

ACCELERATION OF CHARGED PARTICLES BY OWN FIELD IN A NON-STATIONARY ONE-DIMENTIONAL STREAM

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Abstract

The behavior of a non-stationary stream of the charged particles interacting with own field is studied. For the description the integral of the movement received in Meshchersky's works is used. The additional integral of the movement - interfaced to Meshchersky's integral, necessary for completely self-agreed description of a stream of the particles interacting with own field is constructed. The system of the equations reducing a problem to the solution of system of the ordinary differential equations is removed. Private decisions for potential, density of particles and density of current are provided.

INTRODUCTION

In case of a research of self-consistent systems of the charged particles, in physical electronics, in the theory of bundles and in plasma physics movement integrals in some cases play a defining role in the theory of accelerators. It is possible to specify the theory of electronic rings, the theory of rigidly focusing systems, the kinetic theory of quasistationary statuses of bundles (see [1], [2], [3]). The considerable part of the known integrals of movement which are not following from properties of symmetry of system is provided in the operation [4] devoted to the definitely decided nonstationary potentials in a quantum mechanics. Especially fruitful is use of integrals of movement for the description of the particle systems interacting with own fields. In the real operation classical nonstationary ensembles by means of the invariant described in a number of operations are studied (see [5], [6], [7]). For the description of self-consistent systems except this invariant there is necessary use of the conjugate integrals of movement. The possibility of convergence of the nonstationary system described by the same invariant to the system of ordinary differential equations is shown and private numerical solutions of this system are received.

THE INTERFACED INTEGRALS OF THE MOVEMENT

We will consider the simplest one-dimensional stationary case when the system can be described by a Hamiltonian:

$$H = \frac{p^2}{2m} + U(x) \quad (1)$$

Here p - an impulse, m - the weight, $U(x)$ - potential.

We will determine by the integral interfaced to H :

$$J_H^\pm = \pm \int_0^x \frac{dx'}{\sqrt{\frac{2}{m}(H - U(x'))}} - t. \quad (2)$$

We will consider a case of the non-stationary Hamiltonian set in a special way. See [1, 5, 8].

$$H = \frac{p^2}{2m} + \frac{1}{\xi^2(t)} U\left(\frac{x}{\xi(t)}\right) \quad (3)$$

For the system described by a Hamiltonian (3) there is an invariant:

$$I = \frac{m}{2} (\dot{x}\xi - x\dot{\xi})^2 + \frac{m}{2} \lambda \frac{x^2}{\xi^2} + U\left(\frac{x}{\xi}\right), \quad (4)$$

where dimensionless function $\xi(t)$ satisfies to the equation: $\ddot{\xi} = \frac{\lambda}{\xi^3(t)}$, λ - a constant. The solution of the equation for ξ can be presented in the form: $\xi(t) = \sqrt{at^2 + 2bt + c}$, where a, b, c - constants. At the same time $\lambda = ac - b^2$. We will designate $x_* = \frac{x}{\xi(t)}$ and we will enter modern times: $\tau = \int_0^t \frac{dt'}{\xi^2(t')}$. Then

$$I = \frac{m}{2} \left(\frac{dx_*}{d\tau}\right)^2 + \frac{m\lambda}{2} x_*^2 + U(x_*).$$

We will construct the integral interfaced to I . to Similarly stationary case we will receive:

$$J_I^\pm = \pm \int_{x_1}^{x_*} \frac{dx'_*}{\sqrt{\frac{2}{m}(I - U(x'_*)) - \lambda x'^2_*}} - \tau. \quad (5)$$

The lower limit of x_1 can be any.

If the integral of I has dimension of energy, then, it is natural. that the integral of J interfaced to him has dimension of time. For the particles characterized by different sizes of I , remains passing time a particle to x_* , from which sdedut to subtract the current time.

The integral (5) can be used for a research of the self-coordinated non-stationary systems described by a Hamiltonian (4).

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ONE-DIMENSIONAL SELF-CONSISTENT SYSTEM

We will consider a one-dimensional nonstationary particle system with charge q , interacting with own field described by potential $\Phi(x, t)$ satisfying to a Poisson equation. We will enter a potential function of $U(x_*) = \xi^2(t)q\Phi$, here q - an elementary charge. As the density in a one-dimensional configuration is proportional to the reverse length, dimensionality of q is proportional to the electron charge divided into length. Potential depends definitely on coordinate and on time, i.e. depends on a self-similar variable - $x_* = \frac{x}{\xi(t)}$. Poisson equation: $\frac{d^2\Phi}{dx^2} = -4\pi qn(x, t)$, where $n(x, t) = \int d(m\dot{x})f(I, J_I^\pm)$. In this section we will read further that there are only particles moving in the positive direction of an axis of x_* . As

$$d\dot{x} = \frac{1}{m\xi} \frac{dI}{\sqrt{\frac{2}{m}(I-U) - \lambda x_*^2}},$$

Poisson equation can be presented by in the form:

$$\frac{d^2U}{dx_*^2} = -4\pi q^2 \xi^3 \int \frac{dI f(I, J_I^+)}{\sqrt{\frac{2}{m}(I-U) - \lambda x_*^2}} \quad (6)$$

Complete coherence of the task is reached in case the integral in the right part is proportional to by $[\xi(t)^{-3}]$. We will take a distribution function in a look:

$$f = \kappa \delta(I - I_0) \exp\left\{\frac{3}{2\tau_0} J_I^+\right\} \quad (7)$$

Density determined by a distribution function (7) has an appearance:

$$n = \frac{\kappa}{\xi^4 \sqrt{\frac{2}{m}(I_0 - U) + \frac{x_*^2}{4\tau_0^2}}} \times \exp\left\{\frac{3}{2\tau_0} \int_{x_1}^{x_*} \frac{dx'_0}{\sqrt{\frac{2}{m}(I_0 - U(x'_0)) + \frac{x_0'^2}{4\tau_0^2}}}\right\}, \quad (8)$$

where τ_0 - a constant of dimensionality of time. The dependence of integral from τ is defined by multiplier $\exp\{\frac{3}{2\tau_0} J_I^+\}$. If to put $\xi^3 \exp\{-\frac{3\tau}{2\tau_0}\} = const = \xi_0^3$, then both parts of an equation (7) depend only on x_* . It is possible to receive: $\xi = \sqrt{\frac{t}{\tau_0} + \xi_0^2}$. At the same time to the given expression for $\xi(t)$ of corresponds to value $\lambda = -\frac{1}{4\tau_0^2}$. We will mark here that the choice of a distribution function (8) is unique. On the one hand, only the exponential dependence on J allows to consolidate the self-consistent task to self-similar, and the dependence on I in look δ function allows to avoid need of the solution of the integro-differential equation.

At the same time density of current of particles has an

appearance:

$$j = \frac{\dot{\xi}x}{\xi} n + \frac{\kappa}{\xi^5} \exp\left\{\frac{3}{2\tau_0} \int_{x_1}^{x_*} \frac{dx'_0}{\sqrt{\frac{2}{m}(I_0 - U(x'_0)) + \frac{x_0'^2}{4\tau_0^2}}}\right\}$$

In these expressions the size of x_1 has to be defined by physical conditions.

We will enter designations:

$$v_0^2 = \frac{2I_0}{m}, \quad s = \frac{x_*}{2v_0\tau_0}, \quad s_1 = \frac{x_1}{2v_0\tau_0},$$

$$y = \frac{2U}{mv_0^2}, \quad z(s) = \int_{s_1}^s \frac{ds'}{\sqrt{1 - y(s') + s'^2}}.$$

Poisson's equation takes a form:

$$y'' = -\frac{\theta_*}{\sqrt{1 - y(s) + s^2}} \exp\{3z(s)\} = -\frac{\theta_*}{3} \frac{d}{ds} \exp\{3z(s)\} \quad (9)$$

Here $\theta_* = \frac{32\pi q^2 \kappa}{mv}$. If to designate $n_* = \frac{\kappa}{v_0}$, then $\theta_* = \omega_*^2 \tau_0^2$, where $\omega_*^2 = \frac{32\pi n_* q^2}{m} \xi_0^3$. From (8) follows: $y' = -\frac{\theta_*}{3} \exp\{3z(s)\} + C_0$ end the equation can be given by $\exp\{3z(s)\}$ to a look:

$$y'' = (y' - C_0) \frac{3}{\sqrt{1 - y(s) + s^2}}. \quad (10)$$

The equation (9) has the private decision in the form of a square trinomial: $y = \frac{2}{s^2} + bs + c$. Can be received: $a = -16, c = 1 - \frac{b^2}{36}, C_0 = \frac{b}{9}$. I.e. $y = 1 - \frac{b^2}{36} + bs - 8s^2$. Using expression for y' , we will receive: $-16s + \frac{8}{9}b = -\theta_* \frac{s - b/18}{s_1 - b/8}$. From these ratios can be received: $\theta_* = 1, s_1 = \frac{b}{8} + \frac{1}{16}$. These equalities are a condition of existence of the considered private decision. The maximum value of y is reached at $s = \frac{b}{16}, y_{max} = 1 + \frac{b^2}{288}$. At $s > \frac{b}{18}$ of potential decreases with decrease of s under the linear law: $y = \frac{b}{9}(s - \frac{b}{18}) + y_*$, where $y_* = 1 + \frac{b^2}{324}$. Potential can be written down in a look:

$$y = \sigma\left(\frac{b}{18} - s\right) \left(\frac{b}{9}s + 1 - \frac{b^2}{324}\right) + \sigma\left(s - \frac{b}{18}\right) \left(1 - \frac{b^2}{36} + bs - 8s^2\right). \quad (11)$$

where $\sigma(s)$ - the Heaviside function. To such type of potential there corresponds density having an appearance of "step": at $s < \frac{b}{18}$ density is equal to zero whereas at $s > \frac{b}{18}$ density of n_* is defined by equality $\frac{32\pi n_* q^2}{m} \xi_0^3 \tau_0^2 = 1$. Potential has a maximum and changes with growth of s under the parabolic law, and at $s < b/18$ - on linear. Density is constant at positive s and decreases on time, and speed grows with growth of s and also decreases on time. On Fig 1,2 the dependences characterizing the potential, density and density of current corresponding to the provided exact solution, i.e. the solution of the equation for potential at $b = 9, C_0 = 1, y(9/16) = 41/32, y'(9/16) = 0$ are given.

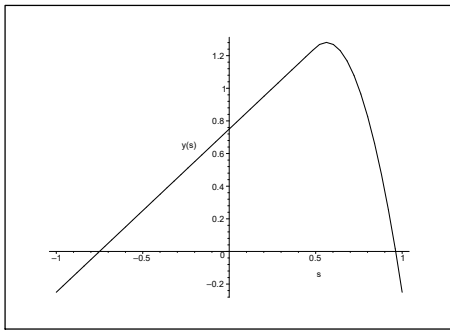


Figure 1: Dependence of potential on coordinate at $C_0 = 1, y(9/16) = 41/32, y'(9/16) = 0$.

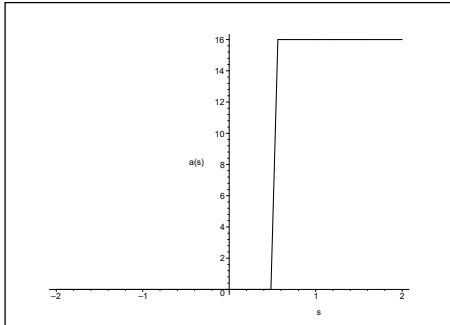


Figure 2: Dependence of density of particles on coordinate at $C_0 = 1, y(9/16) = 41/32, y'(9/16) = 0$.

CONCLUSION

In work, thus, the non-stationary model describing possible acceleration of charged particles by own field is considered. We will note increase in the effective accelerating potential because of composed $\frac{1}{4\tau_0^2}$, determining the additional accelerating force here. We will note here too that "the interfaced integrals" for the first time are removed also to work [8].

REFERENCES

- [1] Chikhachev A. S., Kinetic theory of quasistationary statuses of bundles of the charged particles, M.: Fizmatlit, 2001.
- [2] Kapchinsky I.M., Theory of the linear resonant accelerators. Dynamics of particles, M.: Energoizdat, 1982.
- [3] Sarantsev V. P., Perelstein E.A. Collective acceleration of ions electronic rings, M.: Atomizdat, 1979.
- [4] Efthimiou C.J., Spector D., Phys. Rev. A. 1994. v. 49. N 4. pp. 2101-2112
- [5] V. V, Dodonov, V.I.Man'ko, D.T.Nikonov, Phis.Lett.A, 1992, v.162, 359-364
- [6] MestscherskyJ. Astron.Nachr., 1893, v.132, p.129; Astron.Nachr., 1902, v.159, p.229.
- [7] A.S. Chikhachev, JETP, 2006, t.130, 917-921
- [8] A.S.Chikhachev, Technical Phisics, 2014, vol 59, N 4, pp 487-493.