ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPAK057

SIMULATIONS OF OPTICAL STOCHASTIC COOLING WITH ELEGANT*

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Abstract

Fermilab is pursuing a proof-of-principle test of the Optical Stochastic Cooling (OSC) of 100 MeV electrons in the Integrable Optics Test Accelerator. In support of this we present simulations of horizontal damping with OSC. We find excellent agreement with theory on the amplitude dependent damping rates. Additionally particle tracking is used to confirm the necessity and effectiveness of sextupoles used to correct non-linear path lengthening in the OSC chicane.

INTRODUCTION

The Optical Stochastic Cooling (OSC) is a beam cooling method that holds promise for the cooling of dense particle beams. It is similar in concept to the widely used stochastic cooling with microwaves but by utilizing optical frequencies enables cooling to be done with bandwidth 3-4 orders of magnitude larger than any current stochastic cooling system [1] [2]. At optimum gain, this increase in bandwidth reduces damping time by that same amount. The transition to optical frequencies is done by replacing the traditional 'pickup' and 'kicker' with two identical undulators. A particle radiates a short wave-pulse in the pickup and then, by interacting with its own radiation in the kicker, receives a corrective kick. A magnetic chicane between the undulators simultaneously gives space for the required focusing optics and optical amplifier while also providing the appropriate delay (with respect to the particles wave-packet) necessary for cooling(see Fig.1). Currently, Fermilab is pursuing a proof-of-principle test of the OSC using 100 MeV electrons in the Integrable Optics Test Accelerator (IOTA) [3]. The expected undulator and beam parameters are given in Table 1.

The energy kick, δu, a particle will receive in the kicker is determined by the particles longitudinal displacement, s, relative to the reference particle upon traveling between undulator centers

$$\frac{\delta u}{U_s} = \kappa \sin(k_o s), \tag{1}$$

where κ is the kick amplitude normalized by particles energy U_s, and $k_o \equiv 2\pi/\lambda_o$ where λ_o is the undulator-radiation on-axis wavelength. Although the kick is purely in energy, coupling between longitudinal and horizontal planes in the cooling insertion results in horizontal cooling. In this paper

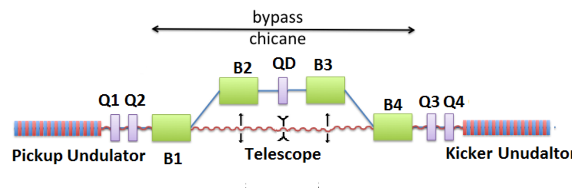

Figure 1: Overview of the OSC beamline (the beam propagates from left to right). The letters Bi and Qj respectively refer to quadrupole and dipole magnets.

we report on simulations of the OSC in the IOTA ring using the particle tracking code ELEGANT [4] and focus on the horizontal phase-space damping.

Table 1: Undulator Parameters for the OSC Test in IOTA with 100 MeV Electrons

Parameter	Value	Unit
undulator parameter, K	1.038	-
length, L_u	77.4	cm
undulator period, λ_u	11.06	cm
number of periods, N_u	7	-
on-axis wavelength, λ_o	2.2	μm
electron Lorentz factor, γ	195.69	-

HORIZONTAL COOLING RATE

The horizontal damping rate from the OSC can be found in a very similar way as that of damping of betatron oscillations from the emission of synchrotron radiation [5]. Coupling between longitudinal and horizontal planes occurs where dispersion in the ring is non-zero. In this case the particles position is given as $x = x_\beta + (u/U_s)D$ and its angle by $x' = x'_\beta + (u/U_s)D'$. Here the β subscript is used to denote displacement from betatron motion, D and D' is dispersion and its derivative and u is the particles difference in energy with respect to the reference particle. The longitudinal displacement of the particle due to its horizontal coordinate in the pickup center is

$$s = M_{51}x + M_{52}x' \tag{2}$$

where M_{5n} are the elements of the transfer matrix from pickup to kicker centers. At first lets assume $k_o s \ll 1$ so that Eq. 1 can be Taylor expanded. During the kick the particles position and angle do not change appreciably, and yet clearly

* Work supported by the US Department of Energy (DOE) contract DE-SC0013761 to Northern Illinois University. Fermilab is operated by the Fermi research alliance LLC under US DOE contract DE-AC02-07CH11359.

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its energy has; hence there must be a corresponding change to x_β and x'_β:

$$\begin{aligned}\Delta x_\beta &= -\kappa D M_{51} k_o x \\ \Delta x'_\beta &= -\kappa D' M_{52} k_o x'.\end{aligned} \tag{3}$$

From these equations a corresponding decrease in the particles Courant-Synder invariant, $\epsilon = \beta x'^2_\beta + \alpha x_\beta x'_\beta + (1 + \alpha^2) x^2_\beta/\beta$ is found. Dividing this change by the revolution period τ_s, and averaging over betatron oscillations yields

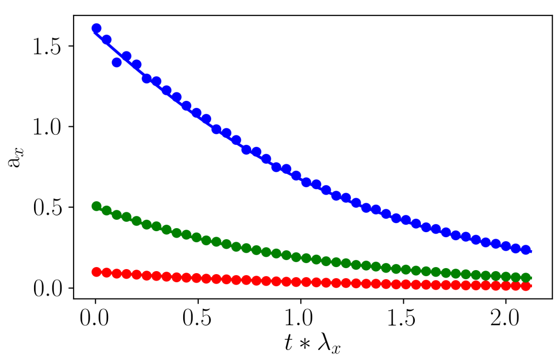
$$\frac{d\epsilon}{dt} = \kappa \frac{k_o}{2\tau_s}(M_{51} D + M_{52} D')\epsilon. \tag{4}$$

and so $\epsilon(t) = \epsilon_o \exp(-t\lambda_x)$ where λ_x is the horizontal damping rate:

$$\lambda_x = \kappa \frac{k_o}{2\tau_s}(M_{51} D + M_{52} D'). \tag{5}$$

For the more general case of arbitrary particle amplitude we re-express Eq. 1 as

$$\frac{\delta u}{U_s} = \kappa \sin\left[a_x sin(\psi_x)\right] \tag{6}$$



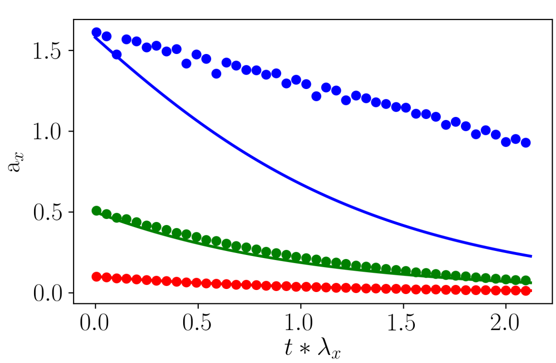


Figure 2: The amplitude of longitudinal displacement $a_x(t)$ for different initial amplitudes. Dots are computed from ELEGANT outputs while solid lines are from numeric integration of Eq. 9. The top pane assumes no separation occurs between particle and radiation while the bottom pane accounts separation for the case of the +**I** telescope.

where ψ_x is the betatron phase and a_x is the amplitude of the particles longitudinal displacement over the course of a betatron oscillation expressed in units of phase of the undulator central wavelength [6]:

$$a_x = k_o \sqrt{\epsilon\left(\beta M_{51}^2 - 2\alpha M_{51} M_{52} + (1+\alpha^2) M_{52}^2/\beta\right)}. \tag{7}$$

The damping time is long compared to the period of the betatron oscillation and so the cooling rate can be averaged over the period yielding the amplitude dependent damping rate:

$$\lambda_x(a_x) = 2\lambda_x J_1(a_x)/a_x \tag{8}$$

and the rate of change of a_x is

$$\frac{da_x}{dt} = -2 J_1(a_x)\lambda_x. \tag{9}$$

In ELEGANT the kick is computed by recording the arrival

Table 2: IOTA Parameters Extracted From or Piven to ELEGANT

Parameter	Value	Unit
Revolution period, τ_s	133	ns
emittance (without cooling), ϵ_σ	2.67	nm
Normalized Kick Amplitude, κ	2.0×10^{-6}	-
OSC damping time, $1/\lambda_x$	22	μs
M_{51}	0.6×10^{-4}	-
M_{52}	4.0	mm
M_{56}	3.8	mm

times of the particle in the pickup and kicker, t_p and t_k respectively. During the first pass through the insertion the average time difference $\langle t_k - t_p \rangle$ of the bunch is computed and used to compute the arrival phase of the reference particle. In subsequent passes through the cooling insertion a particles time of flight $t_k - t_p$ is used to find s and compute the corresponding change in the particles momentum. The top pane in Fig. 2 shows the damping of $a_x(t)$ as computed from ELEGANT using particle positions and angles and Eq. 7. It is in excellent agreement with the damping curved obtained from numerical integration of Eq. 9. Table 2 summarizes lattice and cooling parameters used in simulations[1].

Our model neglects kicks by neighboring particles since we are interested in the dynamics of a single classical particle in the accelerator. This choice is further justified considering that the OSC test in IOTA will first be done passively where the kick amplitude is far from the optimal gain when the incoherent kicks from neighboring particles become non-negligible.

We can account for separation that occurs between the particle and its radiation for a particle that is off of the optical axis. This is particularly important in active OSC. In order to suppress depth of the field effects associated with a finite

[1] In our simulations κ and consequently the OSC damping time can be set arbitrarily. The values given in the Table 2 were chosen to limit the number of turns needed to observe damping. The OSC damping rate in IOTA is expected to be more than three orders of magnitude slower.

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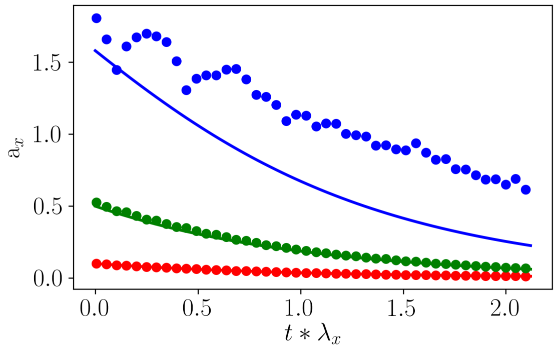

Figure 3: Same setup as Figure 2 but without sextupole to correct the non-linear path lengthening. Separation between light and particle is not accounted.

undulator length the focusing light optics are made to be a telescope with a transfer matrix $\pm\mathbf{I}$ where $\mathbf{I}$ is the identity matrix. For an active test of the OSC in IOTA the positive case must be chosen so that radiation from the pickup can be focused to a reasonable size in the OA crystal making the pump laser power and associated heating manageable. And so if a particle radiates at position x_p the center of its imaged radiation will appear at x_p in the kicker. However the beam optics have negative diagonal elements in the horizontal plane. Approximating the beam optics transfer matrix as $-\mathbf{I}$ then implies the separation between the particle and light is $\Delta x_l = 2x_p$. At the pickup center $\beta = 5.7$ and so a particle with its Courant-Synder invariant equal to the beam emittance will be displaced (at its maximum displacement in x over the course of its betraton oscillation) 240 μm. Using formulas given in [6] the undulator radiation spot size (from center to zero) in the kicker is 590 μm resulting in an 30% reduction in the field amplitude. The bottom pane of Fig. 2 shows the damping $a_x(t)$ assuming a $+\mathbf{I}$ telescope. The particles coordinates in pickup and kicker locations were used directly to compute its separation from the field. As expected particles with large amplitudes are affected most.

NON-LINEAR PARTICLE MOTION

For the case of OSC in IOTA an analysis of the beam optics yielded that the horizontal cooling range $n_x = \sqrt{\epsilon_{max}/\epsilon_\sigma}$ to be[2]

$$n_x = \frac{\mu_{0,1}}{2k_o\Delta s}\sqrt{\frac{D^{*2}}{\epsilon_\sigma\beta^*}}. \tag{10}$$

where β^* and D^* are denoting value at the chicane center and $\Delta s \approx M_{56}/2$.

Additionally a higher order contribution to s was identified

[2] $\mu_{0,1} \approx 2.405$ is the first zero of $J_0(x)$. For the case that the longitudinal beam emittance has been set to zero, as is being considered in this paper, $\mu_{0,1}$ should be replaced with $\mu_{1,1} \approx 3.832$.

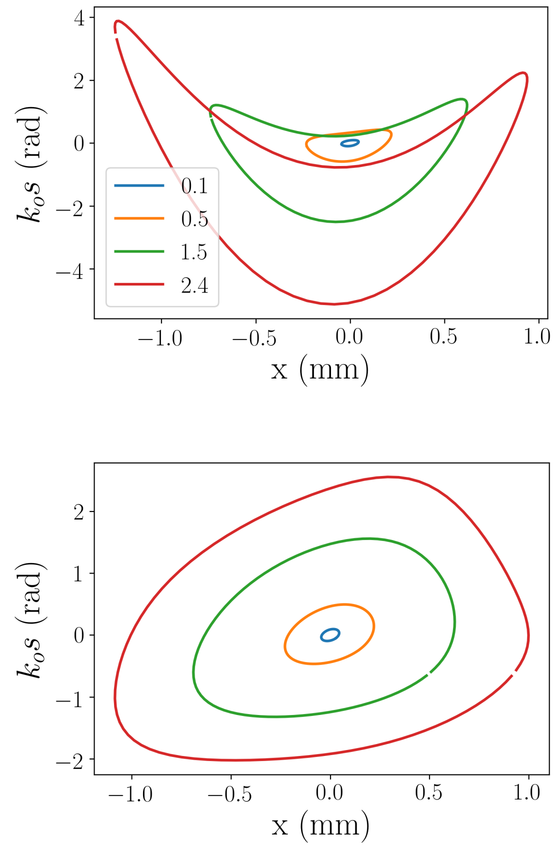

Figure 4: Path lengthening from pickup to kicker centers as dependent on the particles position in the pickup. The contours are for different a_x value with sextupole magnets turned off (top plot) and on (lower plot).

as coming from the particles transverse angles

$$\Delta s_2 = \frac{1}{2}\int \left(x'(z)^2 + y'(z)^2\right)dz. \tag{11}$$

An attempt to optimize the cooling range by minimizing β^* exacerbates the non-linear path lengthening by increasing the particles angle. To remedy this two-pairs of sextupoles, one pair placed between the first and second dipole and the other between the 3rd and 4th dipole of the chicane are implemented. The effect of the nonlinear path lengthening as dependent on the particles horizontal position is shown in the top pane of Figure 4. In the absence of non-linear motion particles with the same a_x should form ellipses. In the bottom pane we see the sextupoles are able to restore the ellipse up to large amplitudes. Finally in Fig. 3 the effect of the non-linear path lengthening on the damping rate can be seen. Again particles with large amplitudes are most affected. Note the that the plots in Fig. 2 include non-linear motion and sextupole corrections.

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CONCLUSION

Using ELEGANT we were able to simulate horizontal cooling with OSC in the IOTA ring. We find good agreement with the damping rates predicted in theory and the ones computed here.

ACKNOWLEDGMENTS

P.P. thanks M. Borland (Argonne National Lab.) for his suggestions and useful discussions.

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