GPT-CSR: A NEW SIMULATION CODE FOR CSR EFFECTS

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Abstract

For future applications of high-brightness electron beams, including the design of next generation FEL's, correct simulation of Coherent Synchrotron Radiation (CSR) is essential as it potentially degrades beam quality to unacceptable levels. However, the long interaction lengths compared to the bunch length, numerical cancellation, and difficult 3D retardation conditions make accurate simulation of CSR effects notoriously difficult. To ease the computational burden, CSR codes often make severe simplifications such as an ultra-relativistic bunch travelling on a prescribed reference trajectory. Here we report on a new CSR model implemented in the General Particle Tracer (GPT) code that avoids most of the usual assumptions: It directly evaluates the Liénard–Wiechert potentials based on the stored history of the beam. It makes no assumptions about reference trajectories, and also takes into account the transverse size of the beam. Example results demonstrating normalised emittance growth in the first bunch compressor of FERMI@Elettra are presented.

INTRODUCTION

Here we add a new alternative to the list of simulation codes for CSR effects. Our version is implemented in the GPT [1] code, and we intentionally steered away from the approaches used by other codes such as Elegant [2, 3]. The goal we had in mind was not to create the 'best' CSR code per-se; we wanted to make a model that could answer the question what the consequences are of the assumptions made by the already existing codes. In order to do so, we created a code that is based on direct evaluation of the retarded Liénard–Wiechert potentials.

The main approximation used in this GPT element is that the bunch is modeled as a parametric curve, sliced in the direction of average propagation. This is typically a very good approximation because the dynamics of the underlying process are governed by the rest-frame properties of the bunch. In this frame, the bunch is elongated by the Lorentz factor compared to the length in the laboratory frame, resulting in a very narrow ribbon of charge. In this GPT model, the ribbon itself is allowed to meander through 3D space, capturing the largest amount of 3D effects possible within the parametric 1D framework. The chosen approach makes use of the fact that if all individual beam segments that have radiated in the past are stored, this information is sufficient to calculate the electromagnetic fields everywhere in 3D space, at any later point in time.

The result of the chosen approach is that changes in bunch shape between the point where radiation is emitted and the interaction point are properly taken into account.

Another advantage is that the usual ultra-relativistic assumptions are not needed. A conceptual simplification compared to some other codes is that the radiation is emitted based on the derivative of the momentum, regardless of the reason: A very strong electrostatic deflection field will give rise to the same emitted radiation as a bend magnet or a heavily misaligned quadrupole as long as the trajectories are the same. Furthermore, because the element is based on electromagnetic fields and not 'effects', radiation fields and the fields of other beamline components can all be correctly added. This allows for beamline components very close to the exit of a bend magnet, where a strong CSR wake can be travelling almost parallel to the bunch over long distances, to be properly simulated without degrading the accuracy of the CSR calculations. Finally, both the transverse and longitudinal fields are calculated, taking into account not only energy changes but also transverse effects that might vary along the bunch causing projected emittance growth.

The implementation of this element consists of two main parts: A history manager that maintains the history of the bunch, and a field calculator that integrates the radiation fields based on this stored history.

HISTORY MANAGER

The history manager of our new CSR code stores, at discrete time intervals, a 1D parameterization of the entire bunch, including basic information about transverse size. The first step is to switch to a new coordinate system, where **u** is the average direction of propagation, **v** lies in the bend-plane, and **w**=**u** x **v**.
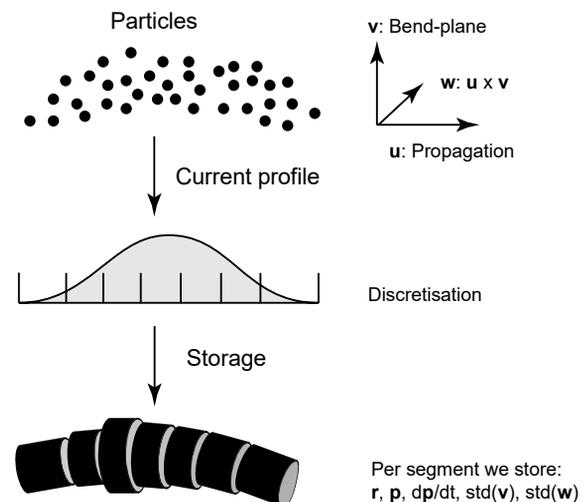
Figure 1: Schematic of the CSR history manager.

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In this frame, an estimate of the current profile is obtained by slicing in **u**, and this in turn defines the non-equidistant charge quantiles of the beam. The recipe is shown schematically in Figure 1.

For each segment the total charge, average position, average momentum, average acceleration and transverse beam sizes are stored. As we will see, this information is all that is needed to calculate the CSR field at any position, at any later point in time. Note that the segmentation is carried out afresh for each time step.

When running simulations with large Lorentz factors and relatively smooth fields, it is possible that the average momentum changes direction over angles (much) larger than $1/\gamma$ during one timestep. This may potentially result in cases where emitted radiation is partially missed due to relativistic beaming, also known as the headlight effect. In that case the history manager forces the Runge-Kutta integrator of GPT to reduce timesteps.

FIELD CALCULATION

At each Runge-Kutta substep in the GPT simulations, the electromagnetic radiation emitted in the past is reconstructed at the current bunch position, as schematically shown in Figure 2. The first step in this process is making a 1D parameterization of the bunch based on non-equidistant charge quantiles of the beam, fully analogous to the history manager. Subsequently, the centers $\mathbf{r}_i$ of each slice i are calculated, thereby avoiding the need for a reference path or trajectory and allowing significant differences in trajectories between the head and the tail of the bunch.

The crucial step in the CSR field calculation is calculating the actual electromagnetic **E** and **B** fields at the current timestep at each of these $\mathbf{r}_i$, by summing the Liénard-Wiechert potentials [4] for all segments $\mathbf{r}_s$ that fulfil the retardation condition $|\mathbf{r}-\mathbf{r}_s|=c\,\Delta t$:

$$\mathbf{E}(\mathbf{r},t) = \sum_{segments} \frac{q_s}{4\pi\varepsilon_0}\left(\mathrm{K} + \frac{\mathbf{n}\times\left((\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right)}{c(1-\mathbf{n}\cdot\boldsymbol{\beta})^3\sqrt{|\mathbf{r}-\mathbf{r}_s|^2+R^2}}\right)_{tr}$$

$$\mathbf{B}(\mathbf{r},t) = \sum_{segments} \frac{\mathbf{n}(t_r)}{c}\times\mathbf{E}(\mathbf{r},t)$$

where

$$\mathbf{n}(t) = \frac{\mathbf{r}-\mathbf{r}_s(t)}{|\mathbf{r}-\mathbf{r}_s(t)|}$$

with $\beta=v/c$, and where q_s is the charge of the segment and where Δt is the time difference between the current timestep and the timestamp of the stored profile. This calculation is always carried out in the global coordinate system. When only a fraction of a stored segment fulfils the retardation condition, only the corresponding fraction of the charge in that segment is used in the equations. The … in the given equation denotes the Coulomb term, this will be discussed later.

In order to include transverse effects, we model each emitting slice by four (or sixteen) independent points, located at plus or minus one standard deviation off-axis. This gives us 4 (or 16) times more emission points $\mathbf{r}_s$ than segments, and because the bunch travels on a curved trajectory

the retardation conditions for these points have to be individually evaluated. We observed that with only four points we are able to capture relevant 3D effects at a relatively low price and increasing this number to 16 did not show any significant differences for our initial test-cases.

The evaluated fields are singular when evaluated at $\mathbf{r}=\mathbf{r}_s$, but under normal circumstances this should not happen since a) the receiver's position **r** is on-axis and the emitting positions $\mathbf{r}_s$ are off-axis, and b) the bunch has moved forward since the last stored profile. Nevertheless, we add a tiny R in the denominator, analogous to the concept of Plummer spheres commonly used in the astrophysical community, as safeguard.

The main challenge writing this CSR model was to prevent interference problems between consecutive interpolations steps. This problem was solved by choosing low-order interpolations to prevent undesired oscillations and using quantiles instead of equidistant arrays to prevent artificial spikes in the Fourier transformed solution. What remains is the challenge that the integrand, tracing 'vertically' back in time, always has two large peaks of opposite sign that almost cancel when integrated. That, however, is the nature of CSR interactions when working directly with the LW-potentials.
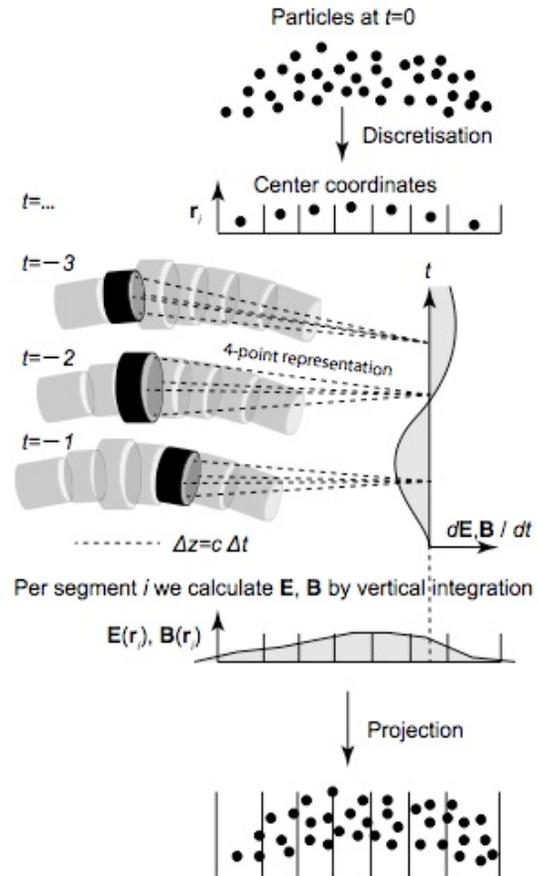

Figure 2: Schematic of field integration.
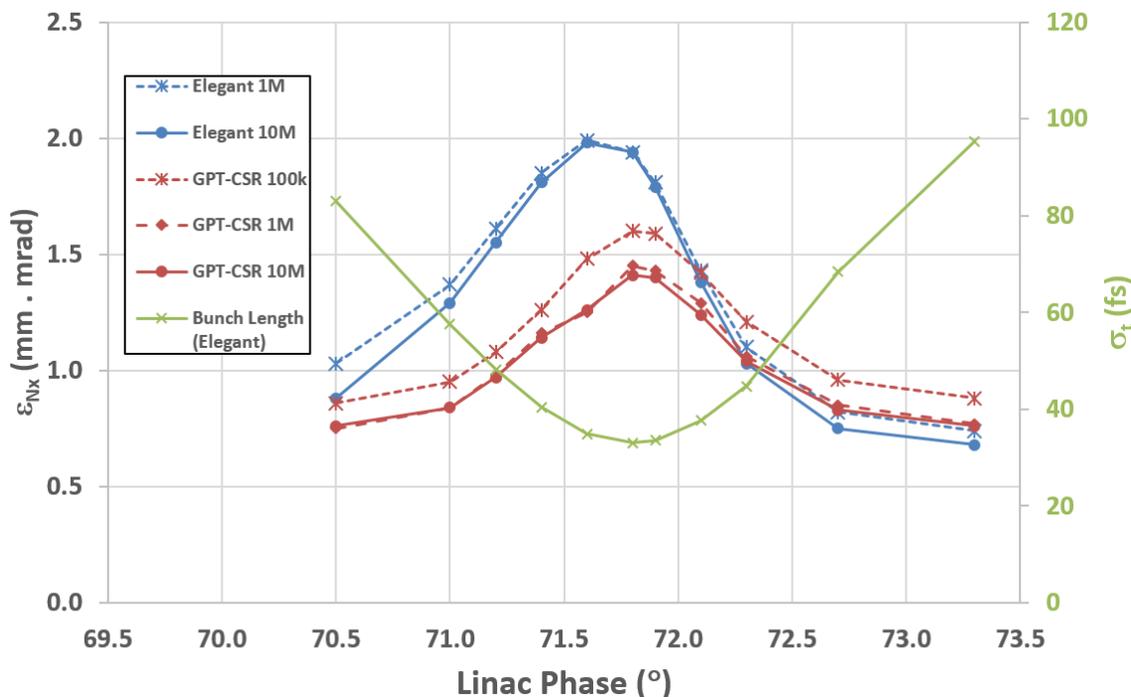
Figure 3: Example convergence study in Elegant and GPT-CSR for normalised emittance after the first bunch compressor of FERMI as a function of preceding linac phase. The energy at this position is 300 MeV, the bunch charge is 100 pC and the R_{56} of the bunch compressor is fixed at 57 mm.

LIMITATIONS

Every simulation code has "sweet spots" in parameter space where it works best, grey areas preferably to be avoided, and no-go zones. In this section we describe the limitations of our code, and offer potential remedies and outlooks where applicable.

Slicing the bunch in the average direction of propagation is problematic in the case of roll-over compression. However, because the slicing problem only happens during very short timescales we are at this point not able to judge how severe the integrated error actually is.

The current implementation includes 3D effects for the emission process by having off-axis emission points. An analogous scheme is however not implemented for the receiving part, resulting in CSR fields that are independent of the transverse coordinate of the receiving particle.

Adding the Coulomb term of the LW potentials to the interaction results in a r^{-2} singularity that sometimes causes numerical instabilities for bunches with very small transverse dimensions.

The current version does not allow for shielding effects. It is foreseen that this will be added using mirror bunches, at the expense of required computing power.

Due to relativistic beaming, the implemented method is not a good match for multi-GeV beams and large deflection angles. It will work, but it will be slow.

TEST-CASE

In preparation for planned experimental time on FERMI@Elettra, convergence studies was performed using GPT-CSR and Elegant. Figure 3 shows one of these, namely a determination of the normalized emittance after the first bunch compressor as a function of preceding linac phase. It shows significant difference in the predicted magnitude of CSR driven emittance growth in comparison with the 1-d model used in Elegant. There is also a shift in the peak to a phase that more closely matches the position of the minimum bunch length. These predictions will be compared to measurements from FERMI in a forthcoming article.

CONCLUSION

We created a new CSR model in the GPT code, based on evaluating retarded Liénard-Wiechert potentials from-first-principles. The model does not assume a reference trajectory, allows the head and tail of the bunch to be on different tracks, takes into account the transverse size of the bunch for the emission process, makes no ultrarelativisitc approximations, and correctly takes into account changes in the current profile between emission and interaction with emitted radiation.

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