HARMONICTRON

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Abstract

The possibility of high intensity hadron accelerator based on a vertical scaling FFAG with harmonic number jump acceleration, named "Harmonictron", has been proposed. The paper gives a principle of this accelerator and also presents a design example of the Harmonictron for accelerating protons from 50 MeV to 500 MeV.

INTRODUCTION

A new type of strong focusing ring accelerator which allows a cw beam acceleration, Harmonictron, has been proposed recently [1] .

A high intensity accelerator, which is able to accelerate hadrons such as proton or deuteron up to the energy of 500 MeV/u–1 GeV/u, is requested to generate intense secondary particles. For reaching intense beam power of more than 10MW, ring accelerators such as cyclotrons and synchrotrons seem to be difficult. The synchrotron is a pulse operated accelerator where the averaged beam current can be limited by the beam duty factor in operation because of varying its magnetic field strength to keep the circular orbit radius constant during beam acceleration. In the cyclotron, because the beam bunch shape is deteriorated by the space charge force, the beam extraction becomes more difficult when the beam current exceeds several mA.

Fixed field alternating gradient (FFAG) accelerator [2] has capabilities for high intensity beam acceleration. In FFAG accelerators, however, either scaling or non-scaling, a continuous wave (cw) operation with radio frequency (rf) field has some difficulties for accelerating non-relativistic energy hadrons. The rf frequency should also be varied to synchronize the particle velocity. Thus, the fixed frequency rf acceleration without losing synchronization is essential for a cw operation.

To overcome these difficulties, an idea of harmonic number jump (HNJ) acceleration has been proposed [3,4]. The scheme of HNJ acceleration, however, has some difficulties to accelerate heavy particles such as protons or deuterons for a wide range of medium (non-relativistic) energy because the transition energy exists where the slippage factor becomes zero. In order to eliminate the transition energy inherently, momentum compaction in beam dynamics must be zero like linear accelerator. A vertical scaling FFAG accelerator makes the momentum compaction zero because of a constant orbit radius whatever the beam energy.

The idea of the vertical scaling FFAG accelerator was originally proposed by Ohkawa [5] in 1955 and analyzed in detail by Brooks recently [6]. The feature that orbit radius is always constant means the zero-momentum compaction and no transition energy exists and it is suitable for applying the HNJ acceleration. We present in this paper, a new scheme of fixed field and cw operation accelerator with HNJ acceleration using a vertical scaling FFAG concept to eliminate the transition energy called "Harmonictron", whose details are shown in our recent paper [1] .

PRINCIPLE OF HARMONICTRON

In the vertical scaling FFAG accelerator, the momentum compaction becomes always zero because the orbit radius is constant during acceleration (Fig. 1). Thus, the transition energy is infinite, in other words, no transition energy exists, and the beam is accelerated always below transition in the vertical scaling FFAG so that many problems caused by the transition energy can be avoided. The magnetic field strength changes exponentially in the vertical direction to keep a zero chromatic beam optics with constant orbit radius in the vertical scaling FFAG shown as,

$$B_y = B_0 exp(my). \tag{1}$$

Here, a characteristic number m is expressed with a field index, n, as,

$$m = n\rho. \tag{2}$$

The linearized particle motion in the transverse direction which is subject to a skew quadrupole magnetic field can be expressed by the betatron equations in skew coordinates under the approximation of no orbit curvature effect ($\rho \rightarrow$ large).

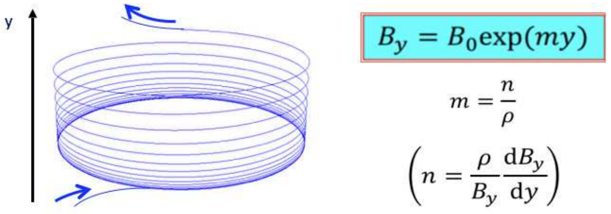

Figure 1: Schematic drawing of vertical FFAG and the magnetic field configuration for vertical direction.

The characteristic number of m specifies the orbit displacement, y_d, between initial momentum(p_i) and final beam momentum (p_e) as $m = (1/y_d)ln(p_e/p_i)$. If p_e/p_i equals 3 and y_d is less than 1 m, then, m should be more than 1.1 m^{-1}. In Table 1, the typical machine parameters of a vertical scaling FFAG accelerator which accelerates proton from 50 MeV to 500 MeV is presented.

In the relativistic energy range, where particle velocity almost equals light velocity, a light mass particle such as the

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Table 1: Basic Parameters of Proton Harmonictron

Physical Parameter	**Value**
particle	proton
injection energy	50 MeV
extraction energy	500 MeV
circumference	264 m
maximum B field	3.2 T
m	$2.0\,\mathrm{m}^{-1}$
number of cells	32
initial harmonics	80
change of harmonics/turn	-1
rf frequency	117.8 MHz

electron can be accelerated with constant frequency rf field in the vertical scaling FFAG accelerator. Thus, Ohkawa named it "electron cyclotron". Even non-relativistic particle such as proton can be accelerated by the rf field which synchronizes a time revolution elapsing around the ring for each turn as in the ordinary proton synchrotron.

Applying HNJ acceleration to the vertical scaling FFAG, heavy particles such as the proton can be accelerated in wide range of the non-relativistic energies with a fixed frequency RF acceleration which allows a cw operation. We name this new type of accelerator based on vertical scaling FFAG with HNJ acceleration as "Harmonictron".

From the synchronization condition of HNJ acceleration, the required energy gain to jump an integer harmonic number $\Delta_i h$ of harmonics between the turns i and i+1 can be expressed with as [4] ,

$$E_{i+1} - E_i = \frac{\Delta_i h}{f_{rf}(\frac{dT}{dE})_{E=E_i}}. \tag{3}$$

Here, T is a revolution time of piecewise linearized around the particle energy.

The term dT/dE of required energy gain in Eq. (8) can be expressed in the following equation with a slippage factor.

$$\frac{dT}{dE} = \frac{\eta \gamma^2 C}{M_0 c (\gamma^2 - 1)^{3/2}}, \tag{4}$$

where C is the circumference of the ring, c is light velocity and M_0 is rest mass energy. Since the momentum compaction is zero in Harmonictron (vertical scaling FFAG), Eq. (4) can be expressed as,

$$E_{i+1} - E_i = -\frac{\Delta_i h M_0 c (\gamma^2 - 1)^{3/2}}{f_{rf}} \frac{c}{C}. \tag{5}$$

As can be seen from this equation, the required energy gain per turn is a function of γ_i since f_{rf} is constant and $\Delta_i h$ should be a negative value for acceleration.

The rf voltage and/or phase have to be changed to satisfy the energy gain per turn shown in Eq. (3) in HNJ acceleration. A couple of schemes have been proposed to change the rf voltage or phase during acceleration by Ruggiero in his original paper [3], however, practical difficulties arise for realizing them. Moreover, in HNJ acceleration of medium energy heavy particle, the energy change per each turn is so large, that adiabatic condition in longitudinal focusing (synchrotron oscillation) may not be satisfied enough to keep within the large longitudinal beam acceptance. Thus, preserving the adiabatic condition of synchrotron oscillation during acceleration is important to keep a large phase space acceptance.

The criterion of adiabaticity for rf acceleration can be expressed as [7],

$$\Omega_s \gg \frac{1}{\Omega_s}\frac{d\Omega_s}{dt}, \tag{6}$$

where $\Omega_s/2\pi$ is a synchrotron frequency. When this condition is satisfied, the particles are well trapped by a rf bucket and accelerated. The above condition can be evaluated with the adiabatic parameter which is given by the following equation when the rf phase is constant π [8, 9].

$$n_{ad} = \frac{\Omega_s T_r}{1 - [V_i/(V_i + \Delta V)]^{1/2}}. \tag{7}$$

Here, V_i is the total rf voltage of i-th turn and ΔV is the increment of rf voltage derived by the rf cavity after i-th turn, T_r is a transit time of the rf cavity gap. The parameter, n_{ad}, counts the adiabaticity of the system showing how slow is the change of the bucket height with respect to the synchrotron frequency. When $n_{ad} \gg 1$, the system can be adiabatic. The adiabatic condition in HNJ acceleration could be satisfied by distributing the multi rf cavities in the ring and tuning the frequency of each rf cavity [4, 10]. If the rf system consists of N rf cavities, the adiabatic parameter shown in Eq. (7) becomes approximately N times bigger than that for a single rf cavity.

The rf frequency of each rf cavity distributed homogeneously around the ring can be obtained with the following equation [4].

$$f_{i,j} = f_{ref}\left[1 + \frac{2j + N + 1}{2N}\frac{\Delta_i h}{h_i}\right]^{-1}, \tag{8}$$

where i is the turn number, j is the cavity number, h_i is a harmonic number and f_{ref} is a reference rf frequency.

As long as h_i is larger than its variation $\Delta_i h$, the frequency of each cavity is independent of the turn number and is approximately given as,

$$f_i \simeq f_{ref}[1 - (\frac{2j + 1}{2N} + \frac{1}{2})\frac{\Delta_i h}{h_i}], \tag{9}$$

Thus, the rf frequency of each cavity is independent of the turn number and increases monotonically when $\Delta_i h$ is negative as a function of the cavity number. Moreover, if $h_i >> \Delta_i h$, then, $f_i \sim f_{ref}$.

When the rf voltage is constant, the rf phase in HNJ acceleration can be varied during beam acceleration as shown in Fig. 2. If the longitudinal adiabatic condition expressed by 3(7) is satisfied during beam acceleration, the particles could be well captured by the rf bucket and accelerated around the stable phase.

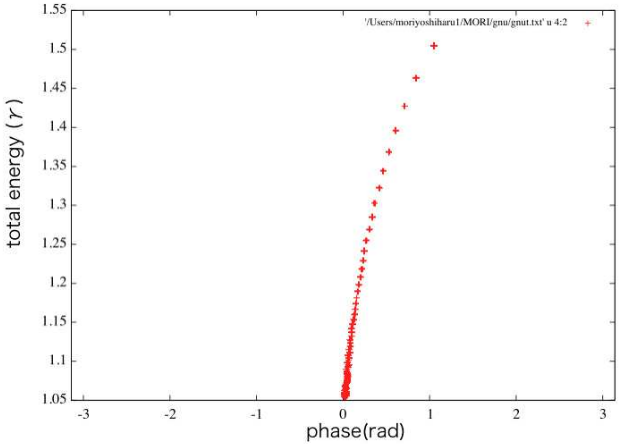

Figure 2: Variation of rf phase in HNJ acceleration during beam acceleration when the rf voltage is constant.

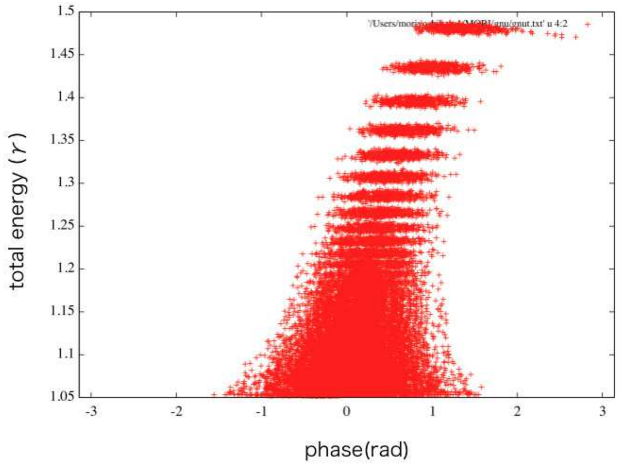

Figure 3: The beam tracking simulation results for the longitudinal beam motions for the phase spread of σ=1 rad at initial beams.

The beam tracking simulation results for the longitudinal beam motions for different phase spread initial beams are presented in Fig. 3. In this case, the number of rf cavities is 32 which are homogeneously distributed around the ring, and the rf voltage of 1.41 MV is constant during the beam acceleration. As can be seen from this figure, the particles are well captured and accelerated up to the maximum energy following the rf stable phase, and the phase acceptance at the beam injection is quite large, which is more than 70% of 2π. This means that an adiabatic beam capture process is fulfilled in the HNJ acceleration using many rf cavities with a small rf voltage which are distributed around the ring. The particles are captured adiabatically and well accelerated in a bucket with harmonic number jump.

SUMMARY

The vertical scaling FFAG accelerator with a harmonic number jump acceleration (HNJ) scheme, named "Harmonictron", is proposed for medium energy heavy particle acceleration in cw mode operation.

Harmonictron has a couple of unique features. Since no transition energy exists in Harmonictron, a wide range of beam energy becomes possible with a monotonic change of harmonic number in HNJ acceleration. By keeping enough adiabaticity in longitudinal motions for capture and acceleration of beam by distributing many rf cavities around the ring, HNJ acceleration with a constant rf voltage becomes possible, so that the cw operation with large longitudinal acceptance can be realized.

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