

# ELECTRON ACCELERATION BY A PLASMA WAVE IN THE PRESENCE OF A TRANSVERSELY PROPAGATED LASER WITH MAGNETIC FIELD

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## Abstract

In this paper, we study the enhancement of electron acceleration by plasma wave in presence of a laser propagated perpendicular to the propagation of the wakefield and a static magnetic field. Electrons trapped in the plasma wave are highly accelerated by the additional field of the laser combined with the wakefield. The additional resonance provided by the laser field as well by the magnetic field contributes to the large energy gain of the electrons. The resonant enhancement of electron acceleration has been validated by single particle simulation. The dependence of energy gain on laser intensity, static magnetic field, laser spot size, initial electron energy and electron trajectories have been investigated.

## INTRODUCTION

Advancement in laser technology has led to the development of very high-power and ultra-short laser pulse, based on chirped pulse amplification. Laser driven plasma wakefields have generated the great interest in last few decades owing to its ability to sustain very high accelerating gradients. It enables compact devices by producing electric fields of the order of 10-100 GeV/m. The concept of laser-driven plasma wakefield was first proposed by Tajima and Dawson [1]. Further experimental achievements led to the production of quasi-mono-energetic electron beams with energies in the range of 100 MeV to 1GeV. The plasma waves utilized for accelerating electrons to ultra-high energies can be excited using different schemes, such as laser wakefield acceleration (LWFA), plasma beat-wave acceleration (PBWA), plasma wakefield acceleration (PWFA), self-modulated wakefield acceleration (SMWFA). In LWFA, single laser pulse having pulse duration comparable to the plasma period is used to excite the plasma wave. The electrons may get trapped in the longitudinal wake of the field and accelerated to GeV energies. While in PBWA, two laser pulses with their frequency difference comparable to plasma wavelength are used to excite the plasma wakefield. Wave particle dephasing is the major limitation of these schemes. Surfatron concept [2] was also utilized, where a magnetic field is applied in the transverse direction to the plasma wave to prevent trapped electrons from outrunning the plasma wave.

The electron acceleration by plasma wave in plasma channel in the presence of guiding magnetic field to stabilize the electron motion and get higher accelerating gradients, was studied in Ref. [3]. In Ref. [4] the electron acceleration in plasma in the presence of static electric and magnetic fields in plasma is also discussed. Sharma and Tripathi [5] observed the electron energy gain up to 100 MeV using laser pulse in magnetized plasma. It was also observed that electron attains maximum energy near Doppler shifted cyclotron resonance.

## ELECTRON DYNAMICS

In an underdense plasma, the nonlinear profile of plasma wave can be taken as:

$$\mathbf{E}_p = \hat{z}A \exp\left[\frac{-x^2}{r_p^2}\right] \cos(\eta) + \hat{x}A \left(\frac{2x}{k_p r_p^2}\right) \exp\left[\frac{-x^2}{r_p^2}\right] \sin(\eta), \quad (1)$$

where  $\mathbf{E}_p = \mathbf{E}_{px}\hat{x} + \mathbf{E}_{pz}\hat{z}$ ,  $r_p$  is the radius of wakefield and  $\eta = \omega t - kz + \delta$  is the phase of the wave. The scalar potential  $\phi$  of the wakefield can be calculated using  $\mathbf{E} = -\nabla\phi$ . Let the plane polarised laser propagates through the plasma in transverse direction. The form of the laser field taken is

$$\mathbf{E}_l = \hat{z}A_0 \exp\left[\frac{-r^2}{2r_0^2}\right] \exp[-i(\omega_0 t - k_0 x)] = \mathbf{E}_z \hat{z}, \quad (2)$$

where  $r_0$  is the laser spot size. The magnetic field associated with the laser can be calculated using Maxwell equations as

$$\mathbf{B}_l = -\hat{y} \left(\frac{k_0}{\omega_0}\right) A_0 \exp\left[\frac{-r^2}{2r_0^2}\right] \exp[-i(\omega_0 t - k_0 x)], \quad (3)$$

A static magnetic field  $\mathbf{B}_0$  is also applied in negative y-direction ( $-\mathbf{B}_0\hat{y}$ ), which is parallel to the laser magnetic field.

In the presence of plasma wave, laser, and constant magnetic field, the relativistic equation of motion and energy gain equation can be written as

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E}_l + \mathbf{v} \times \mathbf{B}) - e\mathbf{E}_p \quad (4)$$

$$\frac{d\gamma}{dt} = -\frac{e}{mc^2} (\mathbf{E}_l + \mathbf{E}_p) \cdot \mathbf{v} \quad (5)$$

where  $\mathbf{p}$  is the electron momentum and  $\gamma$  is the 2-D relativistic Lorentz factor for electrons given by  $\gamma^2 = 1 + (p_x^2 + p_z^2)/m_0^2 c^2$ . Substituting the plasma wave field and laser field in Eqs. (4) and

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(5) and taking components of momentum equation, the electron dynamics can be investigated.

Here we have normalized the physical quantities as  $a_0 = eA_0/m_0\omega_0c$ ,  $a_p = eA/m\omega_p c$ ,  $b_0 = eB_0/m\omega$ ,  $t \rightarrow \omega t$ ,  $r_0 = kr_0$ ,  $r_p = kr_p$ ,  $v_z = v_z/c$ ,  $v_x = v_x/c$ ,  $z \rightarrow k_0 z$ , and  $x \rightarrow k_0 x$ , where  $e$  and  $m$  are the electron charge and rest mass, respectively. The ordinary coupled differential equations are obtained describing single electron dynamics. These equations are solved numerically using relativistic single particle code.

## RESULTS AND DISCUSSION

We solved the relativistic equation of motion and energy equation numerically using fourth order Runge-Kutta method in MATLAB. The dimensionless parameters have been used during entire study. The electron dynamics has been investigated for distinct laser and plasma parameters in presence of static magnetic field  $b_0 = 0.01$  (estimated using  $B_0 = mc\omega/e$ ). These parameters are optimised to get the maximum acceleration. Here,  $a_0=0.25$  corresponds to the laser intensity of  $8.6 \times 10^{16}$  W/cm<sup>2</sup>, given laser wavelength equal to 1 $\mu$ m. Other laser and plasma wave parameters chosen are: laser spot size  $r_0=30$  (corresponds to  $r_0 \sim 8.8\mu$ m), radius of the wakefield  $r_p=50$  (corresponds to  $r_p \sim 14.7\mu$ m), initial positions  $x_0$  and  $z_0$  are 0.6 and 0.4 respectively, initial velocities  $v_{x0}$  and  $v_{z0}$  are 0.5 and 0.8 respectively. The ratio of plasma frequency to the laser frequency ( $\omega_p/\omega = 0.5$ ) has been optimised to a value of 0.5.

To observe the effect of the additional laser field, we have given the estimation of electron energy with laser intensity parameter ( $a_0$ ). Figure 1 represents the variation of electron energy  $\gamma mc^2$  with respect to normalized distance  $z$ . It is observed that the electron gains the energy during acceleration due to the additional acceleration imparted by the laser field. Next, we have kept the plasma wave amplitude constant at 0.8 to estimate the effect of only laser field. Energy gain for different values of  $a_0$  ( $a_0=0.15, a_0=0.20, a_0=0.25$ ) has been plotted both in presence and absence of static magnetic field. It is observed that energy gain increases with  $a_0$  ranging from 0.15 to 0.25. The maximum electron energy reached is about 2 GeV. This gain is due to the combined effect of transverse electric field of laser and longitudinal electric field of plasma wave. The electrons accelerate up to GeV energy level due to the combined accelerating field in the interaction region. It can be understood from momentum and energy equations which shows that due to coupling of transverse electric field of laser  $E_z$  with plasma field, the electrons get additional momentum in the direction of propagation of the plasma wave. The electron energy obtained in presence of magnetic field is approximately two-folds the energy obtained in absence of magnetic field, as shown in Fig. 1. This is due to the fact that the electrons rotates at the cyclotron frequency ( $\omega_c$ ) about the lines of force in the magnetic field. Thus at  $\omega = \omega_c$ , cyclotron resonance occurs, and the resonant increment in electron acceleration is obtained.

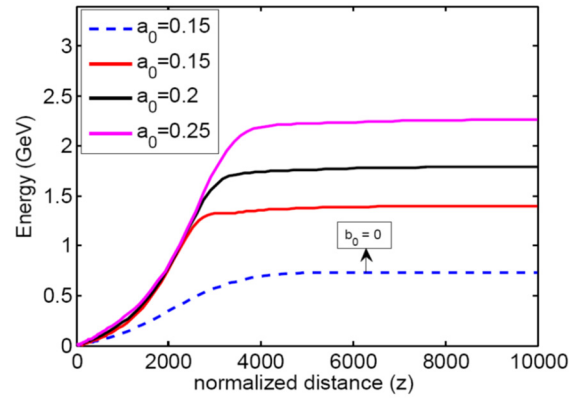


Figure 1: Variation of electron energy ( $\gamma mc^2$  in GeV) as a function of normalized distance  $z$  for electrons originated at  $x_0=0.6$ , and  $z_0=0.4$  with normalized initial velocities  $v_{x0}=0.5$ , and  $v_{z0}=0.8$ . Other parameters are as  $a_p=0.8$ ,  $r_0=30$ ,  $r_p=50$ ,  $\omega_p/\omega = 0.5$ .

One of the worthmentioning observation here is that the energy gain is noted only for values of  $a_0$  less than  $a_p$  ( $a_0 < a_p$ ). This could be due to the reason that at low laser intensity, the laser magnetic field contribution ( $\mathbf{v} \times \mathbf{B}$ ) to the electron motion may be negligible. If we increase  $a_0$ , larger effect of laser magnetic field is experienced by the electron and it is scattered away from the propagation axis.

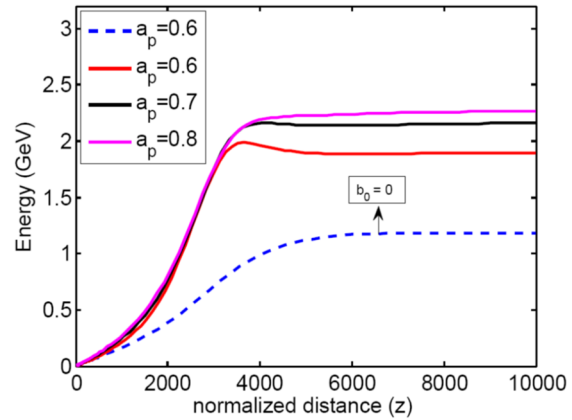


Figure 2: Variation of electron energy ( $\gamma mc^2$  in GeV) as a function of normalized distance  $z$  keeping  $a_0=0.25$ .

Figure 2 shows the variation of electron energy with normalized distance at  $a_0=0.25$  for different plasma intensity parameter ( $a_p=0.6, 0.7, 0.8$ ). Other parameters are same as used in Fig. 1. The laser field ( $a_0$ ) is kept constant to observe the electron acceleration only by the plasma wave. It is observed that varying  $a_p$  leads to an increase in the energy of the accelerated electrons. A higher value of  $a_p$  corresponds to larger longitudinal electric field. Hence electrons trapped in plasma wave experience a larger force in longitudinal direction and gain high energies. In the absence of external static magnetic field, the energy of electron obtained is approximately half of the energy obtained in presence of the magnetic field. This is because of the cyclotron resonance. As for  $\omega > \omega_c$ , resonance

condition is satisfied and electrons attain their maximum energy.

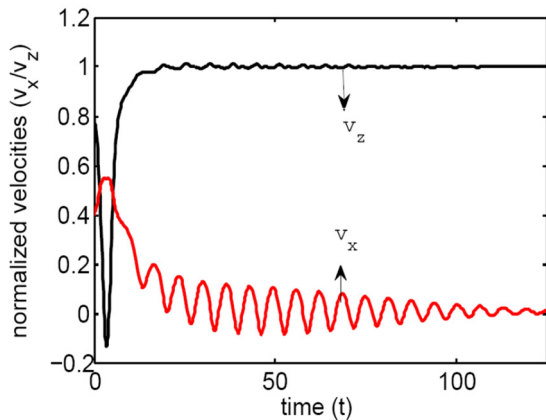


Figure 3: Plot of normalised velocities  $v_x$  and  $v_z$  with time for  $a_0=0.25$ . Other parameters are same as in Fig. 1.

Initial kinetic energy of electron is also an important factor in determining the electron energy gain and trajectories during acceleration. In Fig. 3, variation of normalised velocity of electron in x and z-direction with time is shown for different initial kinetic energies. Other parameters considered are same as in Fig. 1. As the electron is mainly accelerating in z-direction, the velocity of electron in that direction is expected to be increasing with time. Hence it accelerates and attains a maximum velocity approximately equal to the speed of light c.

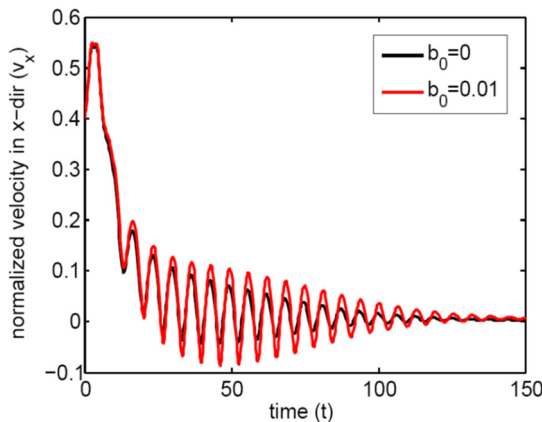


Figure 4: Plot of  $v_{x0}$  with time for  $b_0=0, 0.01$ . Other parameters are same as in Fig. 2.

The initial velocity given to electron in x-direction is  $0.4c$ . Initially the electron velocity increases and then starts decreasing with time. And finally, almost all of the velocity is transferred in z-direction. This indicates that after a certain time the electron traverses in z-direction without much scattering in transverse direction.

In Fig. 4, electron velocity plot is shown both in the presence as well as absence of static magnetic field. Other parameters are same as in Fig. 3. In the presence of magnetic field, electron velocity oscillation amplitude is greater than in the absence of magnetic field. This is due to the additional Lorentz force of external static magnetic field in x-direction.

## CONCLUSION

In this study, we observed a significant increment in the electron energy when a laser is propagated transversely to the plasma wave in the presence of a magnetic field. This electron energy gain is further enhanced in the presence of an external static magnetic field due to cyclotron resonance. We observed approximately twofold increment in the electron energy in the presence of an external constant magnetic field. It is due to an additional  $\mathbf{v} \times \mathbf{B}$  term in z-direction. The role of laser intensity, plasma wave amplitude, and initial electron energies on electron energy has also been observed. Electron energy increases with the laser intensity as long as plasma wave amplitude is kept higher than laser intensity parameter. Initial energy of electrons is also a major factor in determining the electron energy and electron trajectory. It has been noticed that the electrons with higher initial kinetic energies are less scattered. This study may be useful in producing very high energy electrons, which may further be used in medicine, high energy physics, free-electron lasers and so.

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