

# POWER ENHANCEMENT OF FREE-ELECTRON LASER OSCILLATORS WITH THE NATURAL GRADIENT OF A PLANAR UNDULATORS#

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## Abstract

Transverse Gradient Undulator (TGU) has been proposed with the initial purpose of mitigating the gain degradation in free electron laser oscillators (FELO) driven by beams with a large energy spread. However, a special-designed TGU with a fixed transverse gradient is required to enhance the gain. In this paper, we investigate using the natural field gradient of a normal planar undulator instead of a TGU to enhance the FELO power. In this method, the beam is first vertically dispersed and then sent through a normal undulator with a vertical off-axis orbit to experience the field gradient. Theoretical analysis and numerical simulation based on parameters of FELiChEM are presented. The results demonstrate that this scheme can enhance the FEL power with careful optimization of dispersion strength and vertical beam orbit offset, especially when the energy spread is relatively large.

## INTRODUCTION

Free electron lasers (FEL) oscillator has been adopted worldwide in the wavelength range covering from the terahertz to the ultra-violet regions [1-3]. Additionally, FEL oscillator (FELO) also has been proposed to radiate in x-ray region by using the diamond crystal mirror [4].

Given the present TGU has many important applications in different kinds of FELs, including in PEHG-FEL [5], generation of broadband radiation [6], and so on. In fact, it was initially proposed to enhance the small signal gain in conventional FELO while the energy spread is relatively large [7]. In other words, it can be used to lower the design requirements of the electron injector. However, TGU requires the special design and the transverse gradient will be fixed when the tilt angle of magnetic pole fixed.

Currently, an IR FELO is being built at NSRL, covering the spectral range from 2.5 to 200  $\mu\text{m}$  and dedicating for energy chemistry research [3]. Since the radiation wavelength range is broad, in the design, the FEL performance at the two sides is not optimized. Therefore, we investigate using the natural field gradient of a normal planar undulator to enhance the output power of short-wavelength FEL. In this method, the beam is first vertically dispersed by a dogleg and then the dispersed beam passes through a normal undulator with a vertical off-axis orbit [8].

In this paper, we first briefly introduce the small signal gain of a normal undulator and a TGU with the energy spread. Considering that the previous studies of using the transverse or vertical gradient of FELO are mostly focused

on the theoretical analysis, we present the simulation results based on the parameters of FELiChEM. The comparisons of using a TGU and a normal undulator with or without the natural field gradient were given in details. Finally, a summary is given in the last section.

## SMALL SIGNAL GAIN IN FELO

In a FELO the small signal gain is one of the most important parameters determining whether the oscillation can start. We use the 1D FEL model and ignore 3D effects. Considering the energy spread, the maximum gain  $g_N$  can be simplified as [9]:

$$g_N = \frac{g_0}{1 + (5.46 N_u \sigma_\eta)^2} \quad (1)$$

where  $N_u$  is the number of undulator period,  $\sigma_\eta$  is the energy spread, and  $g_0 = 0.27(4\pi N_u \rho)^3$  is the small signal gain which is independent of  $\sigma_\eta$ . Normally,  $\sigma_\eta$  is required to be smaller than  $1/2N_u$ .

However, in a TGU-enabled FELO, the electron beam is first horizontally dispersed by a dogleg before entering into the TGU. The dispersion strength  $\eta$  introduced by the dogleg and the transverse gradient  $\alpha$  introduced by the TGU should follow a well-known relation  $\alpha\eta = (2 + K_0^2) / K_0^2$ . Assuming a dispersed beam passes through a TGU, the maximum gain  $g_{TGU}$  can be approximately simplified as [9]:

$$g_{TGU} \approx g_N \frac{\eta \sigma_\eta}{\sigma_x} \frac{\sqrt{2}}{1 + [\eta/5.46 N_u \sigma_x]^2} \quad (2)$$

where  $\sigma_x$  is the initial RMS horizontal beam size.

Table 1: Main FEL Parameters

Parameter	Specification	Unit
radiation wavelength	2.5	$\mu\text{m}$
undulator parameter $K_0$	0.97	-
undulator period	4.6	cm
period number	50	-
beam energy	50	MeV
beam transverse emittance	10	mm.mrad
beam transverse size	0.5	mm

According to Equations (1) and (2), the small signal gain depends on the energy spread of the normal undulator and TGU, are given in Fig. 1. It can be concluded that TGU can significantly enhance the FEL gain, especially when  $\sigma_\eta$  has a large value. The bigger the energy spread, the bigger

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the improvement of the FEL gain. However, if  $\sigma_\eta$  is small, TGU is insufficient to enhance the FEL power and will be discussed in the next section. The corresponding FEL parameters are given in Table. 1.

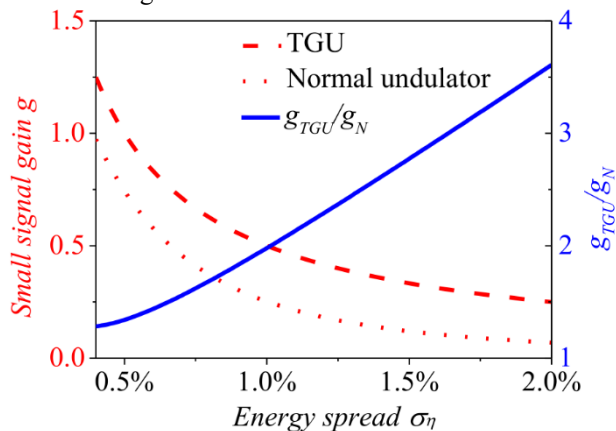


Figure 1: Red: the small signal gain  $g$  depends on  $\sigma_\eta$  for using TGU (dashed), comparing with the normal undulator (dotted). Blue: the ratio of  $g_{TGU}$  and  $g_N$  depends on  $\sigma_\eta$  (solid).

## SIMULATIONS

### Power Enhancement in a TGU

We numerically investigate the FEL performance in a TGU, comparing with a normal undulator based on the modified OPC code [10] and GENESIS 1.3 code [11]. For TGU, we choose that  $\alpha=36m^{-1}$  and  $\eta=86.5mm$  which satisfy  $\alpha\eta=(2+K_0^2)/K_0^2$ .

Fig. 2 is the simulation result which show the corresponding evolution of FELO power with different energy spread for using TGU, comparing with the normal undulator. The efficiency of power enhancement based on TGU markedly improved with the increase of  $\sigma_\eta$ , which is consistent with that shown in Fig. 1. When  $\sigma_\eta$  is comparatively small ( $\sigma_\eta=0.5\%$ ), the FELO saturated power generated by the normal undulator is slightly bigger than using a TGU. However, it is not contradictory with that analysed in Fig. 1. Since the improvement of  $g$  is small with a small  $\sigma_\eta$  and the dispersion of the beam introduced by the dog-leg will increase the beam size, the gain improvement by a TGU is insufficient to cancel the gain degradation caused by a large beam size in the three-dimensional simulation.

In Fig. 2(b), the FELO saturated power generated by TGU is improved by 33% than using a normal undulator. In Fig. 2(c), due to  $\sigma_\eta$  is large and  $g_N$  is small, the growth of FEL power could not overcome the cavity loss when using a normal undulator. However, TGU is more effective to enhance the FEL gain with a large  $\sigma_\eta$ . Comparing Fig. 2(b) with (c), as  $\sigma_\eta$  increases from 0.7% to 0.9%, the enhancement of FELO saturated power by using a TGU becomes

very significant. Therefore the FELO power can be enhanced to a very high level by using a TGU. In other words, TGU oscillator can reach the same saturated power when using a beam with a comparatively large energy spread.

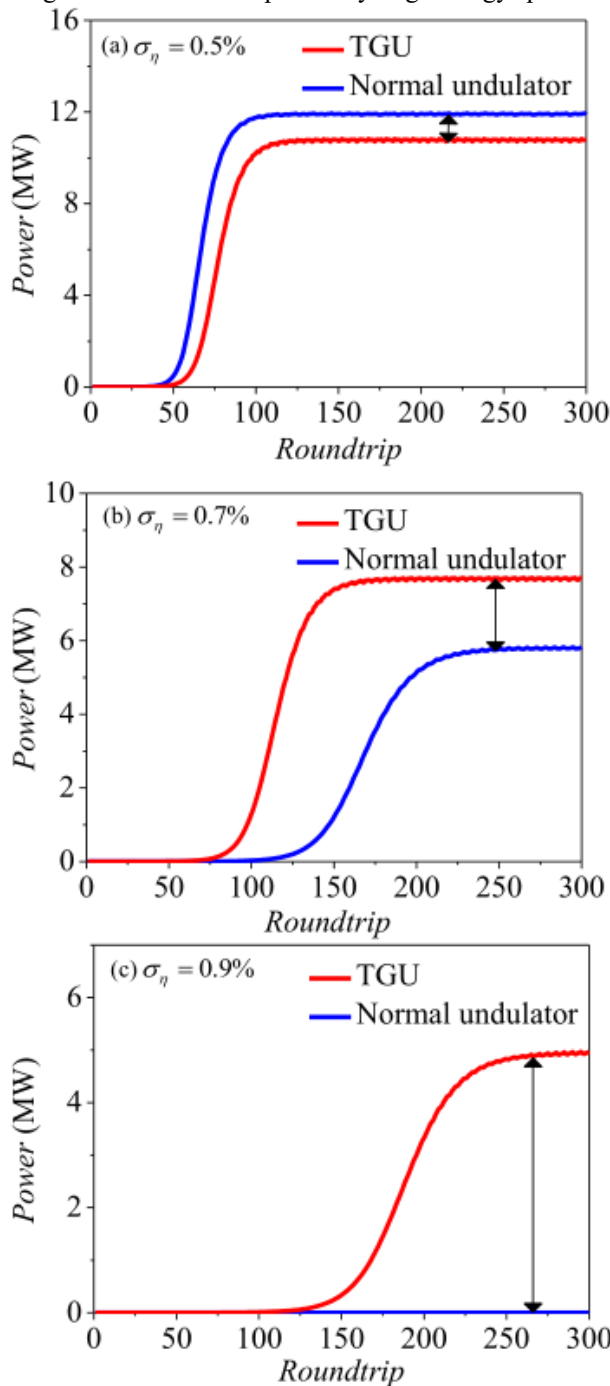


Figure 2: FELO power at  $2.5\mu m$  for using TGU (red), comparing with the normal undulator (blue).

### Power Enhancement when using the Natural Gradient

For a normal planar undulator with a vertical magnetic field, the undulator strength  $K$  along the vertical coordinate can be described as

$$K(y) = K_0 \cosh(k_u y) \quad (3)$$

where  $k_u = 2\pi / \lambda_u$  is the wavenumber and  $\lambda_u$  is the undulator period.

For a small range around an off-axis orbit of  $y = y_c$ , the natural vertical gradient  $\alpha_y$  can be deduced as  $\alpha_y = k_u \tanh(k_u y_c)$  [8].

Therefore, we investigate using the natural field gradient of a normal planar undulator instead of a TGU to enhance the FEL gain. In this method, the beam is first vertically dispersed by a dogleg and then the dispersed beam passes through a normal undulator with a vertical off-axis orbit. In order to comparing the natural gradient (NTG) of a normal planar undulator with a TGU,  $y_c$  is chose to be  $2\text{ mm}$ , then the corresponding gradient  $\alpha_y \approx 36\text{ m}^{-1}$  and the vertical dispersion strength  $\eta = 86.5\text{ mm}$  are same with that used in a TGU. In addition,  $\sigma_x$  should be modified to be  $\sigma_y$  in Eq. (2).

The simulation results are shown as followings.

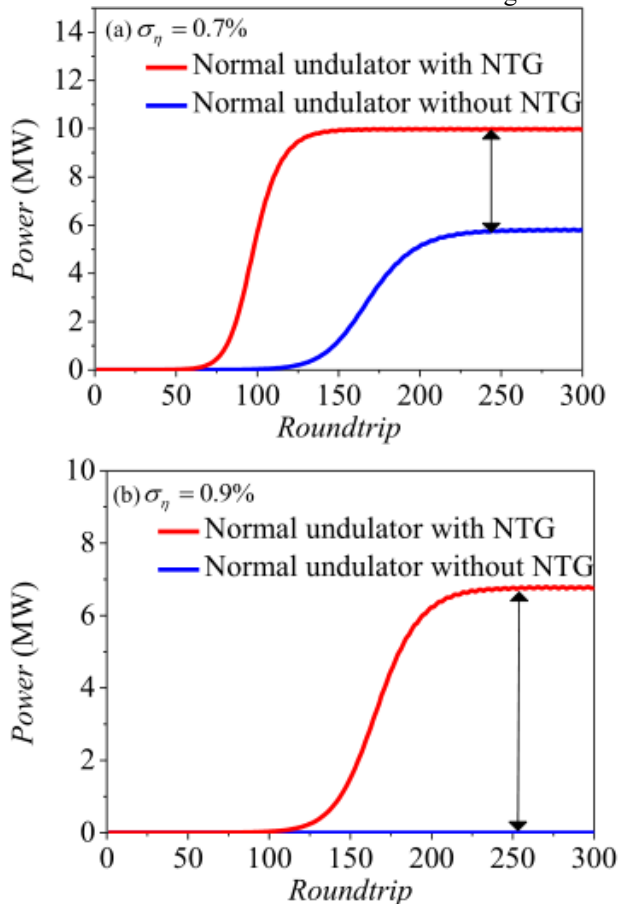


Figure 3: FEL power at  $2.5\mu\text{m}$  for using NTG (red), comparing with that without NTG (blue) .

Fig. 3(a) and (b) show the similar results using NTG of the normal undulator in comparison with that using a TGU. For a beam with comparatively small energy spread, the FEL power generated by a normal undulator with NTG is 74% more improved than without NTG, as shown in Fig. 3(a). The FEL power enhancement by NTG is more than

twice for using a TGU. In Fig.3(b), a significant improvement can be also found when using NTG. Compared Fig. 3(b) with Fig. 2(c), The FEL power generated by NTG is slightly larger than using a TGU. Therefore the normal undulator can reach a rather high saturated power. Moreover, compared with the TGU, using the natural gradient planar undulator has a distinct advantage for the gradient can be conveniently tuned in a quite large range by adjusting the beam orbit offset. Therefore, the natural gradient of a normal planar undulator may be a good choice to replace a TGU to enhance the FEL power.

However, the off-axis beam will experience a betatron oscillation driven by the natural focusing. If the betatron wavelength is smaller than or comparable with the undulator length, the natural focusing may destroy the linear dependence of the electron energy on the vertical coordinate to some extent. Thus the betatron wavelength should be designed to be much bigger than the total undulator length.

## SUMMARY

In this paper, we present the theoretical analysis and numerical simulation of TGU and the normal planar undulator. It shows that TGU can be used to reduce the sensitivity on energy spread and improve the FEL power. In addition, the natural gradient of the normal planar undulator can be used to replace the TGU, so as to improve FEL power when using an off-axis beam with a large energy spread. The corresponding experiment studies will be put into practice at FELiChEM in the future.

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