cavities we would not expect correcting the implicit averaging of the surface resistance to yield a big difference in the final result. However, for more complicated geometries and field configurations such as multi-mode cavities, coaxial half-wave and quarter-wave cavities, reentrant cavities, low-β cavities, deflecting/crabbing cavities, and mushroom cavities, the traditional, implicitly averaged measurement of the surface resistance may be significantly different from the result of the process outlined in this paper.

To demonstrate these differences, Fig. 1 shows as an example a theoretical curve for the surface resistance of nitrogen-doped niobium[3] [1] (solid line) as well as the $R_{av}(H)$ that would be measured from various cavity shapes with this treatment (dashed lines). In general the corrections for elliptical-type cavities are modest but significant. The TESLA single-cells and nine-cells show a maximum difference of about 1%, CEBAF [8] and low-β elliptical cavities [9] (scaled

[3] The theoretical calculations in this paper were made with the following material parameters: $\ell = 10$ nm, $\Delta_0/k_B T_c = 1.97$, $T_c = 9.2$ K, $T = 2$ K.

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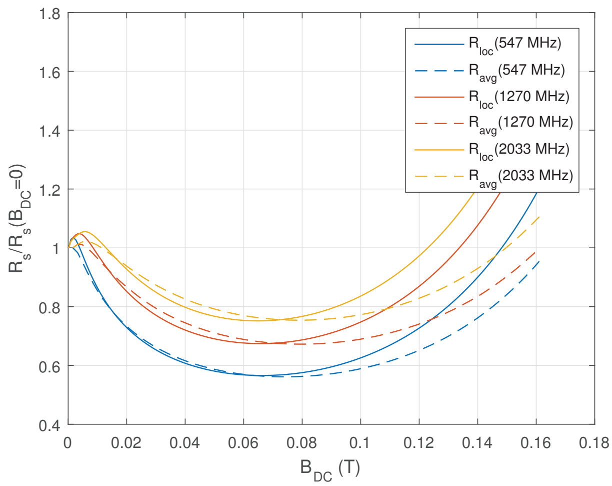

Figure 2: Shown here as a function of applied DC magnetic field are the local (intrinsic) surface resistance (solid lines), generated here from theory [7], and what would be recorded using the traditional (implicit averaging) method for the three modes of the Cornell coaxial sample host cavity (dashed lines).

here to 1.3 GHz) up to 3%, and reentrant (high-gradient) cavities [10] up to 11%.

Figure 2 shows similar calculations, in this case for the forthcoming Cornell coaxial sample host cavity [11, 12]. Here, the differences between the intrinsic R_s and the surface-averaged $R = G/Q$ are quite substantial. Using this method will be crucial for obtaining accurate results for the field-dependent surface resistance from this cavity.

CONCLUSIONS

In this work we have described a mathematical method to improve measurements of the microwave surface resistance as a function of field by accounting for the distribution of the field on the cavity surface. This method can provide modest improvements over the traditional calculation for elliptical TE-mode cavities; more substantial improvements can be gained for more exotic cavity geometries and field configurations.

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