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CAVITY CHARACTERIZATION STUDIES WITH THE LATEST REVISION OF YACS*

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Abstract

YACS is a 2.5D finite element method solver capable of solving for the full 3D eigenfrequency spectra of resonant axisymmetric structures while reducing the computational problem to a 2D rotation plane. The most recent revision of YACS now supports arbitrary order basis functions for the geometry and field discretization. In earlier revisions of YACS spurious modes were introduced by increasing the order of either the geometry or field basis functions. To prevent the emergence of spurious modes, YACS now matches the function spaces of the in-plane and out-plane function bases, and thus yields spurious free solutions. To demonstrate the capabilities of YACS, extensive cavity characterization studies on curved multicell microwave cavities are presented. Due to the combined utilization of the rotation symmetry, higher order basis functions and curved elements, eigenfrequency spectra above 10 GHz for L-band multicell structures can be easily obtained.

INTRODUCTION

Earlier studies already demonstrated the superior convergence rates of YACS compared to modern commercial 3D solvers [1]. It was also reported that solving for eigenmodes with an azimuthal mode number $m \neq 0$, while utilizing higher order basis functions for the field or geometry discretization, introduces spurious modes. Observations since then suggested that this problem was related to the coupling of the out- and Nédélec-type in-plane field discretization function bases [2]. The coupling for $m \neq 0$ is expressed in the coupling block matrix $\underline{\underline{K}}_{ij}^{p\theta}$ within the stiffness matrix $\underline{\underline{K}}$ [3,4]

$$\underline{\underline{K}}_{ij}^{p\theta} = m\langle \mu_p^{-1} r^{-1} \underline{\boldsymbol{\phi}}_i, \underline{\boldsymbol{\nabla}}_p \psi_j \rangle_\Omega .$$

Where $\underline{\boldsymbol{\phi}}_i$ is the Nédélec-type in-plane basis function and $\underline{\boldsymbol{\nabla}}_p \psi_j$ the in-plane gradient of the out-plane basis function. Studies showed that spurious free solutions can only be obtained when both $\underline{\boldsymbol{\phi}}_i$ and $\underline{\boldsymbol{\nabla}}_p \psi_j$ reside within the same function space. As a consequence YACS now forces the out-plane basis to be of one order higher than the in-plane basis when solving for modes with azimuthal mode number $m \neq 0$. For monopole modes, i.e. $m = 0$, the orders for both bases can still be chosen arbitrarily.

CURVED ELEMENTS

Modern superconducting accelerating structures often utilize curved geometry parameterizations, e.g. elliptical or spline parameterizations [5], to counter field limiting effects like multipacting and field emission [6]. Numerical solvers that utilize higher order basis functions only for the field discretization and not for the geometry discretization, e.g. linear triangular elements, ultimately suffer from substantial geometry discretization errors. For low element counts this is easy to observe (Fig. 1).

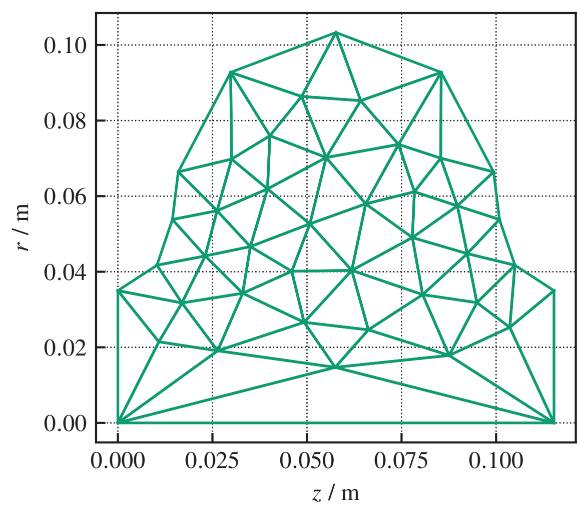

Figure 1: Finite elements of a TESLA-type mid-cell cavity triangulated with GMSH [7] using linear, i.e. first order, elements.

To reduce the geometry discretization error, one common approach is to use curvilinear coordinate transformations and thus curved finite elements (Fig. 2).

Usage Considerations

Using curvilinear or higher order elements does come with some drawbacks. First of all the evaluation of geometry related differential operators, like the Jacobian determinant or the curl and gradient, involve more complex mathematics, best described by tensor analysis [8], and thus more complex computations. One immediate consequence of this is that the aforementioned differential operators aren't constant within the individual elements anymore, and need to be evaluated at every quadrature point. In addition, the curvilinear coordinate transformations effectively increase the order of the field discretization basis, which in turn requires the usage of higher order quadrature rules. This also negatively affects the conditioning of the system matrices, since the orthogonal properties of the function bases can be voided by the transformation into curvilinear coordinate systems.

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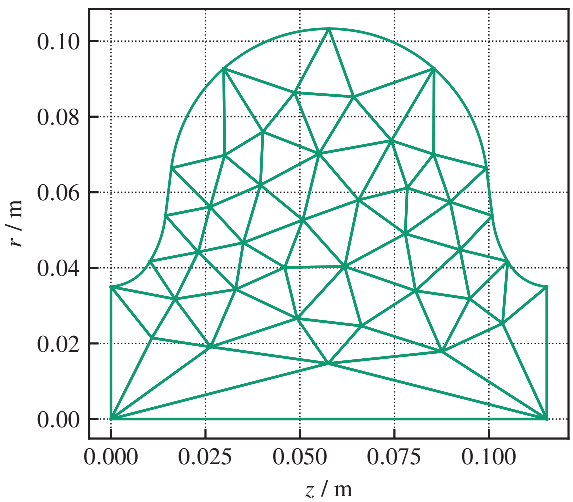

Figure 2: Finite elements of a TESLA-type mid-cell cavity triangulated with GMSH [7] using quadratic, i.e. second order, elements.

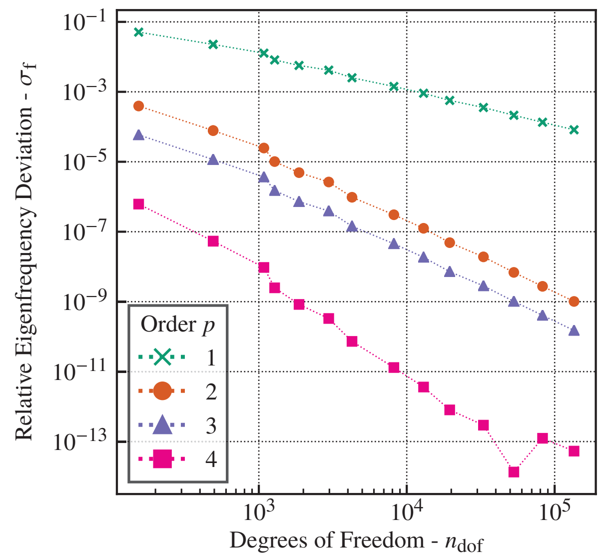

Figure 3: Eigenvalue deviation of the TM_{101}-Mode with respect to the analytical solution of a spherical cavity as a function of degrees of freedom for different geometry discretization orders.

One major benefit of curved elements, aside from vastly reducing the geometry discretization error, is the fact, that an increase in the order of the geometry basis functions does not increase the degrees of freedom of the corresponding generalized eigenvalue problem.

Despite the aforementioned drawbacks the introduction of curved elements can significantly increase the accuracy for a given number of degrees of freedom when approximating cavities that are parameterized by curved geometries. To demonstrate this, Figure 3 shows the obtained eigenvalue accuracies for the TM_{101} mode with respect to the analytical solutions of a spherical cavity as a function of the element order and the number of degrees of freedom. The differences between the achieved accuracies for a given number of degrees of freedom is very significant, ranging from $\approx 10^5$ to $\approx 10^9$ between the lowest and highest order.

CAVITY CHARACTERIZATION

The characterization of a given cavity geometry is a common task during cavity optimization. This typically involves the calculation of different important figures of merit and ideally some sort of perturbation analysis. Using 3D solvers, the exploration of the eigenmode spectrum is often limited to frequencies up to some low multiple of the fundamental frequency, thus losing insight into higher order modes that are especially dangerous for application of superconducting cavities in storage rings [9]. Furthermore the eigenmodes are often polluted by high azimuthal order multipole modes, that are often neglected in the characterization.

To cope with these problems, YACS can be used to elevate those boundaries substantially, enabling the exploration of eigenmodes up to and beyond 10 GHz for L-band multicell cavities.

Characterization of TESLA-Type Cavities

To demonstrate the capabilities of YACS two studies have been carried out with TESLA-type elliptical cavities [10]. The first study involves the evaluation of the longitudinal R/Q [11] for the first 80 monopole eigenmodes of a single mid-type cavity and for the first 720 monopole eigenmodes of a 9-cell mid-type cavity structure.[1] The results can be seen in Fig. 4 and 5.

For the second study, all geometry parameters of a single mid-cell cavity were varied to calculate the Jacobian matrix for different figures of merit (Fig. 6). This type of Jacobian matrix can be a powerful tool, not only for optimization algorithms but also for estimating the influence of production tolerances or geometry deforming transient effects like heat load or Lorentz force detuning.

CONCLUSION

With the most recent revision of YACS the emergence of spurious modes, when solving for eigenmodes with an azimuthal mode number $m \neq 0$ and utilizing higher order basis functions for the field or geometry discretization, could be successfully suppressed. Furthermore, studies on spherical cavities demonstrated a substantial gain in accuracy with respect to a fixed number of degrees of freedom, for cavities that are parameterized by curved geometries when using higher order elements.

To showcase potential areas of application of YACS, studies on TESLA-type cavities were performed up to frequencies of around 10 GHz. This regime is typically opaque to

[1] Since this study only aims to demonstrate the feasibility of calculating broad eigenfrequency spectra, the different geometry parameters for the end cells and the beam tube were omitted.

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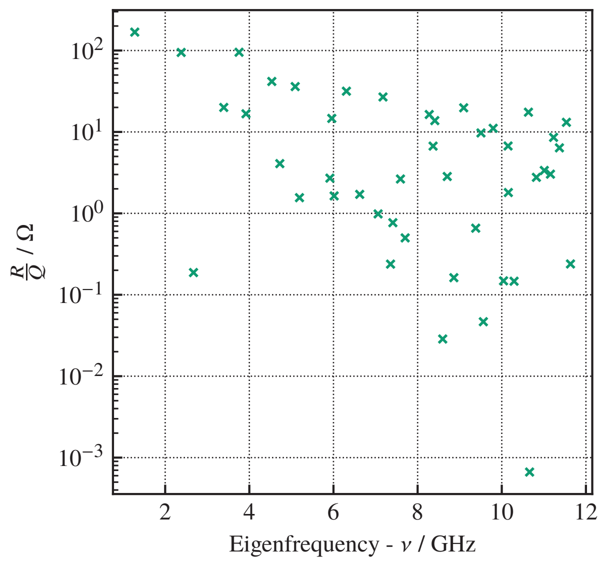

Figure 4: The longitudinal R/Q for the first $n = 80$ monopole eigenvalues of a single TESLA-type mid-cell cavity.

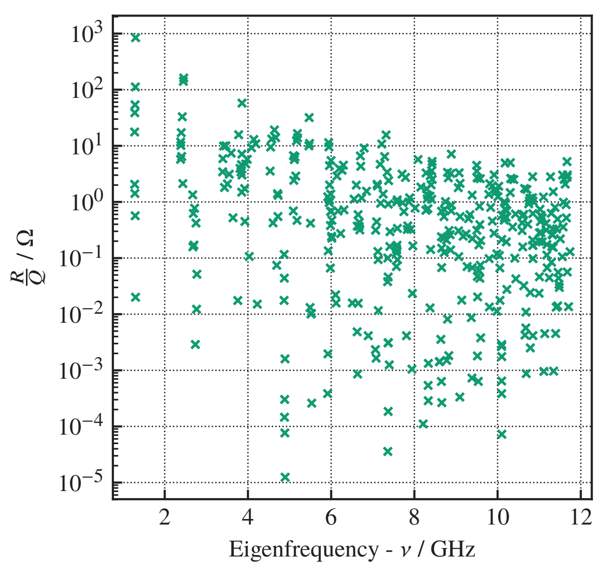

Figure 5: The longitudinal R/Q for the first $n = 720$ monopole eigenvalues of the complete (9-cell) TESLA-type cavity structure.

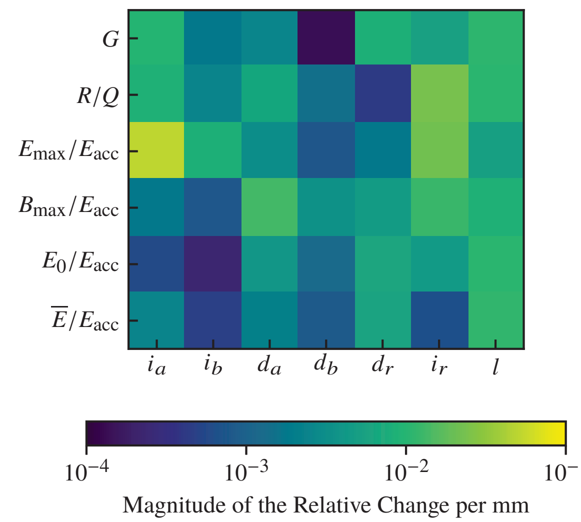

Figure 6: Jacobian of a TESLA-type mid-cell cavity with common figures of merit [11] as a function of the iris ellipse $i_{a/b/r}$, dome ellipse $d_{a/b/r}$ geometry parameters as well as the length of the cell l, where a resp. b do represent the semi-resp. major-half axes of the ellipses and r the r-coordinate of the ellipses centroids.

common commercial 3D solvers, but may otherwise be of importance for real world applications of those cavities, especially for superconducting multicell cavities aimed to be installed in storage rings.

The primary development goal for YACS is to provide a publicly available version as soon as possible. In addition, further enhancements and features are planned to be implemented including adaptive mesh refinement, lossy and perfectly matched layer boundaries. There is also an ongoing effort to convert the remaining Python code into more efficient high level compiled languages.

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