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STUDY OF GF SYMPLECTIC TRACKING METHOD AND COMPENSATION FOR THE EPU104 AT THE HLS-II

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Abstract

An elliptically polarized undulator (EPU) was applied to obtain high-brightness coherent synchrotron radiation at the upgraded Hefei Light Source, HLS-II. However, the EPU has serious dynamic effects on the beam performances including close orbit, emittance and dynamic aperture etc. when installed at the storage ring. In order to understand the effects, a Taylor expanded generating method was adopted to generate a fast and symplectic map for particle tracking. As for the compensation of the EPU, striplines were equipped above and below the vacuum chamber to reduce the nonlinear effects. With the symplectic tracking routine and the surface fitting method, different parameters such as dynamic aperture and the driving terms, could be set as the objective function to accomplish the optimization of the EPU.

INTRODUCTION

Undulators play an important role in the 3rd generation synchrotron radiation sources, and the use of these devices allows reaching high spectral brightness. However, magnetic field imperfections can result in the reduction of the beam dynamic aperture and lifetime. And the tracking study is the prerequisite for learning the beam dynamic of the whole ring and the influence of the insertion devices. In this paper, a symplectic particle tracking method based on generating functions (GF) is presented [1]. Such method allows a larger integration step size compared to other integration methods, which reduces the calculation time greatly. An analytic field description which can be differentiated and integrated is needed while implementing this symplectic tracking method. Thus, one field model especially suitable for APPLE II type undulator is adopted to rebuild the internal space magnetic field. In this way, the magnetic field consists of a series of Fourier coefficients, which can be derived from fitting with numerical data.

At last, we should focus on the compensation of the EPU. The surface fitting method [2] is convenient and easy for the Lie operation, but the calculation time for tracking is much longer than the GF method. Such two methods can be verified against each other. For compensation, striplines carrying different current are added above and below the vacuum chamber to optimize the dynamic nonlinear properties [3]. Both two methods can be applied to choose the optimal parameters. We can use surface fitting method to obtain the effective Hamiltonian [4] of the whole ring, which can be analyzed through normal form method [5]. And the results of the dynamic aperture can be derived from the GF tracking routine.

Table 1: Parameters of EPU104

Type	APPLE-II
Period length	104 mm
Period number	31
Structure length	3224 mm
Vertical/Horizontal gap	30~80 mm/2.8 mm
Remanence	1.25 T
Magnetic material	NdFeB N38SH
Standard magnetized block	32mm x 32mm x 26 mm
End-magnetized block1	32mm x 32mm x 6.5 mm
End-magnetized block2	32mm x 32mm x 19.5mm

ANALYTIC REPRESENTATION OF THE MAGNETIC FIELD

The main parameters of EPU104 at HLS-II are listed in Table 1. A RADIA [6] software package specially designed and optimized for solving problems of insertion devices was applied to get the three-dimension magnetic field data of EPU104. And one model based on a Fourier series expansion of the field of one magnet row is describes as following form:

$$
\begin{aligned}
B_x &= \sum_{i=0}^{n}\sum_{j=1}^{m}\frac{k_{xi}}{k_{yi,\tilde{j}}}c_{i,\tilde{j}}\sin(k_{xi}x)\exp(-k_{yi,\tilde{j}}y)\cos(k_{\tilde{j}}z+\varphi) \\
&\quad\times\exp(-k_{yi,\tilde{j}}\Delta g/2) \\
B_y &= \sum_{i=0}^{n}\sum_{j=1}^{m}c_{i,\tilde{j}}\cos(k_{xi}x)\exp(-k_{yi,\tilde{j}}y)\cos(k_{\tilde{j}}z+\varphi) \\
&\quad\times\exp(-k_{yi,\tilde{j}}\Delta g/2) \\
B_z &= \sum_{i=0}^{n}\sum_{j=1}^{m}\frac{k}{k_{yi,\tilde{j}}}c_{i,\tilde{j}}\cos(k_{xi}x)\exp(-k_{yi,\tilde{j}}y)\cos(k_{\tilde{j}}z+\varphi) \\
&\quad\times\exp(-k_{yi,\tilde{j}}\Delta g/2) \\
\tilde{j} &= 1+l(j-1) \qquad k_{xi}=k_{x1}i=(2\pi/\lambda_{x0})i \\
k_{yi,\tilde{j}} &= \sqrt{k_{\tilde{j}}^2+k_{xi}^2} \qquad k_{\tilde{j}}=k\cdot\tilde{j}
\end{aligned}
\tag{1}
$$

We can find a series of solutions for the Fourier coefficients with the numerical magnetic field data calculated by RADIA, which minimize the variance between calculation data and fit data. The magnetic field results based

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on such a model with $n = 40, m = 20$ are shown in Figure 1, Figure 2 and Figure 3.

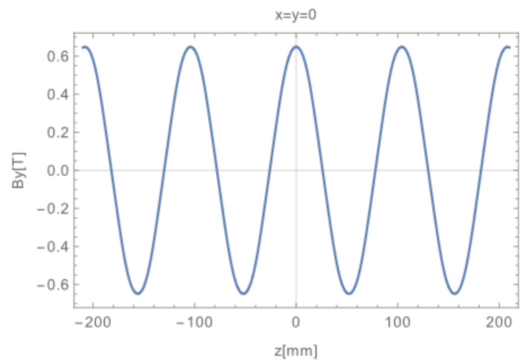

Figure 1: On-axis vertical magnetic field distribution along longitudinal direction (4 periods) in horizontal polarization mode.

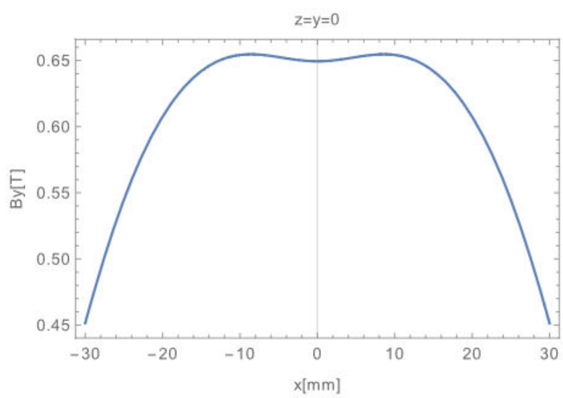

Figure 2: On-axis vertical magnetic field distribution along horizontal direction in horizontal polarization mode.

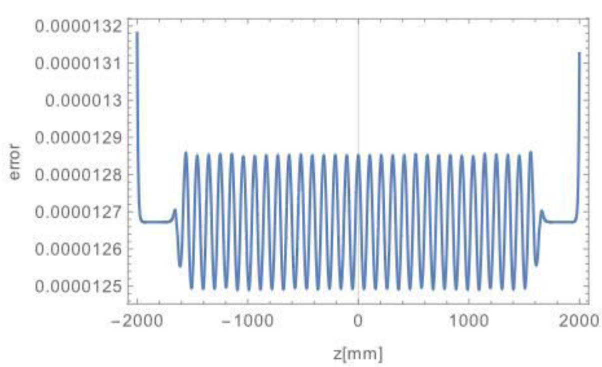

Figure 3: Errors of vertical magnetic field obtained by surface fitting method and Fourier expansion model.

GENERATING FUNCTION TRACKING METHOD

With the resulting analytic magnetic field representations, a symplectic mapping routine for particle tracking can be produced by GF method, which is a solution of the Hamiltonian-Jacobi equation. The 2nd order tracking map is shown in this section.

Starting with the Hamiltonian-Jacobi equation, we choose the third form of the canonical transformation and the 2nd order tracking formulation is derived as:

$$\frac{\partial F_3}{\partial z} + H = 0$$
$$F_3(x, y, p_{xf}, p_{yf}) = \sum_{ijk} f_{ijk} p_{xf}^i p_{yf}^j x_3^k \tag{2}$$

$$x_f = -\frac{\partial F_3}{\partial p_{xf}} = x + p_{xf} z_f - f_{101}$$
$$y_f = -\frac{\partial F_3}{\partial p_{yf}} = y + p_{yf} z_f - f_{011}$$
$$p_x = -\frac{\partial F_3}{\partial x} = p_{xf} - f_{101x} p_{xf} - f_{011x} p_{yf} - f_{002x} - f_{001x}$$
$$p_y = -\frac{\partial F_3}{\partial x} = p_{yf} - f_{101y} p_{xf} - f_{011y} p_{yf} - f_{002y} - f_{001y} \tag{3}$$

The variable x_3 is to count the order of the vector potential. The f_{ijk} are functions of the local position variables (x, y, z) and have the expression as follow:

$$f_{001} = \int A_z dz \qquad f_{101} = \int (A_x + \int (\frac{\partial A_z}{\partial x}) dz') dz$$
$$f_{011} = \int (A_y + \int (\frac{\partial A_z}{\partial y}) dz') dz$$
$$f_{002} = -\frac{1}{2} \int [(A_x + \int (\frac{\partial A_z}{\partial x}) dz')^2 + (A_y + \int (\frac{\partial A_z}{\partial y}) dz')^2] dz \tag{4}$$

Implementing above implicit formulation Eq. (3), we can obtain the particle tracking results as shown in Figure 4 and Figure 5. From Figure 4, we can find the EPU has an impact on the horizontal displacement.

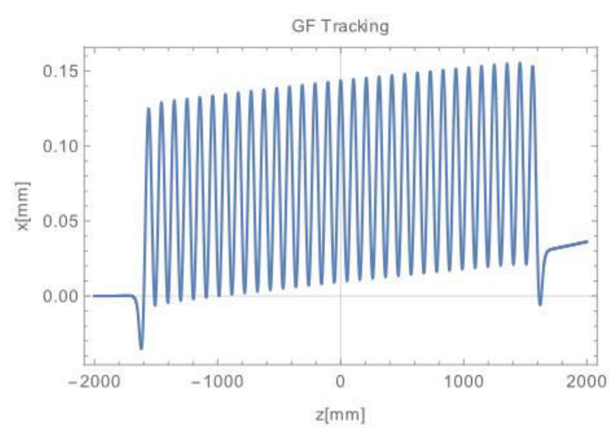

Figure 4: Trajectory of horizontal direction of 800-MeV electron with 2nd order GF tracking method.

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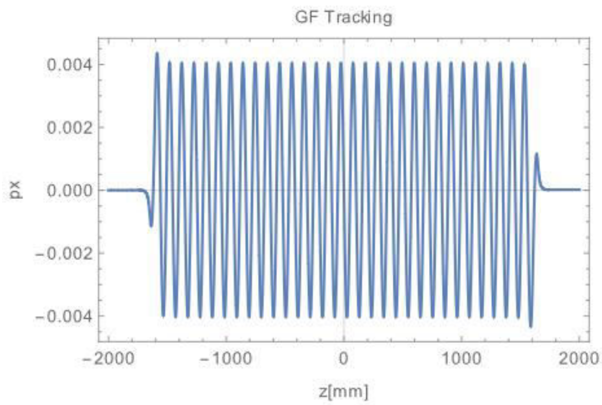

Figure 5: Normalized momentum trajectory of horizontal direction of 800-MeV electron with 2nd order GF tracking method

COMPENSATION WITH STRIPLINES

Eight striplines are distributed averagely and symmetrically above and below the vacuum chamber for beam dynamic compensation. The striplines can produce specific 3D magnetic field with different currents, which should be adjusted in accordance with the objective functions of the nonlinear effects of EPU104, such as magnetic field integrals and dynamic aperture. The stripline is $4\text{mm} \times 2\text{mm} \times 4000\text{mm}$ in volume, and the gap between striplines on the same horizontal plane is 1 mm.

Applying the tracking map, we could determine the optimal parameters for minimizing the integral field error. On the other hand, we could use the effective Hamiltonian derived from the surface fitting method to check the nonlinear effects with normal form method. The trajectories after the compensation with striplines are shown in Figure 6 and Figure 7. Figure 8 shows the normal form analysis of the whole ring with the surface fitting method.

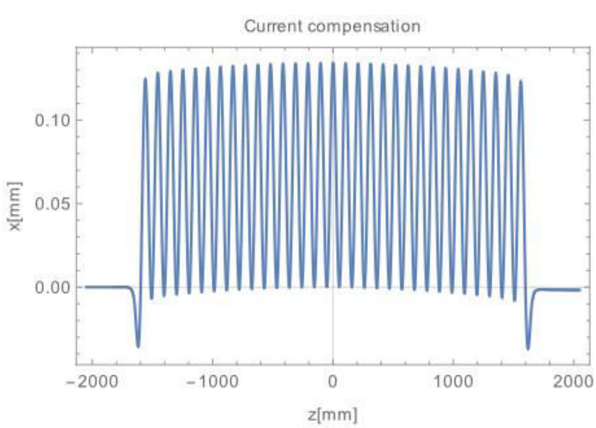

Figure 6: Normalized trajectory of horizontal direction of 800-MeV electron after current compensation.

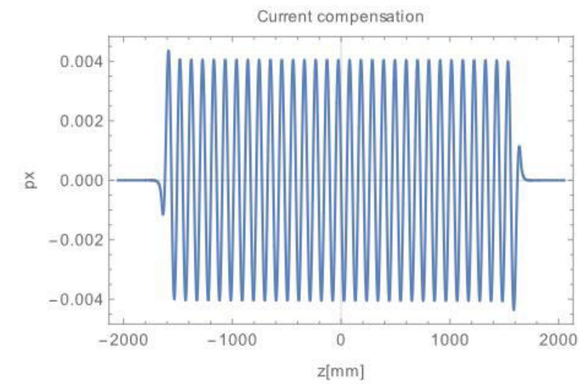

Figure 7: Normalized momentum trajectory of horizontal direction of 800-MeV electron after current compensation.

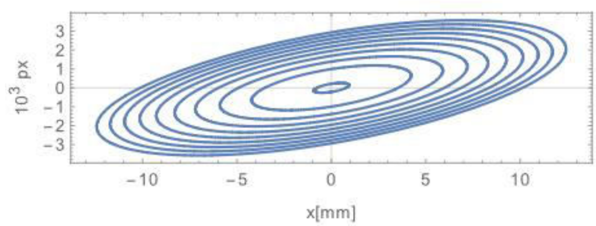

Figure 8: Normal form analysis of the whole ring after current compensation.

CONCLUSION

The APPLE II type magnetic field model described in this paper can give an analytic representation of magnetic field with enough accuracy. And the GF tracking method shows great advantages in calculation speed and symplecticity. The future work is mainly about the compensation of nonlinear effects of the EPU.

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