

# CONTROL OF COLLECTIVE EFFECTS BY ACTIVE HARMONIC CAVITY IN AN MBA-BASED LIGHT SOURCE WITH APPLICATION TO THE PETRA UPGRADE

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## Abstract

Based on the reference lattice for PETRA IV [1] we investigated collective effects with non-zero current. Out of many possibilities we firstly computed the intrabeam-scattering (IBS) effects on the emittance as well as lifetime as a function of current. The result indicated that PETRA IV would benefit from the reduced peak current when the harmonic cavity lengthens the bunch. The operating point of harmonic cavity was explored by tracking simulations as well as analytic formula. In order to compute the energy spread and bunch length we had used the known impedance function of the APS [2]. In this way more realistic estimation of IBS effects was expected. However, because of the complex nature of PETRA IV lattice, which includes achromatic cells for undulators, arc cells of octants and straight sectors for damping wigglers, we simplify the longitudinal dynamics by assuming the ring made of 92 multi-bend-achromat (MBA) cells. The optics is approximated as a linear-chromatic transfer map enabling fast tracking and the ring impedance is concatenated into the one location. The detailed collective effects with and without harmonic cavities are presented in the paper.

## INTRODUCTION

The lattice of insertion device (ID) cell considered in this paper is a hybrid seven-bend achromat (H7BA), patterned after the European Synchrotron Radiation Facility (ESRF) design [3]. In PETRA IV there will be 22 ID cells and the rest of the ring will consists of arcs and straight-sections. Since the details of lattice configuration is still evolving, the investigative study of harmonic cavity presented in this paper assumes a ring made of 92 H7BA cells which can fit into the PETRA III tunnel. Even though this is a surrogate model of the full lattice described in Ref. [1], we believe that the information presented in this paper will be useful in assessing the collective effect including intrabeam scattering (IBS) and bunch lengthening with a harmonic cavity.

## INTRODUCTION TO HARMONIC CAVITIES

The equations of motion of a single electron are

$$\frac{d\tau}{dt} = \alpha\varepsilon, \quad (1)$$

$$\frac{d\varepsilon}{dt} = \frac{e}{ET_0} [V(\tau) - V_s], \quad (2)$$

where  $V_s$  is the synchronous voltage,  $\tau$  is the arrival time of the electron with respect to the synchronous particle,  $\varepsilon$

is the fractional energy deviation,  $E$  is the energy, and  $\alpha$  is the momentum compaction factor. The combined voltage from the main and harmonic rf system is given by

$$V(\tau) = V_1 \sin(\omega_1 t + \phi_1) + V_h \sin(h\omega_1 t + \phi_h), \quad (3)$$

where  $h$  is a harmonic number,  $V_1$  and  $V_h$  are the fundamental and harmonic cavity voltage, respectively, and similarly for the rf phase  $\phi_1$  and  $\phi_h$ . The synchronous voltage of double cavity system is then

$$V_s = V_1 \sin \phi_1 + V_h \sin \phi_h. \quad (4)$$

From the equation of motion for  $\tau$  we can define the potential as

$$\Phi(\tau) = -\frac{\alpha e}{ET_0} \frac{V_1}{\omega_1} \left\{ \frac{\cos \phi_1 - \cos(\omega_1 \tau + \phi_1) + r}{h} [\cos \phi_h - \cos(h\omega_1 \tau + \phi_h)] \right\}, \quad (5)$$

where  $r$  is the voltage ratio  $V_h/V_1$ . If we expand the potential in time

$$\Phi(\tau) = a\tau^2 + b\tau^3 + c\tau^4 + H.O. \quad (6)$$

the coefficients  $a$ ,  $b$ , and  $c$  are given by

$$a = \frac{\alpha e \omega_1}{2ET_0} (V_1 \cos \phi_1 + hV_h \cos \phi_h), \quad (7)$$

$$b = -\frac{\alpha e \omega_1^2}{6ET_0} (V_1 \sin \phi_1 + h^2 V_h \sin \phi_h), \quad (8)$$

$$c = -\frac{\alpha e \omega_1^3}{24ET_0} (V_1 \cos \phi_1 + h^3 V_h \cos \phi_h). \quad (9)$$

For the maximum bunch lengthening we set  $a=b=0$ . Then the rms bunch length for the quartic potential  $\Phi(\tau)=c\tau^4$  can be estimated as

$$\sigma_\tau = 0.69 (\alpha^2 \sigma_\varepsilon^2 / c)^{1/4}. \quad (10)$$

Together with the synchronous condition of Eq. (4), we found the harmonic voltage and phase as

$$r = \sqrt{\frac{1}{h^2} - \frac{(V_s/V_1)^2}{h^2 - 1}}, \quad (11)$$

and

$$\tan \phi_h = \frac{hV_s/V_1}{\sqrt{(h^2 - 1)^2 - (hV_s/V_1)^2}}. \quad (12)$$

The synchronous phase of the fundamental rf system needs to be changed with the new phase

$$\sin \phi_1 = \frac{h^2}{h^2 - 1} \frac{V_s}{V_1}. \quad (13)$$

The 3<sup>rd</sup> harmonic cavity satisfying Eq. (11)-(13) will lengthen the bunch according to Ref. [4, 5].

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## APPLICATION TO A H7BA LATTICE

The lattice of ID cell is a hybrid seven-bend achromat (H7BA). For this study we considered the ring which consists of 92 H7BA cells with the total length close to the circumference of PETRA III. The lattice parameters are shown in Table 1.

Table 1: H7BA Lattice Parameters

Cell			
Length	$L_{\text{cell}}$	25.2	m
Emittance	$\epsilon_x$	17.6	pm
Energy Spread	$\sigma_\epsilon$	$6.82 \times 10^{-4}$	m
Momentum Compaction	$\alpha_0$	$2.085 \times 10^{-5}$	
Radiation Loss	$\Delta U_0$	0.0214	MeV
Ring			
Length	$L_{\text{ring}}$	2336.8	m
Radiation Loss	$U_0$	1.97	MeV
Rf Voltage	$V_1$	4.0	MV
Rf Frequency	$\omega_{\text{rf}}/2\pi$	499.912	MHz

According to Eq. (11, 12) we set the harmonic cavity at  $V_h=1.14$  MV and  $\phi_h=-12.5$  degree with the harmonic number  $h=3$ . The predicted bunch length will be 10.45 mm by Eq. (10), which shows the addition of a harmonic cavity is effective in lengthening the bunch because it is five times longer than the one with fundamental cavity only.

For specification of cavity requirements we need to scan harmonic voltage. For this purpose we used the program elegant where the second order moment of beam distribution was computed with synchrotron radiation effect included [6]. In the simulation the harmonic voltage was varied while the harmonic phase is fixed at  $\phi_h=-12.5$  deg for which the synchronous condition is not satisfied except at  $V_h=1.14$  MV. The result is shown in Fig. 1, where the symbol indicates the result predicted by Eq. (10).

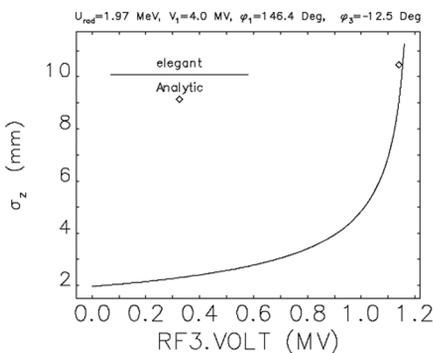


Figure 1: The bunch lengthening simulated by *elegant* momentum tracking as a function of harmonic cavity voltage.

Since the momentum tracking by *elegant* is fast and reliable, we could investigate various parameter ranges and found that the harmonic phase can be fixed at zero without degrading the optimum performance. This also alleviates the change of synchronous phase with the harmonic voltage variation. Because of these reasons we set the harmonic phase zero in the following sections.

## INTRABEAM SCATTERING

A severe limitation for low emittance lattices is the effect of intrabeam scattering (IBS), which is a single bunch collective effect limiting the density of the particle beam. The emittance  $\epsilon_{x,y}$  and energy spread  $\sigma_\epsilon$  of the beam will be an equilibrium between radiation damping, quantum excitation, and IBS [7]:

$$\epsilon_{x,y} = \epsilon_{x0,y0} \left( 1 - \frac{\tau_{x,y}}{T_{x,y}} \right)^{-1}, \quad \sigma_p^2 = \sigma_{p0}^2 \left( 1 - \frac{\tau_p}{T_p} \right)^{-1}, \quad (14)$$

where  $\epsilon_{x0,y0}$  is the zero current emittance,  $\sigma_{p0}$  the zero current energy spread,  $\tau_{x,y,p}$  are the damping times and  $T_{x,y,p}$  are the IBS growth times. Assuming a 10% coupling ratio, we estimated IBS effect by using the program *ibsemittance* developed at Advanced Photon Source. The equilibrium horizontal emittance due to IBS for the reference lattice is shown in Fig. 2. In the figure the legend V3 means the 3<sup>rd</sup> harmonic cavity voltage. Without a harmonic cavity the emittance increased significantly even at very low single bunch current.

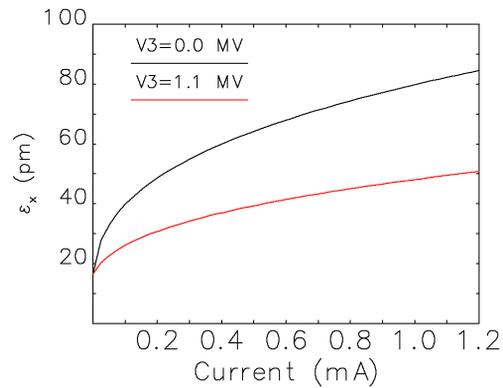


Figure 2: The emittance increase due to IBS effect with and without the 3<sup>rd</sup> harmonic cavity.

We could use damping wigglers to mitigate IBS effects which provide the additional damping; however, in general, it also increases the energy spread. If we want to preserve the small energy spread of Table 1, we can use the harmonic cavity which does not alter the energy spread. By setting the harmonic voltage  $V_3$  to 1.1 MV, we found the IBS growth time increased significantly and its emittance reduced by a factor of two in high current regime. The emittance with and without harmonic cavity is shown in Fig. 2 for comparison.

## POTENTIAL WELL DISTORTION AND MICROWAVE INSTABILITY

In addition to the harmonic voltage there is another source of bunch lengthening, which is longitudinal impedance of the ring. The imaginary part of impedance modifies the rf potential and it lengthens the bunch without causing instability. The real part of impedance is responsible for the microwave instability for the current above the threshold. Since the measured impedance of PETRA III is similar to the APS, namely  $Z/n=0.15 \Omega$ , we simply assume that the impedance of PETRA IV will be

twice the APS impedance in Ref. [2]. With this impedance effect included we simulated the effect of harmonic cavity on longitudinal dynamics. The phase spaces of 1-mA bunch with 200k macro-particles are shown in Fig. 3, where each snapshot corresponds to  $V_3=[0, 0.1, 0.2, \dots, 1.8, 1.9, 2.0]$  MV respectively from top-left to bottom-right in sequence. It shows the instability in motion all range of harmonic voltage and the bunch is bifurcated into two bunch-lets starting from  $V_3=1.5$  MV.

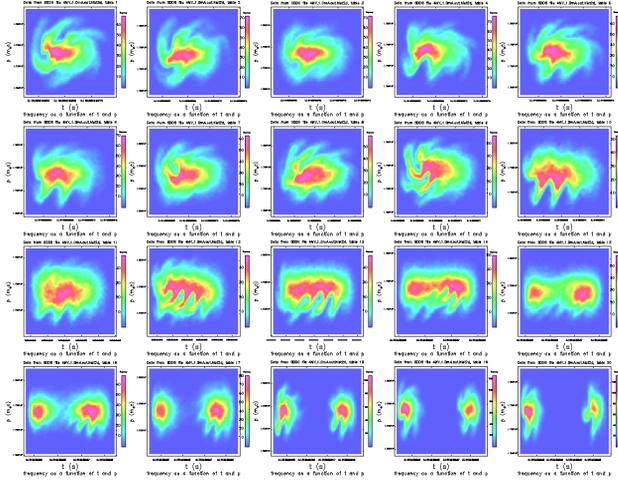


Figure 3: The phase of time and momentum of a 1-mA bunch in a combined rf-bucket of the fundamental and 3rd harmonic cavity with the harmonic voltage from 0.0 MV to 2.0 MV.

The bunch lengthening by impedance effect is depicted in Fig. 4 with and without harmonic voltage. We note that the bunch length at 1 mA is now about 20 mm with  $V_3=1.1$  MV, which is again doubled from the value predicted by Eq. (10). So total lengthening by harmonic cavity and impedance effect is ten times longer than the natural bunch length. Overall impact on the emittance is shown in Fig. 5.

### SINGLE BUNCH CURRENT LIMIT

We attempt to set a preliminary impedance budget for PETRA IV upgrade based on the assumption that the single bunch current will be limited by transverse mode coupling instability (TMCI). The formula for the threshold current is [7]

$$I_{th} = \frac{4\sqrt{\pi}(E/e)\omega_s v_\beta \sigma_\tau}{RZ_{t,eff}}, \quad (15)$$

where  $R$  is the radius of the ring,  $Z_{t,eff}$  is the effective transverse impedance,  $E$  is the beam energy,  $\omega_s$  is the synchrotron frequency,  $v_\beta$  is the betatron tune, and  $\sigma_\tau$  is the rms bunch length in time. According to Ref. [1] the requirement for a timing mode is 1 mA per bunch. In order to satisfy the threshold current less than 1 mA, the impedance budget should satisfy :

$$Z_{t,eff}(\sigma_z, \omega_\xi) = 0.055 \times \sigma_z, \quad (16)$$

where  $Z_{t,eff}$  is in  $\text{M}\Omega/\text{m}$  and  $\sigma_z$  is the rms bunch length in mm. Since the harmonic cavity can increase the bunch

length to 20 mm at 1 mA as shown in Fig. 4, we may set the impedance budget as  $1.2 \text{ M}\Omega/\text{m}$  for PETRA IV.

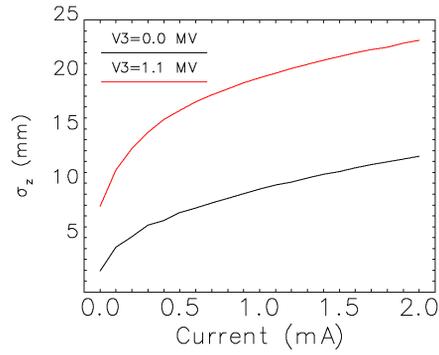


Figure 4: The bunch lengthening with the impedance taken into account.

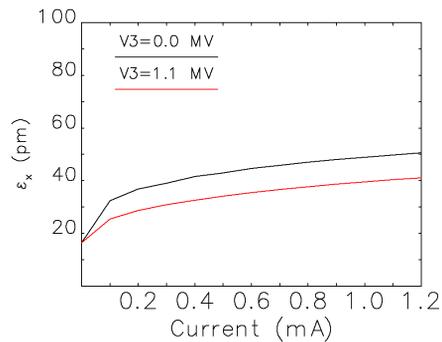


Figure 5: The emittance increase due to IBS effect including the total bunch lengthening with the harmonic cavity and impedance effect.

### CONCLUSION

We found a 3<sup>rd</sup> harmonic cavity above 1.0 MV gap voltage can mitigate the IBS effect effectively and has a beneficial effect in raising the single bunch current limit.

### REFERENCES

- [1] C. Schroer *et al.*, “The Future Ultra-low Emittance Source at DESY”, *J. Synchrotron Rad.*, submitted for the publication.
- [2] Y.-C. Chae and Y. Wang, “Impedance Database II for the Advanced Photon Source Storage Ring”, in *Proc. PAC’07*, Albuquerque, USA, pp. 4336-4338.
- [3] L. Farvacque *et al.*, “A Low-emittance Lattice for the ESRF”, in *Proc. of PAC’13*, Pasadena, USA, p. 79.
- [4] R.A. Bosch *et al.*, “Robinson Instabilities with a Higher-harmonic Cavity”, *Phys. Rev. ST. Accel. Beams*, vol. 4, p. 074401, 2001.
- [5] J.M. Byrd and M. Georgsson, “Lifetime Increase using Passive Harmonic Cavities in Synchrotron Light Sources”, *Phys. Rev. ST. Accel. Beams*, vol. 4, p. 030701, 2001.
- [6] M. Borland, “Implementation and Performance of Beam Moments Computations in Elegant”, APS, Argonne, USA, Rep. OAG-TN-2008-002, Feb., 2008.
- [7] A. W. Chao and M. Tigner, “Handbook of Accelerator Physics and Engineering”, 2<sup>nd</sup> Printing, world scientific, 1999, p. 212.