# FAST READOUT ALGORITHM FOR CYLINDRICAL BEAM POSITION MONITORS PROVIDING GOOD ACCURACY FOR PARTICLE BUNCHES WITH LARGE OFFSETS* 

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## Abstract

A simple, analytically correct algorithm has been developed for calculating fully relativistic beam coordinates using the signals from an ideal cylindrical beam position monitor (BPM) with four pickup electrodes (PUEs) of infinitesimal widths. Results from realistic BPMs with finite-width PUEs are then simulated. Small, empirically determined corrections result in excellent accuracy. Good accuracy is also obtained with non-relativistic beams. The algorithm is then tested with BPM data from the Cornell photo-injector. High data acquisition rates are demonstrated with a new FPGA-based BPM readout system.

## INTRODUCTION

A closed-form algorithm has been developed for determining the exact beam position in cylindrical 4-button BPMs for the ideal case of very small pick-up PUEs and fully relativistic round beams [1]. With Particle Studio [2] simulations, we show how realistic results deviate from the ideal case and to what extent they can be corrected to improve the agreements for cases of wide PUEs and nonrelativistic beams. We then apply the new algorithm to data measured with a BPM at the Cornell photo-injector [3]. Finally, we describe an implementation using FPGAs to achieve high data acquisition rates.

## THE CLOSED-FORM ALGORITHM

Equations (1) through (6) below describe the algorithm derived [1] for the ideal case of a cylindrical BPM with four infinitesimally small PUEs traversed by a fully relativistic charged-particle bunch parallel to the axis of the cylinder. The correction coefficients $b$ and $\varepsilon$ used below, are for improving agreement in non-ideal cases. Calling $A_{x}, B_{x}$ and $A_{y}, B_{y}$ the signal amplitudes measured at opposite PUEs in the horizontal and vertical planes respectively, we define the usual normalized signal differences:

$$
\begin{equation*}
Q_{x}=\frac{A_{X}-B_{X}}{A_{X}+B_{X}} \quad \text { and } \quad Q_{y}=\frac{A_{y}-B_{y}}{A_{y}+B_{y}} \tag{1}
\end{equation*}
$$

And the modified values, where $b=0$ in the ideal case:

$$
\begin{equation*}
Q_{x}^{\prime}=Q_{x}+b Q_{x}\left|Q_{y}\right| \quad \text { and } \quad Q_{y}^{\prime}=Q_{y}+b Q_{y}\left|Q_{x}\right| \tag{2}
\end{equation*}
$$

[^0]We then define:

$$
\begin{equation*}
Q=\sqrt{Q_{x}^{\prime 2}+Q_{y}^{\prime 2}} \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
\rho=\frac{1}{Q}-\sqrt{\frac{1}{Q^{2}}-1} \tag{4}
\end{equation*}
$$

And we finally obtain the beam coordinates X and Y :

$$
\begin{equation*}
X=a(1+\epsilon) \rho \frac{Q_{x}^{\prime}}{Q} \quad \text { (5) } \quad Y=a(1+\epsilon) \rho \frac{Q_{y}^{\prime}}{Q} \tag{5}
\end{equation*}
$$

where $\epsilon=0$ in the ideal case and $a$ is the radius of the cylindrical BPM. Eqs. (1) to (6) define the algorithm and could be written as one equation for $X$ and one for $Y$ as functions of the four signal amplitudes, but these expressions would be too long and cumbersome.

## PARTICLE STUDIO SIMULATIONS FOR A 4-BUTTON BPM

The model used for the simulations is shown In Fig. 1. Simulations were performed for beam positions from 0 to 20 mm in 5 mm steps in both dimensions. The assumed bunch charge was 1 nC and the Gaussian bunch length was 30 mm RMS. The simulations are performed for fully relativistic beams $(\beta=v / c=1)$.


Figure 1: Perspective view and cross-section of the BPM model used for the simulations. The BPM diameter is 2 a $=60 \mathrm{~mm}$ and the button diameters are 10 mm . The beam position shown is $\mathrm{X}=17.5 \mathrm{~mm}, \mathrm{Y}=17.5 \mathrm{~mm}$.
A graphic representation of the results is shown in Fig. 2. The RMS distance between calculated (circles) and nominal positions (dots) is $29.2 \mu \mathrm{~m}$, which makes position errors barely visible given the scales of this graph. The values used for the correction parameters used are $\varepsilon=0.0234$ and $b=-0.0144$.

In contrast, we show in Fig. 3 the result of using the same Particle Studio (PS) simulated data with the readout approach presently in use [4] at the Relativistic Heavy Ion

Collider (RHIC) and many other facilities. In this approach, the horizontal and vertical data are processed independently. The position is calculated from the ratios $\mathrm{Q}_{\mathrm{x}}$ and $\mathrm{Q}_{\mathrm{y}}$ as the sum of a linear and a cubic term.


Figure 2: Simulation results obtained with the model of Fig. 1. The RMS distance between calculated (circles) and nominal positions (dots) is $29.2 \mu \mathrm{~m}$.


Figure 3: The circles represent positions obtained by using third order polynomial calibrations applied individually to each axis, while the black dots represent the beam positions used as input to the simulation.

The poor agreement for large offsets occurs because the Taylor series expansion of the underlying function Eq. (4) converges very slowly. Cross terms could be used to improve agreement along the diagonals, but the new approach is much better.
Another comparison of the old approach to the new one with and without the correction terms is shown in Fig. 4. We see that for beam offsets larger than $\sim 3 \mathrm{~mm}$, the new approach provides much more accurate results.


Figure 4: Errors along the diagonal of the 60 mm diameter BPM computed with the conventional cubic polynomial approach and with the new equation with and without correction terms. The lower plot is a vertically expanded view of the upper one.

## PERFORMANCE FOR NONRELATIVISTIC BEAMS

Deviations that occur for values $\beta=\mathrm{v} / \mathrm{c}<1$ (see derivation of the algorithm in ref. [1]) are shown in Fig. 5. PS simulations with the same 60 mm diameter BPM model shown in Fig. 1 were used.


Figure 5: Position errors as function of distance from the center for beams of different velocities obtained from simulations with the 60 mm BPM model shown in Fig. 1.

Table 1 shows the proton energies corresponding to the $\beta$-values used as well as the $\varepsilon$ an $b$ correction parameters and the maximum errors obtained.

Table 1: Simulation Parameters and Maximum Errors

| $\mathbf{v} / \mathbf{c}$ | Proton <br> energy | $\boldsymbol{\varepsilon}$ | $\boldsymbol{b}$ | Max. error <br> for $\mathbf{r}=\mathbf{2 0} \mathbf{~ m m}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | $(\mathrm{MeV})$ |  |  | $(\mathrm{mm})$ |
| 1 |  | 0.022 | -0.0125 | 0.03 |
| 0.9 | 1214.3 | -0.0013 | -0.033 | 0.11 |
| 0.7 | 375.6 | -0.035 | -0.062 | 0.24 |
| 0.5 | 145.2 | -0.057 | -0.084 | 0.35 |

## TESTING WITH DATA FROM THE CORNELL PHOTOINJECTOR

To test how well this algorithm works in practice, we performed a brief test using data from a stripline BPM at the Cornell Photoinjector [3]. The inner diameter of the pipe is 34.9 mm , and the striplines are 66 mm long, roughly 7.5 mm wide. All measurements were performed with $\sim 5 \mathrm{pC}$ bunch charge with $<1 \mu \mathrm{~A}$ of average current and a kinetic electron energy of $5.5 \mathrm{MeV}(\beta=0.9964)$. Fig. 6 shows the result of using the data to calculate positions with three different values of the parameter $b$ and comparing with the simple difference/sum method.


Figure 6: Reconstructed beam positions with (a) the simple difference/sum method, and for the present algorithm with (b) $b=-0.06$, (c) $b=-0.08$ and (d) $b=-0.10$. For this diameter of pipe and stripline width, a value of $b=-0.08$ best corrects the nonlinear curvature of the data.

## IMPLEMENTATIONS USING FIELD PROGRAMMABLE GATE ARRAYS

A recently developed, FPGA-based readout system [4] was modified to implement the new algorithm. Fig. 7 is a simplified block diagram of the added blocks where the correction term implementation isn't shown. The maximum position calculation rate is 14.3 MHz (a new position every 70 ns ) with a latency of 510 ns .


Figure 7: Diagram of the FPGA-based calculations used to implement the new algorithm. Each block performs a specific operation using the IEEE-754 single precision floating point representation using Xilinx [5] LogiCORE Floating-Point IP Blocks.

## CONCLUSIONS

The accurate position determinations for beams that are far from the center of the beam-pipe are of particular importance in cases where normal operation requires such orbits. That, for example, is the case for the CBETA project [6] that may serve as a recirculating electron Linac prototype for beam cooling in a future electron-ion collider [7]. The usual cubic approximation is totally inadequate in this case, even when the beam is in a plane defined by two of the PUEs. This can now be understood by performing the Taylor expansion of equation (4) and noting that the convergence is very slow. We showed that the better approach is to use the new analytic expression and to apply, simple, empirically determined corrections for the non-ideal cases.

The present approach offers significant accuracy and speed improvements for cylindrical BPM applications where possible beam offsets are sufficiently large to justify corrections to the linear approximation.

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