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HIGH ORDER IMAGE TERMS AND HARMONIC CLOSED ORBITS AT THE ISIS SYNCHROTRON

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Abstract

ISIS is the spallation neutron source at Rutherford Appleton Laboratory in the UK. Protons are accelerated from 70 to 800 MeV in a 50 Hz rapid cycling synchrotron. Due to the intense beam, space charge forces are high during the first part of the acceleration cycle.

The vacuum vessel in the synchrotron has a rectangular shape where the apertures are conformal to the design beam envelopes. At high intensities image forces interact with the beam, especially when the closed orbit is large.

An analysis of image forces has been made and used to classify higher order image terms. These have been identified using simulations of round beams in rectangular vacuum vessels. The higher order image terms from harmonic closed orbits have been used with single particle resonance theory, taking account of the coherent nature of the beam response. Several predictions of beam resonance have been made.

A simulation study has been carried out using a smooth focusing lattice and uniform density beams. Resonant beam behaviour has been observed and explained by the proposed theory.

PENCIL BEAMS BETWEEN PARALLEL PLATES

Laslett's image co-efficients [1] are expressed as $\epsilon_1 = \frac{\pi^2}{48}$ for images due to the offset of the field point from the centre of the beam ($\hat{y}$), and $\xi_1 = \frac{\pi^2}{16}$ for images due to the offset of the beam from the midpoint of the parallel plates ($\bar{y}$), giving for the first order approximation electric field due to images (E_y)

$$E_y \simeq -\frac{\lambda}{\pi\varepsilon_0 h^2}\left(\epsilon_1\hat{y} + \xi_1\bar{y}\right), \tag{1}$$

where λ is the line charge density, ε_0 is the permittivity of free space and h is half the plate separation.

For an off-centred beam the higher order image terms can become significant. Baartman [2] describes general higher order image coefficients κ

$$\frac{E_{yimage}}{4\lambda} = \frac{1}{4\pi\varepsilon_0}\left(\epsilon_1\frac{\hat{y}}{h^2} + \xi_1\frac{\bar{y}}{h^2} + \kappa_{30}\frac{\bar{y}^3}{h^4} + \kappa_{21}\frac{\hat{y}\bar{y}^2}{h^4} + \kappa_{12}\frac{\hat{y}^2\bar{y}}{h^4} + \kappa_{03}\frac{\hat{y}^3}{h^4} + \ldots\right) \tag{2}$$

The terms in this equation will be used as a basis for understanding the strength and effect of image driving terms using numerical methods in the rest of this paper.

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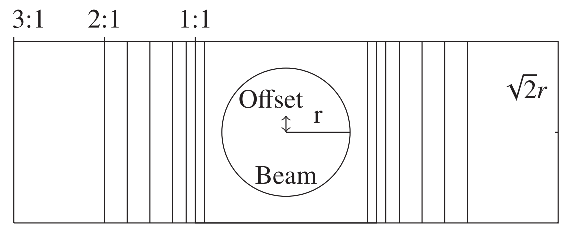

Figure 1: Schematic of simulation showing range of offset and aspect ratio.

SIMULATION OF ROUND BEAMS IN RECTANGULAR APERTURES

Simulation Setup

A set of 2D electrostatic simulations was performed in which the vacuum chamber aspect ratio was swept from 0.9 to 3 by varying the horizontal aperture while keeping the vertical constant at 100 mm. The beam was offset between 0 and 10 mm in steps of 1 mm in the vertical direction for each of these cases ($\bar{y}$ = 0 - 10 mm). The beam was a uniform round distribution. The beam radius was 0.7 of the vertical half beam pipe size. These parameters are illustrated in Figure 1. The electric potential and fields were generated for each case, specifically calculated inside the beam ($E_y(x, y, 0, \bar{y})$).

Mathematica [3] was used to fit the image field in several steps. A third order polynomial function with respect to y (the field point) was fitted to the electric field within the beam obtained from the simulations.

$$E_{iy}(x, y) = A + By + Cy^2 + Dy^3 \tag{3}$$

In Equation 3, E_{iy} is the vertical image field and A, B, C and D are variables to be determined from the simulation data.

Further fitting was carried out for A, B, C and D to obtain functions with respect to $\bar{y}$ (the beam offset) in order to obtain the values of the κ terms from Equation 2, i.e. A was fitted to an equation for $E\bar{y} + F\bar{y}^3$, where E and F are further constants to be determined from the simulations.

Fitting in this manner averaged out variation in the horizontal plane, but allowed comparison with 1D theory. Different regions of the beam were explored to see whether there was better agreement with the 1D case.

In the following results, image coefficients will be shown plotted for the whole beam, and for the region −20 mm ≤ x ≤ −20 mm, −40 mm ≤ y ≤ −40 mm, labelled "subset" in Figure 2, as well as compared to the values calculated for pencil beams in parallel plate geometry by Baartman.

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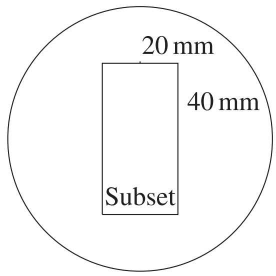

Figure 2: Schematic illustrating subset within beam.

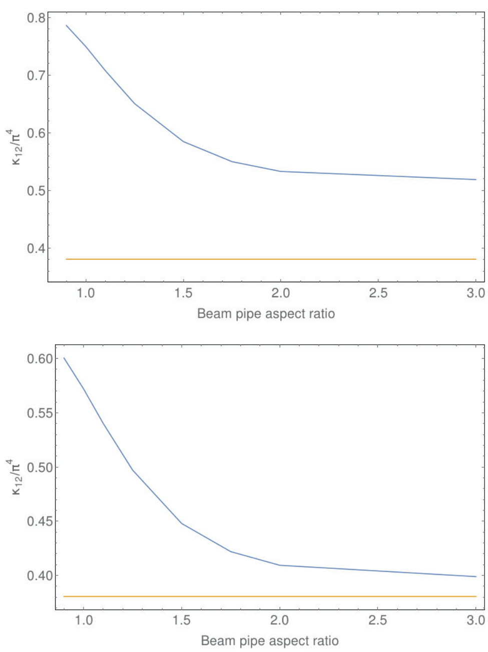

Figure 3: κ_{12} image term in blue for (top) the whole beam, (bottom) a section of beam $-20\,\mathrm{mm} \leq x \leq -20\,\mathrm{mm}$, $-40\,\mathrm{mm} \leq y \leq -40\,\mathrm{mm}$, parallel plate value in yellow.

Image Term κ_{12} (Sextupole Term, $y^2\bar{y}$ Dependence)

Image term κ_{12} is plotted in Figure 3 for both the whole beam (top) and the beam subset (bottom). The straight line is the value of Baartman's κ_{12} image term for a pencil beam in parallel plate geometry. The value of the coefficient approaches Baartman's parallel plate value for the beam subset, while the whole beam value tends to a larger number, due to horizontal variation in the term. The value of the coefficient at an aspect ratio of one is considerably larger, so will provide a larger driving term.

SINGLE PARTICLE HAMILTONIAN INCLUDING IMAGE DRIVING TERMS

If the existence of a high intensity closed orbit can be established, the effects of images induced by this closed orbit can then be assessed. The equation for single particle transverse betatron motion, with respect to the closed orbit, can be expressed as

$$y'' + ky = F_D + F_I \tag{4}$$

where the prime indicates differentiation with respect to s, k is the smooth focusing strength, F_D contains the direct space charge terms and F_I the indirect (i.e. image) terms. For a KV beam, F_D is linear inside the beam. Both F_D and F_I are included in the simulations that follow, however it is the effect of F_I that is of interest here. Closed orbits will create image forces that act back on the beam. These will modify the closed orbits themselves, however this will be treated as a small perturbation. Therefore the single particle motion about the closed orbit may be treated taking the closed orbit motion itself as constant. Direct space charge forces will modify the particle motion but not the closed orbit.

The one dimensional single particle Hamiltonian, associated with Equation 4 can be written [4]

$$H(y, P_y, s) = \frac{1}{2}P_y^2 + \frac{1}{2}ky^2 + V_D + V_I \tag{5}$$

where H is the Hamiltonian, P_y is the canonical momentum, V_D and V_I are potentials representing F_D and F_I and the other variables are as above.

Writing the indirect space charge forces as Baartman's image terms in potential form:

$$V_I = \frac{1}{\gamma m_0 \beta^2 c^2}\frac{\lambda}{\pi\varepsilon_0}\left[\epsilon_1\frac{y^2}{2h^2} + \xi_1\frac{y\bar{y}}{h^2} + \kappa_{30}\frac{y\bar{y}^3}{h^4} + \kappa_{21}\frac{y^2\bar{y}^2}{2h^4} + \kappa_{12}\frac{y^3\bar{y}}{3h^4} + \kappa_{03}\frac{y^4}{4h^4} + \dots\right] \tag{6}$$

where γ and β are the usual relativistic parameters, m_0 is the particle rest mass and c is the speed of light in vacuum.

Transform the Hamiltonian to action-angle coordinates

$$y = \sqrt{\frac{2J}{\omega}}\sin\phi \tag{7}$$

where J and ϕ are the action and angle variables respectively, and substitute a harmonic closed orbit solution as

$$\bar{y} = a_n \cos n\theta. \tag{8}$$

To obtain

$$H(J,\phi) = \omega J + V_I(J,\theta) + V_D(J). \tag{9}$$

Hamiltonian Including κ_{12}

The κ_{12} image term can be inserted into the Hamiltonian as $T_{12}y^3\bar{y}$ where T_{12} is a constant absorbing κ_{12} and other constant terms. Changing y to action-angle variables and substituting for $\bar{y}$:

$$H = \omega J + T_{12}\left(\frac{2J}{\omega}\right)^{\frac{3}{2}}\sin^3\phi\; a_n\cos n\theta + V_0(J) \tag{10}$$

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$V_0(J)$ is a non-linear term in J due to direct space charge and other image terms.

$$H = \omega J + a_n T_{12} \left(\frac{2J}{\omega}\right)^{\frac{3}{2}} \left(\frac{3}{4} \sin\phi - \frac{1}{4} \sin 3\phi\right) \cos n\theta + V_0(J) \tag{11}$$

$$H = \omega J + a_n T_{12} \left(\frac{2J}{\omega}\right)^{\frac{3}{2}} \left(\frac{3}{4} \sin\phi \cos n\theta - \frac{1}{4} \sin 3\phi \cos n\theta\right) + V_0(J) \tag{12}$$

$$H = \omega J + a_n T_{12} \left(\frac{2J}{\omega}\right)^{\frac{3}{2}} \left(\frac{3}{8}\left(\sin(\phi - n\theta) + \sin(\phi + n\theta)\right) - \frac{1}{8}\left(\sin(3\phi - n\theta) + \sin(3\phi + n\theta)\right)\right) + V_0(J) \tag{13}$$

This suggests that the κ_{12} term has potential resonances at $Q = n$ and $3Q = n$.

SIMULATION RESULTS FOR κ_{12} IMAGE TERM

Resonances were investigated using self-consistent particle-in-cell simulations of a coasting beam through a smooth focusing lattice. Simulations were run for 100 turns, which was sufficient for high intensity resonant behaviour to manifest. A beam of radius 50 mm was generated randomly for the start of each simulation. The vacuum vessel was a constant square aperture with a half aperture of 100 mm. Uniform distributions of 5×10^4 macroparticles were RMS matched to the envelope and closed orbit on the first turn.

A closed orbit was created with a series of kicks, to create a 13th harmonic closed orbit. An example is shown in Figure 4, for an intensity of 7.6×10^{13} ppp. The orbits are over-plotted every 10 turns for 100 turns.

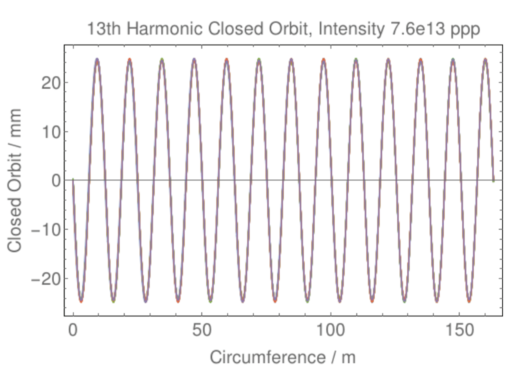

Figure 4: Closed orbit plotted at intensities of 7.6×10^{13} ppp, over-plotted in each case every 10 turns. Simulations were with a distributed angular kick to produce a 13th harmonic closed orbit.

Beam behaviour due to the κ_{12} image term was investigated. This term has the form $y^2\bar{y}$. It is a sextupole term. The Hamiltonian in action-angle coordinates suggests that there is a resonance at $3Q_H = n$, where n is the azimuthal harmonic of the driving term. Beam intensity was then increased, so that the coherent sextupole tune was depressed towards resonance.

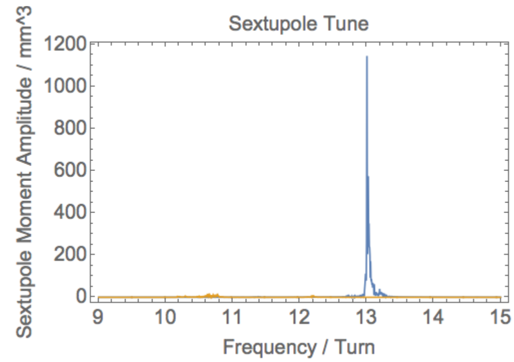

Figure 5: Sextupole moment spectra with a 13th harmonic closed orbit driving the κ_{12} higher order image term, at an intensity of 7.6×10^{13} ppp. Blue: horizontal, yellow: vertical.

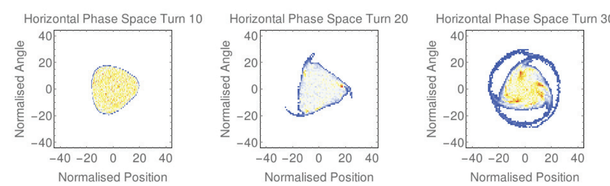

Figure 6: Horizontal phase space density plots on turns 10, 20 and 30 at an intensity of 7.6×10^{13} ppp, for a KV beam in a smooth focusing lattice driven with a 13th harmonic closed orbit.

The sextupole moment frequency of the beam is shown in Figure 5 for an intensity of 7.6×10^{13} ppp. With zero intensity there is no driving term as images are intensity dependent. At 7.6×10^{13} ppp the beam is at resonance caused by the image driving term.

Figure 6 shows horizontal phase space on turns 10, 20 and 30 for the case when the intensity is 7.6×10^{13} ppp. There is clearly a three fold structure in phase space indicating a third order resonance.

SUMMARY

An analysis of 1D image fields from pencil beams in parallel plate geometry has been made and used to create a basis for understanding other geometries. Simulations of round uniform beams in rectangular vacuum vessels have provided values for the image coefficients for a range of aspect ratios. These have been compared with the theory. For the whole beam and square aspect ratio the image coefficients are significantly larger than predicted.

Hamiltonian formalism has been used to derive resonance conditions for particles undergoing harmonic closed orbits. Particle-in-cell simulations have shown that such resonance conditions do indeed lead to resonances of the type indicated.

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