

## SINGLE RING PERMANENT MAGNET LENS\*

K. Jayamanna, R. Baartman<sup>†</sup>, I. Bylinskii, T. Planche, TRIUMF, Vancouver  
 R. Simpson, UBC, Vancouver, Canada, M. Corwin, U. of Waterloo, Canada

### Abstract

A permanent magnet lens has been designed to be a non-powered alternative to solenoids for low energy beam transport. The lens consists of a single ring of 12 sectors, each sector with poles directed inward. This forms an axial field that reverses sign at the midpoint, somewhat like two opposing short solenoids. A prototype lens optimized to decrease the magnetic material required while also reducing aberration, has been built and tested for a 25 keV  $H^-$  beam. Emittance figures measured downstream of the lens are compared with theory.

### INTRODUCTION

At TRIUMF, we are in process of building a hot spare  $H^-$  ion source terminal for the 500 MeV cyclotron. As the ion source is in a Faraday cage biased at -300 kV with respect to ground, it is advantageous to minimize the services required by the source and its attendant optics. The optics consists of one focusing device and deflector plates used to pulse the beam at 1 ms. As the beam is naturally space charge neutralized by positive ions resulting from interactions with the background gas, a decision was made early on that the optics must be magnetic. Conventionally, and since it is highly desirable to maintain the axial symmetry of the ion source beam, the focusing device is a solenoid. Typically, the power required for the solenoid would be  $\sim 3$  kW and this impacts the capacity of the cooling system. In this way, we were led to consider optics based upon permanent magnets.

Initially, we considered the ‘PMS’ lens of Iwashita [1]. This consists of two rings each of permanent magnet sectors: one with N pole on the outside surface, S pole on the inside, and the other vice versa. But detailed calculations revealed that a single ring requires less magnetic material and moreover has slightly smaller aberrations than a double ring.

For a single ring, the on-axis magnetic field changes sign at the midpoint (for the Iwashita lens, there are two changes of sign), but for a solenoid, the field varies more smoothly without changing sign. See Fig. 1. By Ampère’s fundamental law, the integral on axis is zero for the permanent magnet case and therefore the beam suffers no net coupling between the transverse coordinates, and no rotation. On the other hand, because the third order aberration is proportional to  $\int B^2 dz$ , it was anticipated that aberrations would be larger for the permanent magnet lens than for a solenoid of similar volume. This is indeed the case.

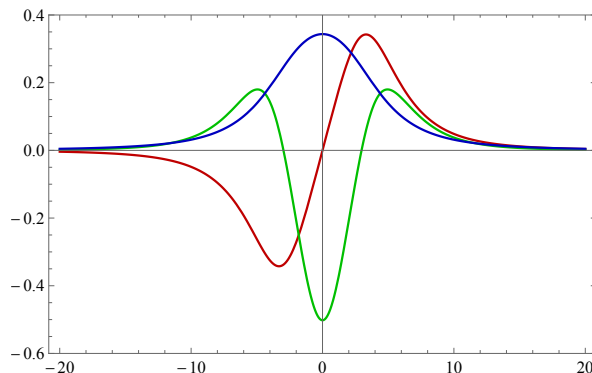


Figure 1: Field in Tesla along axis in cm for 3 cases, all with identical focal strength  $f=25$  cm for 25 keV protons. **Blue** – conventional solenoid with coil inner radius 3 cm, outer radius 6 cm, length 6 cm, **red** – single ring  $B_0=1.2$  T permanent magnet of same dimensions as the coil for the solenoid, **green** – Iwashita lens; this lens also has  $B_0=1.2$  T magnets and same inner and outer radii as the single ring, but is slightly longer at 3.85 cm per ring.

### THEORY

A great virtue of these kinds of permanent magnet lenses is that their on-axis fields are easily given to high accuracy by simple formulas. (See Fig. 3.) Let us define a function  $F$ :

$$F(s) = 1/\sqrt{s^2 + 1} - \log\left(\sqrt{s^2 + 1} + 1\right). \quad (1)$$

Then the on-axis field  $B(z)$  is given by:

$$B(z) = \frac{B_0}{2} \left[ F\left(\frac{z+l/2}{R_1}\right) - F\left(\frac{z+l/2}{R_2}\right) - F\left(\frac{z-l/2}{R_1}\right) + F\left(\frac{z-l/2}{R_2}\right) \right] \quad (2)$$

for the case of magnet remanence strength  $B_0$ ,  $R_1$  inner radius and  $R_2$  outer radius and length of  $l$ . It is straightforward to code this into COSY- $\infty$  [2] and obtain transfer map optics to all orders.

For this project, we needed a  $90^\circ$  phase space rotation and this is achieved with a drift-lens-drift combination where the drift lengths are equal to the lens’s focal length  $f$ , set to 0.25 m. Combinations are tested by setting  $R_1$  and  $R_2$ , for the known  $B_0$ , and fitting the lens length  $l$  to obtain the first order transfer matrix with zeros on the diagonal. The results are shown in Table 1.

The fifth column labeled  $a_3$  is the aberration coefficient ( $x'|x'^3$ ). For example, an initial  $x' = 30$  mrad reaches  $x = 7.5$  mm in the lens, and with  $a_3 = 34$  suffers an aberrant  $\Delta x' = 34 \times 0.03^3 = 0.92$  mrad. The emittance growth is

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<sup>†</sup> baartman@triumf.ca

Table 1: Length  $l$  and volume  $V$  of permanent magnetic material versus inner and outer radii  $R_1$ ,  $R_2$ . The rightmost columns are the COSY-calculated aberration coefficient and the emittance growth.

$R_1$ (cm)	$R_2$ (cm)	$l$ (cm)	$V$ (cm <sup>3</sup> )	$a_3$	$\Delta\epsilon/\epsilon$
<b>Single ring</b>					
2	3.8	11.0	362	35.0	1.46%
2	4	6.35	238	49.8	3.00%
2	5	3.58	236	77.3	7.03%
2	6	2.91	293	80.5	7.70%
2	7	2.58	365	78.5	7.70%
3	5.25	11.6	679	19.7	0.47%
3	5.5	8.10	540	26.3	0.80%
3	5.75	6.77	511	30.9	1.20%
3	6	5.98	507	34.2	1.50%
3	6.25	5.44	514	36.5	1.70%
3	6.5	5.05	527	38.0	1.80%
3	6.75	4.74	544	39.0	2.00%
3	7	4.49	564	39.7	1.90%
4	7	9.42	976	17.1	0.38%
4	8	6.39	963	22.8	0.66%
4	9	5.26	1075	24.3	0.74%
<b>Iwashita (double) ring</b>					
3	4.5	13.1	463	44.7	2.20%
3	5	9.7	490	64.9	5.05%
3	6	7.7	653	80.2	7.90%
<b>Solenoid</b>					
3	6	6	0	14.1	0.30%

calculated by a Monte Carlo method of passing a distribution of particles through the COSY map. The distribution used is thermal as expected directly from the ion source: 1 mm radius uniform in  $x$  and Gaussian with rms of 15 mrad in  $x'$ . For comparison, we have also included the case of a solenoid, CMST in COSY [2]. This solenoid has the same dimensions for the coil as that of the magnets for the finally chosen single ring case  $R_1 = 3$  cm,  $R_2 = l = 6$  cm. It's admittedly very small and its power would be 2.2 kW for copper.

The table shows that the aberrations are larger for the double ring than for the single, for the same volume of magnetic material and inner radius. We have therefore selected the single ring as opposed to the Iwashita design.

Further, that there is an optimum on efficiency of using magnetic material, at a ratio of  $R_2/R_1 \sim 2$ . To minimize aberrations, though, thinner and longer designs are preferred; in the limit, the length diverges, making for a design that would be difficult to construct. The reason is that unlike a solenoid which is stronger as it is longer, the long ( $l \gg R_2$ ) single ring permanent magnet lens (SPML) would have axial fields only at entrance and exit.

For the optimum ratio, we find empirically that for our chosen fixed focal length of 25 cm,  $a_3$  is roughly  $(32 \text{ cm}/R_1)^{3/2}$ .

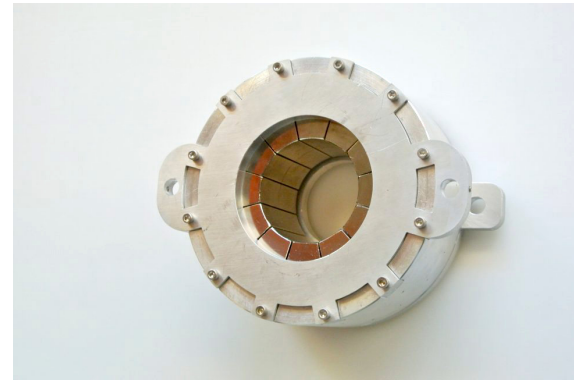


Figure 2: 12-sectored SPML. The outer diameter of the assembly is 14.4 cm, the inside diameter is 5.6 cm, magnet sector length is 5.1 cm

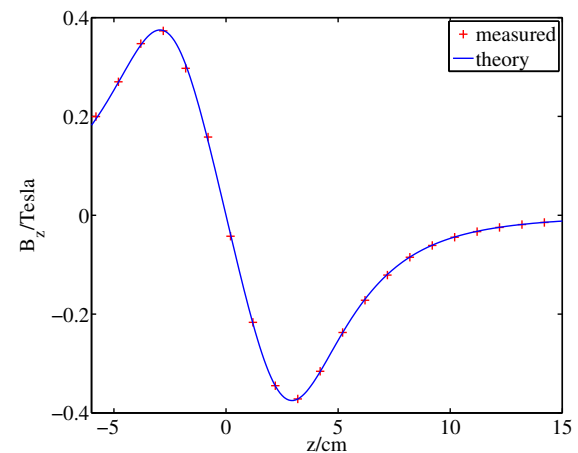


Figure 3: Comparison of theoretical on-axis field according to Eq. 2 (blue curve) with measured field (red + symbols).

## LENS DESIGN

Intending to fabricate the lens from uniformly magnetized NdFeB, the ring is necessarily subdivided into segments. At first using 6 rectangular sectors, we find the sixth harmonic of the magnetic field too large, and the field strength also was insufficient, finally we converged to 12 space-filling circular sectors. It was further discovered that though any gap between sectors reduces the lens strength, such gaps on the other hand reduce the 12<sup>th</sup> harmonic dramatically, becoming zero for 1.5 mm gaps.

Based on keeping emittance growth of a  $4\epsilon_{\text{rms}} = 30 \mu\text{m}$  beam below 2% and for compactness and ease of construction, we chose  $R_1 = 2.79$  cm,  $R_2 = 6.35$  cm, and  $l = 5.06$  cm.

The lens was constructed of 42UH NdFeB which has an advertised remanence of 1.28 T to 1.33 T. Owing to the gaps and that the fields are not exactly radial, it was expected that the lens axial field would deviate from Eq. 2, in particular the field was expected to be smaller.  $B_0$  was therefore fitted and

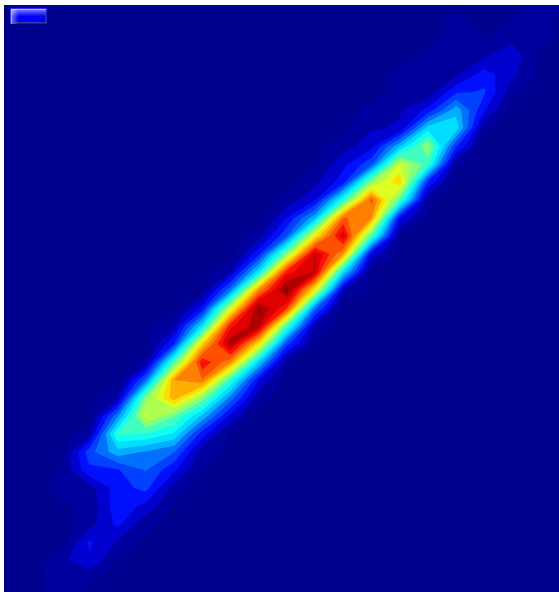


Figure 4: Phase space measured upstream of the SPML. Scale: the width ( $x$ ) of the area is 14 mm, and the height ( $x'$ ) is 30 mrad.

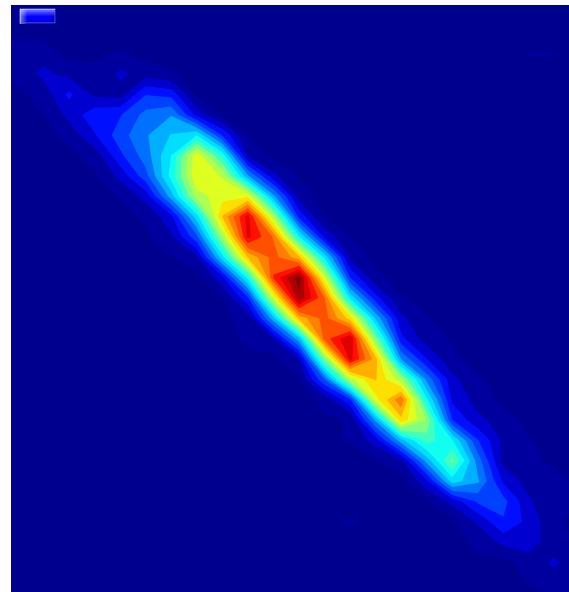


Figure 5: Phase space measured upstream of the SPML but transformed to the location of the scanner downstream of the SPML. Scale: the width ( $x$ ) of the area is 7 mm, and the height ( $x'$ ) is 60 mrad.

was found to be 1.225 T. But then the fit was impressively good, see Fig. 3, and fully justifies the use of Eq. 2 in the COSY model.

## EXPERIMENTAL RESULTS

Our  $H^-$  source test stand was set up with the SPML centre 77 cm downstream of the source exit. Two identical Allison emittance scanners were positioned with their entrances at respectively 21 cm upstream and 22 cm downstream of the centre of the SPML. With only magnetic fields present and a vacuum of  $1 \times 10^{-6}$  Torr, there is near 100% space charge compensation [3].

The technique was therefore to transform the emittance measured upstream of the SPML, using the linear transfer matrix without space charge from the upstream scanner, through the SPML, and to the downstream scanner, and to compare it to the measured downstream emittance.

The results are shown in Figs. 5 and 6. The figures' size and tilt match within experimental error, thus validating the linear optics model. The upstream and downstream measured 4RMS emittances are respectively:  $(17.6 \pm 1.0) \mu\text{m}$  and  $(18.1 \pm 0.5) \mu\text{m}$ . These agree within measurement uncertainty, consistent with the COSY-predicted growth of  $< 1\%$ .

## CONCLUSION

The single ring consisting of 12 permanent, uniformly magnetized sectors directed radially, is an effective replacement for solenoids as low energy ion focusing elements. It has the advantage of requiring no electrical power and no cooling, and further does not rotate the ion beam. On the other hand, it is not variable in strength, and aberrations are 2 to 3 times larger than for a similarly-sized solenoid.

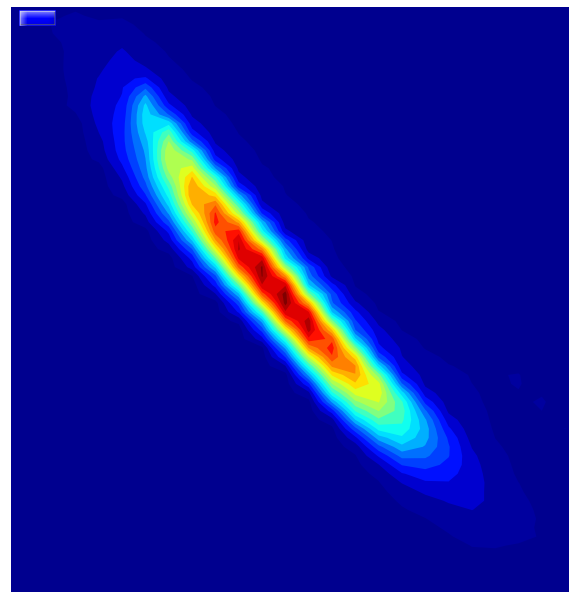


Figure 6: Phase space measured by the scanner downstream of the SPML. Scale: the width ( $x$ ) of the area is 7 mm, and the height ( $x'$ ) is 60 mrad, identical to Fig. 5, making the two figures directly comparable.

## REFERENCES

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