A NEW METHOD TO MEASURE THE BETA FUNCTION IN A PAUL TRAP

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Abstract

The Simulator of Particle Orbit Dynamics (S-POD) is a linear Paul trap at Hiroshima University, Japan, used to study beam physics. S-POD has so far been used to study resonances in high intensity beams, predominantly using a simple alternating gradient lattice configuration. Recently a similar apparatus, the Intense Beam Experiment (IBEX), has been constructed at the Rutherford Appleton Lab in the UK. To use either of these experiments to study beam dynamics in more complex lattice configurations in the future further diagnostic techniques must be developed for Paul traps. Here we describe a new method to measure the beta function and emittance at a given time in a Paul trap.

INTRODUCTION

The Simulator of Particle Orbital Dynamics (S-POD), at Hiroshima University, Japan, is a linear Paul trap (LPT) which confines ions in an rf electric quadrupole potential. It has been shown that the Hamiltonian of ions in the laboratory frame of the Paul trap is equivalent to that of particles moving in a high intensity beam traveling through an alternating gradient lattice, so that transverse motion in the two systems is equivalent [1]. This allows a LPT to simulate the transverse dynamics of intense particle beams.

One current limitation of the use of LPTs in accelerator physics is the lack of beam diagnostics for the trapped ions. To use LPTs to study more complex accelerator physics problems it is necessary to measure the same parameters that are measured in accelerators. This paper explores a new method to extract the emittance and beta function experimentally at a given time in the S-POD LPT. We show that the measurement method allows the beta function at any point in time to be extracted, equivalent to measuring the beta function at any point, s, in an accelerator.

S-POD: A LINEAR PAUL TRAP

In a LPT ions are confined to a small region in space by an electrical quadrupole potential. The potential is created by applying an alternating voltage to four cylindrical trapping rods at a suitable rf frequency. The configuration of these trapping rods is shown in Fig. 1 and Fig. 2. The alternating voltage creates a potential between the rods of the form

$$U = \frac{V_{rf}(t)}{2r_0^2}(x^2 - y^2),\qquad(1)$$

where r_0 is the radial distance from the axis to the rods and V_{rf} the peak to peak amplitude of the applied voltage [2].

The overall motion of ions trapped in such a time dependant potential is characterised by a secular motion, equivalent to betatron oscillations in an accelerator, as the ions move in the pseudo potential created by the trap. Detailed descriptions of the equivalence of the LPT and accelerator can be found in [1], [3] and [4].

In S-POD argon gas is introduced into the vessel, which is otherwise in ultra high vacuum. An electron gun points downwards into the trapping region, ionising argon atoms between the confining rods.

Figure 1 shows the endcaps (End A, End B and Gate). These shorter sections of cylindrical rod are not electrically connected to the central rods (IS). The endcaps are held at a positive DC potential to confine the ions longitudinally and, owing to the relatively small inscribed radius compared to the length of the rods in the confining section, the ion distribution can be assumed to be longitudinally homogeneous.

To study the dynamics of the trapped ions two detectors are currently used, a Faraday cup (FC) and a Micro-Channel Plate (MCP), which collect the ions as they leave the trap. When the DC potential on the end caps at one end of the trap (End A or Gate) is dropped to zero the ions are no longer confined longitudinally. Ions exit the trap towards either the FC or the MCP (see Fig. 1). Extracted ions produce a current on the FC and from this current the number of ions that hit the FC can be inferred. Further details on the experimental setup of the S-POD LPT can be found in [1] and [5].
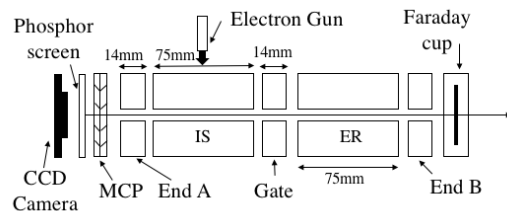

Figure 1: Schematic side view of the S-POD Paul trap with dimensions. Adapted from [1].

So far in S-POD experiments the FC has been used to study beam loss near resonance. Near a resonance beam loss leads to fewer ions remaining in the trap and so fewer are extracted. The MCP has also been used to image the extracted ions in a number of experiments [6]. The problem with both of these measurements is that they obtain a

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time averaged image of the trap, so that calculating the beta function and emittance is challenging.

BETA FUNCTION MEASUREMENT

We have developed a new method to measure the beta function and emittance at a specific point in time in the S-POD LPT by applying a dipole kick to the trapped ions.

In accelerators a dipole kick causes centroid oscillation about the equilibrium orbit [7]. The same is therefore true in a LPT, where the oscillations are no longer a function of distance, s, but of time, t.

When a single kick is applied, the equation describing the resulting transverse motion is

$$x_2 = \theta\sqrt{\beta_1\beta_2}\sin(\phi_{12}), \tag{2}$$

where x_2 is transverse coordinate of the centre of the trapped ions at t_2 (a point in time after the dipole kick has been applied) β_1 is the beta function at the time of kick (t_1) and ϕ_{12} the betatron phase difference between the two points.

The angular deflection, θ, of the trapped ions is given by

$$\theta = \frac{-q\zeta}{2mc^2 r}V_D ct, \tag{3}$$

where V_D is the applied dipole voltage and t is the length of time that it is applied for. ζ is a factor to take into account the geometry of the rods, in S-POD $\zeta = 0.795$.

The transverse trajectory of the ions therefore depends on the strength of the dipole kick applied and the beta function at the point where the kick is applied.

To use the FC to measure the beta function through orbit distortion the ions must move enough transversely to scrape on the rf rods. Some ions will be lost and fewer will be extracted from the trap. Applying the dipole perturbation at different points in the confining potential leads to a varying transverse displacement and therefore varying beam loss, allowing a beta function measurement. To do this a gaussian beam is assumed, this has been validated previously in [8].

Experimental Limitations

In this experiment we apply the main rf sinusoidal voltage to one pair of quadrupole rods (at twice the voltage amplitude required if both pairs are powered) and apply the dipole kick to the remaining pair (Fig. 2).

The simplest method of generating the dipole kick is to directly output it from a wavefunction generator. In an ideal experiment the dipole kick applied to the beam would be a delta function, this is impossible, instead the pulse will have a finite width based on the rise and fall time of the wave function generator (15 ns). The maximum kick possible using this method is 10 V.

However, in normal S-POD operation (applying a voltage between 10 V and 100 V at a frequency of $\sim$ 1 MHz) a beta function on the order of 300 m is expected. Using Eq. (2) and Eq. (3) and assuming a dipole kick of length 100 ns (short enough to achieve good resolution in the reconstruction and long enough that the kick strength can be reduced) a voltage
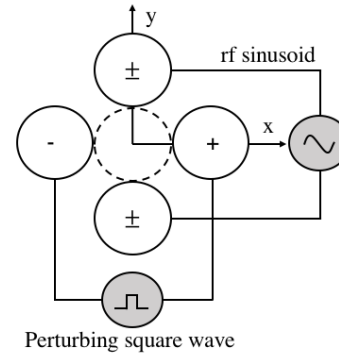

Figure 2: Schematic end view of SPOD rods showing the voltages applied in the beta function measurement.

of $\sim$ 250 V is required to kick the ions to such an extent that all the ions are lost.

We applied a series of dipole kicks to the ions. If the kicks are applied in phase the strength of the kicks add linearly, producing the same effect as a single kick at a larger voltage. To be able to provide a series of dipole kicks at both the same phase advance and beta function the cell tune of the trap must be $\frac{1}{n}$, where n is an integer. As a proof of principle experiment we used alternating gradient lattices with cell tunes of $\frac{1}{5}$, $\frac{1}{7}$ and $\frac{1}{9}$ and a series of 25 dipole kicks.

MEASUREMENT METHOD

When converting extracted ion number into a value for the beta function the ion loss in phase space must be considered. In the case of an integer tune, or a tune of $\frac{1}{n}$, where n is an integer, the phase space rotates about the origin in such a way that the same side of the phase space distribution continuously collides with the confining rods.

Accounting for this asymmetric loss in the phase space, the error function was used to calculate the fraction of ions lost. This method has been previously used and shown to match closely the measured ion loss [8]. After applying a dipole kick the fraction of ions remaining is

$$\xi \approx \frac{1}{2} + \frac{1}{2}\mathrm{erf}(\frac{r_0 - x_{max}}{\sqrt{2}\sigma_x}), \tag{4}$$

where x_{max} is the maximum transverse position of the centre of the trapped ions, σ_x their standard deviation and erf the error function.

In the experiment three alternating gradient lattices were investigated, with tunes of $\frac{1}{5}$, $\frac{1}{7}$ and $\frac{1}{9}$. For each of the three lattices the number of ions extracted was recorded when 25 dipole kicks were applied in phase and at the same time (t_1) in an rf period. The process was repeated at a number of different values of t_1. At each t_1 the number of ions extracted was recorded as the dipole perturbation voltage was varied between 0 V and 10 V.

The experiment was conducted at an ion number of 10^6. Ideally the experiment would be conducted at a lower ion number, where space charge effects are not present, however,

Table 1: The Results of Fitting Eq. (6) to the Curves of Kick Strength against Beam Loss.

Cell tune	Emittance (mm mrad)	β_{max}(m)
$\frac{1}{5}$	3.79E-3 ± 0.19E-3	451 ±6
$\frac{1}{7}$	6.14E-3 ± 0.17E-3	698 ±14
$\frac{1}{9}$	5.05E-3 ± 0.52E-3	621 ±17

at lower ion numbers the signal to noise ratio was found to be too small for an accurate beta function reconstruction. The storage time of the ions before extraction from the trap was set to 1 ms.

RESULTS

By substituting Eq. (2) and Eq. (3) into Eq. (4) and noting that for maximal ion loss $\sin(\phi_{12}) = \pm 1$, so that $x_{max} = \theta\beta_1\beta_{max}$, the fraction of ions remaining is found to be

$$\xi \approx \frac{1}{2} + \frac{1}{2}\mathrm{erf}(\frac{r_0 - \theta\sqrt{\beta_1\beta_{max}}}{\sqrt{2\epsilon\beta_{max}}}). \qquad (5)$$

In the special case where the dipole kick is at the maximum beta function ($\beta_1 = \beta_{max}$) this reduces to

$$\xi \approx \frac{1}{2} + \frac{1}{2}\mathrm{erf}(\frac{r_0 - \theta\beta_{max}}{\sqrt{2\epsilon\beta_{max}}}). \qquad (6)$$

Firstly, the data from the dipole kicks at the value of t_1 which resulted in the greatest ion loss was fitted to using Eq. (6). Figure 3 shows an example of one of these fits, where β_{max} and ϵ are the fitting parameters. The values found for β_{max} and ϵ are shown in Table 1.

Using these values of β_{max} and ϵ the ion loss curves from different kick locations can be fitted to with Eq. (5), extracting values for β_1 from each curve.

The result of this fitting for the three lattices that were studied is shown in Fig. 4.
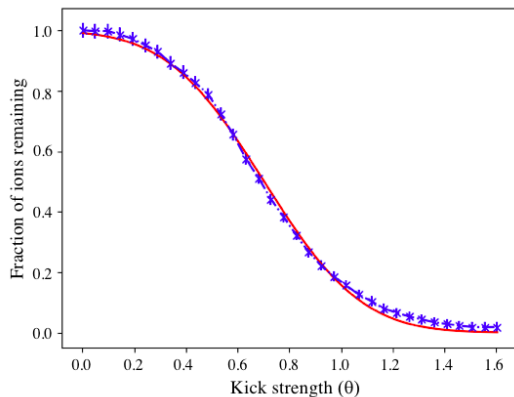

Figure 3: Fit to the curve showing maximum ion loss. The blue points are experimental data points. The red curve is the fit to the data, weighed for the errors.
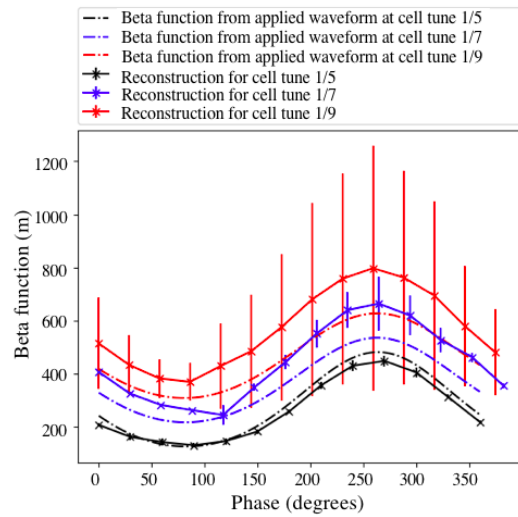

Figure 4: Comparison of reconstructed beta function for tunes $\frac{1}{5}$, $\frac{1}{7}$ and $\frac{1}{9}$ with the beta functions expected from the applied waveforms. Error bars are calculated from the error in the fits to the curves of kick strength against ion loss (Fig. 3).

DISCUSSION

Figure 4 shows the reconstructed beta functions with error bars, in comparison with the beta functions associated with the applied rf waveforms, where the optics of the periodic cell was calculated taking into account the transfer matrix associated with each voltage datum in the waveform. This shows that the reconstruction for a cell tune of $\frac{1}{5}$ worked very well, with the reconstructions diverging from the true beta functions with increasing size of n.

This suggests that the deviation may be due to de-phasing between kicks or detuning of the ions due to nonlinear fields at large transverse amplitudes. For the $n = 5$ case the 25 kicks can be accomplished in 125 rf periods. However, for the case of $n = 7$ this is extended to 175 rf periods and for $n = 9$, 225. These longer kicking times will amplify any error in the kick timings, resulting in kicks that are not precisely in phase. For these out of phase kicks the kick strength no longer adds linearly and the total kick felt by the ions will be reduced, this results in the upwards shift of the reconstructed beta functions in the $n = 7$ and $n = 9$ cases.

The reconstruction of the beta function at a tune of $\frac{1}{5}$ strongly suggests that if a larger single voltage kick is available then this method is capable of reconstructing any beta function. The study also suggests that voltage does not need to be so high that the ions are lost completely, successful fitting occurred even when only 20% of the ions were lost.

If a large enough voltage can be applied to the LPT in a single kick then this method can be used to extract the beta function and emittance from any lattice, not only when n is an integer, allowing more complex lattices to be studied in a LPT.

REFERENCES

[1] H. Okamoto *et al.*, Radio-frequency quadrupole trap as a tool for experimental beam physics, *NIM A*, vol. 485, p. 224, 2002.

[2] W. Paul, Electromagnetic traps for charged and neutral particles, *Nobel Lecture*, Dec. 1989.

[3] D. J. Kelliher *et al.*, Study of beam dynamics in linear Paul traps, in *Proc. HB2014*, East-Lansing, MI, USA. 2014, paper MOPAB26, pp. 86–88.

[4] R. Davidson *et al.*, A Paul trap configuration to simulate intense non-neutral beam propagation over large distances through a periodic focusing quadrupole magnetic field, *Physics of Plasmas*, vol. 7, 2000.

[5] K. Ito *et al.*, Coherent resonance stop bands in alternating gradient beam transport, *PRAB*, vol. 20, 2017.

[6] K. Ito *et al.*, Tune depression of Ion plasmas observed in a linear Paul trap, *J. Plasma Fusion Res. SERIES*, vol. 8, 2009.

[7] USPAS, `http://uspas.fnal.gov/materials/12MSU/corrlect.pdf`

[8] K. Moriya *et al.*, Experimental study of integer resonance crossing in a nonscaling fixed field alternating gradient accelerator with a Paul ion trap, *PRAB*, vol. 18, 2015.