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EXCITATION OF PLASMA WAVE BY LASERS BEATING IN A COLLISIONAL AND MILD-RELATIVISTIC PLASMA

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Abstract

Excitation of plasma wave by two lasers beating in a collisional dominated relativistic plasma is investigated. We study the energy exchange between a plasma wave and two co-propagating lasers in plasma including the effect of relativistic mass change and electron-ion collisions. Two lasers, having frequency difference equal to the plasma frequency, excite a plasma beat wave resonantly by the ponderomotive force, which obeys the energy and momentum conservation. The relativistic effect and the electron-ion collision both contribute in energy exchange between the interacting waves in the beat-wave acceleration mechanism. Our study shows that the initial phase difference between interacting waves generates a phase mismatch between lasers and plasma wave, which alters the rate of amplitude variations of the interacting waves and, hence, affects the energy exchange between the interacting waves. This study may be crucial to design a compact plasma accelerator in low-intensity regime.

INTRODUCTION

The propagation of intense laser pulses in plasmas is relevant to many applications including laser-driven fusion, electron or ion acceleration, x-rays or terahertz radiations etc. Laser-driven Plasma-based accelerators were originally proposed by Tajima and Dawson [1] in which particles are accelerated by the electric field of a plasma wave driven by intense laser fields having acceleration gradient of GV/m. The plasma beat-wave accelerator (PBWA) is proposed as an alternative to short-pulse laser wakefield accelerators due to non-availability of ultrashort and ultra-high power lasers. The excitation mechanism of the plasma wave is a three-wave resonant nonlinear interaction. When two lasers of slightly different frequencies are propagated together in underdense plasma, there is formation of beat envelope due to interference of their transverse electric field. The interference or beating of two lasers produce density perturbation which generates plasma waves. The ponderomotive force, which is proportional to gradient of laser intensities, excites the plasma wave behind the laser beat envelope called the plasma wake with the phase velocity. The plasma wave grows significantly when the resonance between the plasma frequency and laser frequencies is fulfilled [2,3]. The lasers lose energy due to their non-linear interaction with the plasma and become saturate on an amplitude level.

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We have studied the interaction between lasers and plasmas by beating two lasers of frequency difference equal to plasma frequency. We establish the modified dispersion relation, including the effect of electron damping coefficient and relativistic mass change and derived the coupled equations for the amplitude of lasers and plasma waves and their respective phases in a plasma. The dispersion relation will be modified due to relativistic mass change and the damping coefficient. The modified dispersion relation will affect the temporal evolution of lasers and plasma waves.

ENERGY EXCHANGE BETWEEN INTERACTING WAVES

Let us consider two high intensity lasers with electric field given by

$$\boldsymbol{E_j} = \hat{x}A_j(t)\exp\left[-i\left(\omega_j t - k_j z\right)\right], \quad (1)$$

where $j = 1,2$, co- propagating in a cold plasma with density of n_0 and an electron temperature T_e. In the PBWA process, two co-propagating lasers of frequency ω_1 and ω_2 exert a ponderomotive force on electron at frequency difference $\omega_1 - \omega_2$. When frequency difference of two lasers is equal to plasma frequency $\omega_1 - \omega_2 = \omega_p$, this force drives a large amplitude plasma wave (ω, k). The electric field corresponding to Langmuir waves is

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi = \hat{x}A(t)\, exp[-i(\omega t - kz)], \quad (2)$$

where $\omega = \omega_1 - \omega_2 - \delta\omega$, $k = k_1 - k_2 - \delta k$, $\delta\omega$ and δk are the mismatches in the frequency and wave vector. The beating of two lasers generates two Langmuir waves $(\omega_1 \pm \omega_2,\ k_1 \pm k_2)$, out of which Langmuir wave $(\omega_1 + \omega_2,\ k_1 + k_2)$ is off resonant.

From the linear perturbation theory [4], the oscillatory velocities of electrons produced by interaction of laser and plasma waves are $\boldsymbol{v}_1 = e\boldsymbol{E_1}/mi\gamma\omega_1$, $\boldsymbol{v}_2 = e\boldsymbol{E_2}/mi\gamma\omega_2$, and $\boldsymbol{v} = e\boldsymbol{E}/mi\gamma\omega$. When the amplitudes of interacting waves are comparable to each other , the effective relativistic factor may be taken as $\gamma_0 = (1 + a_1^2/2 + a_2^2/2)^{1/2}$, where $\boldsymbol{a}_1 = eA_1/m\omega_1 c$, $\boldsymbol{a}_2 = eA_2/m\omega_2 c$, and $\boldsymbol{a}_0 = eA/m\omega c$, -*e* and *m* are the electron's charge and mass, respectively, and c is the velocity of light in vacuum.

The lasers exert a ponderomotive force on the electrons at (ω, k), $\boldsymbol{F}_p = -e\nabla\phi_p$, where $\phi_p = -e\boldsymbol{E}_1\boldsymbol{E}_2^*/2m\omega_1\omega_2\gamma_0$. The electron density perturbation at (ω, k) due to ϕ and ϕ_p is

$$n_\omega = k^2\varepsilon_0/e\chi_e\left(\phi + \phi_p\right), \quad (3)$$

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where $\chi_e = -\omega_p^2/(\omega^2\gamma_0 - k^2 v_{th}^2)$ is the electron susceptibility, $\omega_p = (n_0 e^2/m\varepsilon_0)^{1/2}$ is the plasma frequency, and $v_{th} = (2T_e/m)^{1/2}$ is the electron thermal velocity.

Using the expression in the Poisson's equation $\nabla^2\phi = -en_\omega/\varepsilon_0$, we obtain

$$\varepsilon\phi = -(k_p^2/k^2)\chi_e\phi_p, \quad (4)$$

where $\varepsilon = 1 + \chi_e$. Using Maxwell's equations, the electron density perturbation at (ω, k) couples with the oscillatory velocity due to laser 2 produces a nonlinear current density due to laser 1 as $\boldsymbol{J}_1 = -en_0\boldsymbol{v}_1 - en_\omega\boldsymbol{v}_2$. Similarly, nonlinear current density due to laser 2 is $\boldsymbol{J}_2 = -en_0\boldsymbol{v}_2 - en_\omega\boldsymbol{v}_1$. In the presence of damping coefficient $\boldsymbol{v}_{ei}$ due to electron-ion collisions, the dispersion relation of electromagnetic wave for laser 1 would be

$$D_1\boldsymbol{E}_1 = (\boldsymbol{v}_{ei} - i\omega_p^2/\omega_1\gamma_0)exp[-i(\omega_1 t - k_1 z)] + (k^2 e\chi_e/m\gamma_0)(\phi + k_p^2/k^2\,\phi_p)\boldsymbol{E}_2, \quad (5)$$

where $D_1 = \omega_1^2 - \omega_p^2/\gamma_0 - k_1^2c^2 + i\nu_{ei}\omega_1$. The damping coefficient of electron-ion collision changes the dispersion relations of waves. In the absence of pump ($D_1 = 0$), the dispersion relation of the electromagnetic wave can be obtained as $\omega_1^2 = \omega_p^2/\gamma_0 + k_1^2c^2 - i\nu_{ei}\omega_1$. The behavior of first electromagnetic wave can be represented by

$$\partial\boldsymbol{A}_1/\partial t = \frac{i\omega_1\zeta}{(\omega_p^2 - i\nu_{ei}\omega_1\gamma_0)}\left[\Phi exp[-i(\delta\omega t - \delta kz)] + k_p^2/k^2\,\Phi_p\right]\boldsymbol{A}_2, \quad (6)$$

where $\zeta = k^2 e\chi_e/m$, and $\Phi_p = -eA_1A_2^*/2m\gamma_0\omega_1\omega_2\ exp\,[-i(\psi_1 - \psi_2)]$. Similarly, we can solve the equation representing the behavior of the electromagnetic wave for second laser and plasma wave.

Expressing $\Phi = (mc^2/e)a_0(t)\,exp\,[-i\psi(t)]$, $A_1 = (mc\omega_p/e)a_1(t)\,exp\,[-i\psi_1]$, and $A_2 = (mc\omega_p/e)a_2(t)\,exp\,[-i\psi_2]$, in the above equations and separating the real and imaginary parts, one can obtain the following normalized equations

$$\partial a_1/\partial\tau = -\alpha_1[\omega_p^2 a_0 a_2 \sin\theta + \nu_{ei}\omega_1\gamma_0\{a_0 a_2 \cos\theta - \eta a_1 a_2^2\}], \quad (7)$$

$$\partial a_2/\partial\tau = -\alpha_2[\omega_p^2 a_0^* a_1 \sin\theta + \nu_{ei}\omega_2\gamma_0\{a_0^* a_1 \cos\theta - \eta a_2 a_1^2\}], \quad (8)$$

$$\partial a_0/\partial\tau = \alpha_3\eta a_1 a_2 \sin\theta, \quad (9)$$

$$\partial\psi_1/\partial\tau = -\alpha_1[(a_2 a_0/a_1)\omega_p^2 \cos\theta - \eta a_2^2\omega_p^2 - \nu_{ei}\omega_1\gamma_0(a_2 a_0/a_1)\sin\theta], \quad (10)$$

$$\partial\psi_2/\partial\tau = -\alpha_2[(a_1 a_0^*/a_2)\omega_p^2 \cos\theta - \eta a_1^2\omega_p^2 + \nu_{ei}\omega_2\gamma_0(a_1 a_0^*/a_2)\sin\theta], \quad (11)$$

$$\partial\psi/\partial\tau = -\alpha_3\eta(a_1 a_2/a_0)\cos\theta, \quad (12)$$

where $\tau = \omega_p t$, $\alpha_1 = (\omega_1 k^2c^2\chi_e)/\omega_p(\omega_p^4 + \nu_{ei}^2\omega_1^2\gamma_0^2)$, $\alpha_2 = (\omega_2 k^2c^2\chi_e)/\omega_p(\omega_p^4 + \nu_{ei}^2\omega_2^2\gamma_0^2)$, $\alpha_3 = (\omega^2\gamma_0 - k^2v_{th}^2)/(4\omega_p\omega_1\gamma_0^2)$, $\eta = (k_p^2/k^2)(\omega_p^2/\omega_1\omega_2)$, $\theta = \psi_1 - \psi_2 + \psi + \delta\omega t - \delta kz$ and ψ_1, ψ_2 and ψ are the phases of laser 1 &2 and plasma waves, respectively. Equatios. (7)-(12) are coupled differential equations to study the temporal evolution of interacting waves.

RESULTS AND DISCUSSION

Introducing the normalized parameters for laser frequencies $\Omega_1 = \omega_1/\omega_p$, $\Omega_2 = \omega_2/\omega_p$, we have solved eqs. (7)-(12) numerically. We consider cold homogeneous plasma of density $n_0 = 1.1\times10^{24}m^{-3}$ $(\omega_p = 5.9\times10^{13}s^{-1})$ and electron temperature $T_e = 10\ KeV$. We chose the frequency of two lasers, $\omega_1 = 2.9\times10^{14}s^{-1}$, and $\omega_2 = 2.4\times10^{14}s^{-1}$. The initial conditions are $a_1 = 0.3$, $a_1 = 0.3$, $a_0 = 0.001$ and $\theta = 0°$ at initial normalized time ($\tau = 0$). The normalized parameters for laser frequencies are $\Omega_1 = 5$ and $\Omega_2 = 4$.

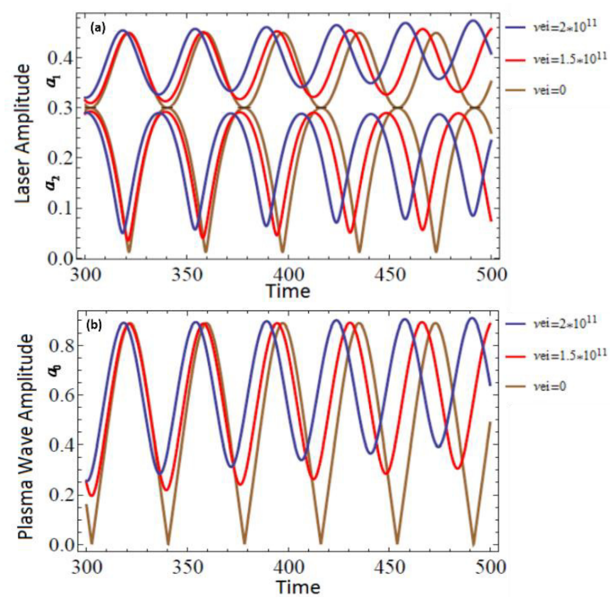

Figure 1: Temporal behavior of normalized amplitude of (a) two lasers a_1, a_2 and (b) plasma wave a_0 for different electron-ion collisional frequencies in plasma of density $n_0 = 1.1\times10^{24}m^{-3}$, $T_e = 10\ KeV$, $\Omega_1 = 5$ and $\Omega_2 = 4$.

Figure 1(a) and (b) shows the time behavior of the amplitudes of two lasers and plasma wave for different electron-ion collisional frequencies, respectively. The electron-ion collision frequency has a deep impact on the evolution of lasers and plasma waves. It can be understood from Eq. (7)- (12) that the variation of amplitude of lasers and plasma wave with time is directly dependent on the electron-ion collision frequency, and laser and plasma frequencies. If there is no collision between electrons and ions, the two lasers beat resonantly to excite a large amplitude plasma wave. As the collisional frequency increases, the lasers dissipate more energy, and hence, the amplitude of lasers decreases. Because of the energy dissipation from lasers, the transfer of energy from lasers to plasma wave

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decreases. Hence, the amplitude of plasma wave decreases as the electron-ion collisional frequency increases as shown in Fig. 1(b). With further increase in time, the lasers no longer beat resonantly, and therefore, the amplitude of the excited plasma wave diminishes. These results are in accordance with the simulations carried out in [5].

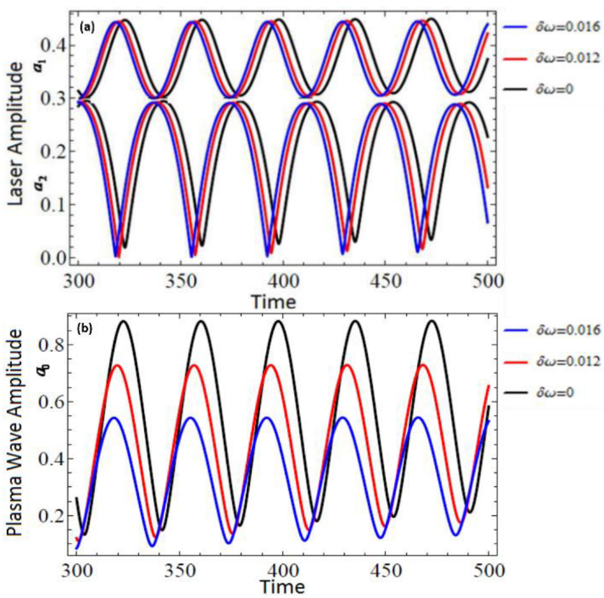

Figure 2: Temporal behavior of normalized amplitude of (a) two lasers a_1, a_2 and (b) plasma wave a_0 for different values of frequency mismatch in a collisional plasma of density $n_0 = 1.1 \times 10^{24} m^{-3}$, $T_e = 10\ KeV$, $\Omega_1 = 5$ and $\Omega_2 = 4$ with electron-ion collisional frequency $\nu_{ei} = 2 \times 10^{11} s^{-1}$.

In Figure 2 (a) and (b) we observe the effect of relative frequency mismatch between lasers frequency and plasma frequency on the amplitude of lasers and plasma wave in the presence of damping coefficient of electron-ion collisions. Due to relativistic effect, the electron quivers with the velocity comparable to the speed of light, and the mass of electrons increase. Due to this relativistic mass change of the electrons, the plasma frequency will be decreased. This leads to relative frequency mismatch between the lasers beat frequency and plasma frequency. The amplitude growth of the waves will slow down and saturation will eventually be occurred[5]. The amplitude of plasma waves with increasing frequency mismatch saturates at a level below than the amplitude when there is no frequency mismatch or both the lasers beat resonantly with the plasma frequency. It is important for the lasers to resonant perfectly with negligible frequency mismatch in order to excite the large amplitude plasma wave. Thus, both the damping coefficient of electron-ion collisions and relative frequency mismatch affect the maximum energy transfer between lasers and plasma wave.

CONCLUSION

We have presented the effect of electron-ion collision and relative frequency mismatch on the temporal evolution of lasers propagating in a collision-dominated plasma and the excitation of plasma wave. With the increase in electron-ion collision frequency and relativistic frequency mismatch, the transfer of energy from lasers to plasma decreases due to more energy dissipation from lasers. The efficiency of the accelerators in order to gain energetic accelerated bunch of electrons is affected by the energy transfer between laser and plasma wave and this can be verified experimentally. This study is also useful in laser-driven fusion studies where numbers of lasers are used to pre-heat the fusion pallet.

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