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COHERENT TRANSITION RADIATION GENERATED FROM TRANSVERSE AND LONGITUDINAL ELECTRON DENSITY MODULATION

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Abstract

Coherent Transition radiation (CTR) of a given frequency is commonly generated with longitudinal electron bunch trains. In this paper, we present a study of CTR properties produced from simultaneous electron transverse and longitudinal density modulation. We demonstrate via numerical simulations a simple technique to generate THz-scale frequencies from mm-scale transversely separated electron beamlets formed into a ps-scale bunch train. The results and a potential experimental setup are discussed.

INTRODUCTION

Coherent transition radiation (CTR) is commonly used in radiation generation setups and temporal bunch profile diagnostics [1–7]. Conventionally, the radiation spectral content in these experiments is determined by the bunch longitudinal modulation. It was demonstrated before, that transverse electron density modulation of ultra-short pulses can lead to additional spectral harmonics localized off-axis from the main radiation cone [8]. Recently, the formation and propagation of transversely modulated photoelectron bunches was investigated at the Argonne Wakefield Accelerator (AWA) facility [9]. In brief, the transverse modulation was imposed using microlens array (MLA) shaping technique and the spacing between resulting beamlets was controlled by solenoid and quadrupole lenses. Alternatively, electron beamlets can be generated via intercepting masks or other transverse shaping techniques in the accelerator. In this paper, we present a numerical simulation of CTR spectral properties, generated from the beams with simultaneous transverse and longitudinal modulation.

ANALYTICAL MODEL

A detailed analytical derivation of the transition radiation from a point charge as a solution of Maxwell's equations between two media can be found in classical textbooks [10,11]. The electromagnetic field of a point charge falling onto a metallic plate can be calculated using the "method of images" [12], where for every charge incident on an infinite conducting plane $(q, \vec{e}_i)$ there is a "mirror" charge $(q', \vec{e}_i')$ behind the plane forming a pair of real and image charges.

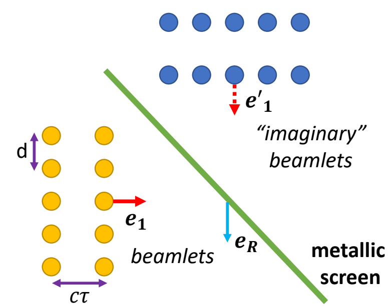

Figure 1: Simplified schematics of the "line" electron beamlet arrangement with spacing d.

When a virtual pair of charges approaches the plane, it emits radiation that is mathematically equivalent to the transition radiation from a point charge [11]. It is assumed that both real and imaginary charges instantly slow down and vanish upon hitting the plane. The Fourier transform of the magnetic component of a radiation field can be written as [11] [1]:

$$\vec{H}_\omega = \frac{q}{2\pi c}\sum_{i=1}^{N}\left(\frac{\vec{e_{iR}}\times\vec{e_i}}{1-\vec{e_{iR}}\cdot\vec{e_i}} - \frac{\vec{e'_{iR}}\times\vec{e'_i}}{1-\vec{e'_{iR}}\cdot\vec{e'_i}}\right)\times \times\frac{\exp\left(i\omega R_i/c + i\omega t_i\right)}{R_i}, \quad (1)$$

where N is the number of charges q, $\vec{e}_i$ is the velocity vector for i-th electron, $\vec{e_{iR}} = \vec{R}_i/R_i$ is the vector from the point of incidence to the detector, R_i is the distance from the point of incidence to the detector, t_i is the time of arrival of i-th charge at the metallic plate, ω is the radiation frequency. The spectral-angular density of the radiation can then be computed as [11]:

$$\frac{d^2W}{d\omega d\Omega} = cR^2|\vec{H}_\omega|^2 \quad (2)$$

The detector is assumed to be located in the far zone, such that $R \gg \sigma$ and $R \gg \lambda$, where σ is the electron beam size

[1] Additionally, an analytical expression for a TR EM-field in case of a finite metallic plane can be found in [13].

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and λ is the generated radiation wavelength. Under these assumptions, $\vec{R}_i$ can be considered as $\vec{R}_i \approx \vec{R}$ or in spherical coordinates $R_{ir} \approx R$, $R_{i\theta} \approx \theta$ and $R_{i\phi} \approx \phi$, where the origin is set to the beam incidence point. Then the bracket term in Eq. 1 can be rewritten according to:

$$1 - \vec{e_{iR}} \cdot \vec{e_i} = 1 - \vec{e_R} \cdot \vec{e_i} \qquad 1 - \vec{e_{iR}'} \cdot \vec{e_i'} = 1 - \vec{e_R'} \cdot \vec{e_i'},$$

where $\vec{e_R} = \vec{R}/R$. In case of a relativistic electron beam $\vec{e_i} = \vec{\beta}$ and $\vec{e_i'} = \vec{\beta}'$, where $\beta \approx 1$. Thus, Eq. 1 can be simplified as:

$$\begin{aligned} \vec{H_\omega} &\approx \frac{q}{2\pi c R} \vec{f} \sum_{i=1}^{N} \exp\left(i\omega R_i/c + i\omega t_i\right), \\ \vec{f} &= \left(\frac{\vec{e_R} \times \vec{\beta}}{1 - \vec{e_R} \cdot \vec{\beta}} - \frac{\vec{e_R} \times \vec{\beta}'}{1 - \vec{e_R} \cdot \vec{\beta}'} \right) \end{aligned} \tag{3}$$

Additionally, one can introduce bunching factor as:

$$F = \left| \sum_{i=1}^{N} \exp\left(i\omega R_i/c + i\omega t_i\right) \right|^2 \tag{4}$$

It can be shown that Eq. 3 is correct when $\psi \gg \sqrt{2\sigma/R \cos\alpha}$, where $\cos\psi = \vec{e_R} \cdot \vec{\beta}'$, $\cos\alpha = \vec{\beta} \cdot \vec{n}$ and $\vec{n}$ is the normal to conducting the plate.

Note, that the path differences ΔR_i remain finite when $R_i \to \infty$. The resulting spectral-angular density can be then rewritten in a shorter form:

$$\frac{d^2 W}{d\omega d\Omega} \approx \frac{q^2}{4\pi^2 c} f^2 F \tag{5}$$

So far we haven't made any assumptions on the form of the charge transverse distribution. Now let's consider a case of equidistant point charges (beamlets) with spacing d arranged in a "line" formation. Additionally, the incidence angle is $\alpha = 45°$. Figure 1 illustrates such an arrangement for both "real" and "imaginary" charges.

If number of charges N and spacing matches the condition $\omega (Nd)^2/cR \ll 1$, transverse bunching factor $F_\perp(\omega)$ can be computed analytically as

$$F_\perp \approx \frac{\sin^2 N\xi/2}{\sin^2 \xi/2}; \quad \xi = \frac{\omega d}{c \cos\alpha} \left(\sin\alpha - \sin\theta\cos\phi\right), \tag{6}$$

where ω is radiation frequency. This expression can be used for spectral density calculation when $R \to \infty$. Additionally, Eq. 6 has its j-th harmonic maximum at:

$$\omega_{\perp j} = \frac{2\pi j c \cos\alpha}{d\left(\sin\alpha - \sin\theta\cos\phi\right)}, \tag{7}$$

which yields that $F_\perp(\omega)$ will present modulation when $\theta \neq \alpha$ while $\phi \neq 0$. Thus, a transverse modulation in charge density results in narrow-band harmonics in spectral density.

NUMERICAL SIMULATIONS

We consider a 50 MeV electron beam formed with transverse electron beamlets with the spacing d = 3 mm and temporal length of σ_t=0.333 ps. Additionally, beamlets form a bunch train with τ=3.33 ps duration. If no transverse density modulation was introduced, such a bunch train yields $\omega/2\pi$=333 GHz narrow band signal; see Fig. 2. Let us investigate the implications of the aforementioned beam parameters on the resulting CTR spectral content. To perform numerical simulations, a code that computes radiation spectral density (Eq. (2)) with exact expression for $\vec{H_\omega}$ (Eq. (1)) and approximation (Eq. (3)) was developed. Code validation and initial simulations are discussed in [8].

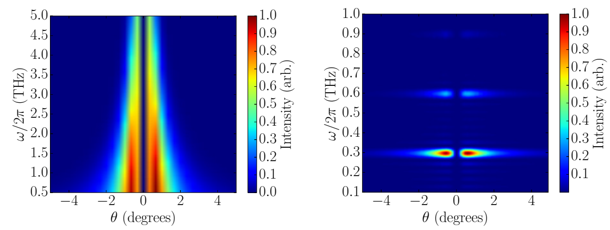

Figure 2: Transverse Gaussian spectrum of a single beamlet with σ_t=0.333 ps (left) and single beamlet bunch train spectrum with τ=3.33 ps (right).

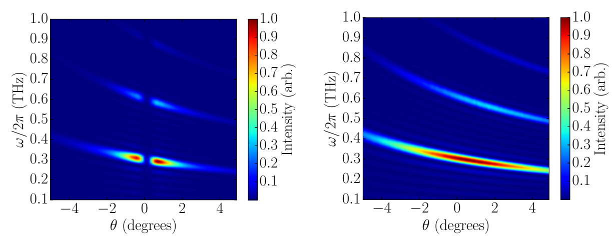

Figure 3: Resulting spectrum of the beamlet formation bunch train at $R \approx 5$ m and $\phi = 0°$ (left) and $\phi \approx 4°$ (right).

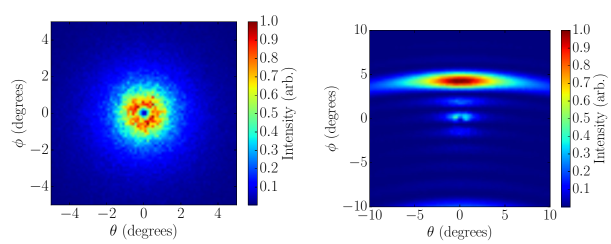

Figure 4: Spatial localization of the CTR in case of single Gaussian beamlet (left) and the extened CTR source formed by the beamlet bunch train (right) at the first harmonic frequency of $\omega/2\pi$=333 GHz.

Transition radiation forms a notable "donut"-shape radiation cone in case of a regular gaussian electron transverse density distribution. The absense of radiation inside the cone is attributed to the fact that it is not emitted along the mirror charge velocity [10, 11]. Such a spectrum is displayed in Figs. 2,4 for the case of single gaussian beamlet. The maximum of the radiation is contained around $1/\gamma \approx 0.57°$ which

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is in agreement with the electron beam energy. Hereafter we assume the detector in the far-zone meaning $R \gg 8$ mm and the electron beam has very small divergence at the metallic screen location. $N = 8000$ particles were used in simulations of both transverse Gaussian bunches and beamlets configurations.

The spectral content of the radiation generated by 8 gaussian beamlets arranged in a bunch train is depicted in Fig. 3. The first harmonic frequency is now dependent on the detector position $R(r, \theta, \phi)$. This effect is the result of the interference of the radiation cones from individual beamlets, and the addtional time delay R_i/c associated with the detector position. At the first spectral harmonic ceases to be localized in space, additionally the CTR "donut" cone transforms into an extended source; see Figs. 3,4. However, its frequency tends to remain relatively narrow-band, therefore motivating the experimental attempt.

TENTATIVE EXPERIMENT

An experimental setup to register the radiation with the aforementioned properties will invoke a beamline with a bunch compressor to generate longitudinal microbunching. Additionally, an optical setup similar to the one recently comissioned at Argonne Wakefield Accelerator [9, 14] will generate the necessary transverse modulation. The beam dynamics study associated to the propagation of the transversely modulated beams through the bunch compressor will be performed as a continuation of this work. Some preliminary studies were recently done at AWA facility [15].

CONCLUSIONS

In conclusion, transverse density modulation can lead to the peculiar spectral content of electron bunch trains. We demonstrated, via numerical simulations, that the first harmonic frequency of the CTR associated to the transversely modulated bunch train can be dependent of the detector location. Additionally, within a given solid angle $\Delta\Omega$ the produced CTR has a "color gradient". The presented study can be extended to the cases of non-periodic trasnverse modulation and ramped longitudinal current profiles.

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