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LANDAU DAMPING AND TUNE-SPREAD REQUIREMENTS FOR TRANSVERSE BEAM STABILITY

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INTRODUCTION

There is a wide experience of using octupole magnets to cure the transverse instabilities, for example since the seventies at CERN PS [1]. In the LHC, the Landau octupoles are routinely combined with the chromaticity and the transverse feedback system in order to ensure the bunch stability through the cycle [2, 3]. Dedicated octupole magnets should be employed in the FCC-hh as a cure against transverse collective instabilities [4], in addition to the transverse feedback system. We estimate the stability provided by an octupole scheme in FCC-hh using the stability diagrams for two-dimensional betatron frequency spreads. Using the estimations for the impedance-driven tune shifts, and comparisons with the LHC experience, we try to provide suggestions for the adequate octupole scheme in the FCC-hh.

The design of the Landau octupole system in LHC [5] has been done using the dispersion relation [1, 5, 6]. This relation is formally for a 2D case of a coasting beam, and can be interpreted for the rigid dipole oscillations in bunches. As it was discussed in [5], applicability of this dispersion relation to higher-order modes (or intra-bunch oscillations) should be investigated.

Here, we consider head-tail modes with different mode index k and study Landau damping due to octupole magnets. We use particle tracking simulations [7, 8] and compare the results with the predictions of the dispersion relation [1, 5, 6]. Therefore, we validate the applicability of the dispersion relation to the $k = 0$ head-tail mode, in the case it is not a rigid-bunch mode because of the chromaticity, and to the higher-order head-tail modes.

DISPERSION RELATION AND OCTUPOLES

For an analytic description of transverse oscillation Landau damping due to a two-dimensional tune spread, the following dispersion relation [1, 3, 6] is often used,

$$\Delta Q_{\rm coh} \int \frac{1}{\Delta Q_{\rm oct} - \Omega/\omega_0} J_x \frac{\partial \psi_\perp}{\partial J_x} dJ_x dJ_y = 1. \qquad (1)$$

Here, $\Delta Q_{\rm coh}$ is the coherent tune shift of an eigenmode produced by the machine impedance, in the case without the tune spread and Landau damping. The incoherent $\Delta Q_{\rm oct}(J_x, J_y)$ is a two-dimensional function of the octupole-induced amplitude-dependent tune shift. The collective mode frequency Ω is the solution for the given impedance, tune spread, and the beam distribution function $\psi_\perp(J_x, J_y)$. This is the dispersion relation for the horizontal plane, the corresponding form is for the vertical oscillations.

The octupole magnets provide the incoherent tune shifts [5],

$$\begin{aligned} \Delta Q_x &= a_x J_x - b_{xy} J_y \\ \Delta Q_y &= a_y J_y - b_{xy} J_x . \end{aligned} \qquad (2)$$

OCTUPOLES FOR FCC-HH

The basic scaling of the complex tune shift of a coherent mode is

$$\Delta Q_{\rm coh} \propto \frac{Z^\perp \hat{\beta}}{\gamma}. \qquad (3)$$

Assuming equal impedances per length in FCC and LHC, we obtain the ratio of the instability growth rate for the collision energy in FCC to that in LHC,

$$\frac{\Delta Q_{\rm coh}^{\rm FCC}}{\Delta Q_{\rm coh}^{\rm LHC}} = \frac{L_{\rm FCC}}{L_{\rm LHC}} \times \frac{\hat{\beta}_{\rm FCC}}{\hat{\beta}_{\rm LHC}} \times \left(\frac{\gamma_{\rm FCC}}{\gamma_{\rm LHC}}\right)^{-1} \approx 1 \qquad (4)$$

We conclude that in the low-order estimations the expected instability growth rates in $\Delta Q_{\rm coh}$ units in FCC can be assumed to be similar to the LHC case.

The total octupole strength as a function of the tune shift scales as

$$N_{\rm oct} O_3 L_{\rm m} \propto \frac{\gamma^2}{\hat{\beta}^2 \varepsilon} \Delta Q_{\rm oct}, \qquad (5)$$

which means that an equal tune shift $\Delta Q_{\rm oct}$ for the collision energy in FCC compared to the collision energy in LHC requires the total octupole strength

$$\frac{(N_{\rm oct} O_3 L_{\rm m})^{\rm FCC}}{(N_{\rm oct} O_3 L_{\rm m})^{\rm LHC}} \approx 22 \, . \qquad (6)$$

We assume an octupole configuration which is similar to the LHC case. Two families of the Landau octupoles are located close to the main arc quadrupoles, with the opposite beta-functions relations. As an example, we consider the FCC-hh ring optics from [9], and we use the beta-function at the main arc quadrupoles, $\hat{\beta}_x^F = 360.96$ m, $\hat{\beta}_y^F = 65.14$ m, $\hat{\beta}_x^D = 66.22$ m, and $\hat{\beta}_y^D = 359.65$ m. Here we assume the identical octupole magnets to the LHC case, $O_3 = (63100 \times I_{\rm oct}[{\rm A}]/550)$, $L_{\rm m} = 0.32$ m.

Figure 1 shows the stability diagram (the right plot) for the nominal FCC beam at the collision energy, $E = 50$ TeV, $\varepsilon = 2.2\,\mu$m. The octupole magnet current is $I_{\rm oct} = +500$ A for all the magnets. The left plot shows the tune footprint for the particles with the transverse action $(J_x + J_y) < 6\varepsilon$. Since the tune shift distribution is symmetric in x and y in

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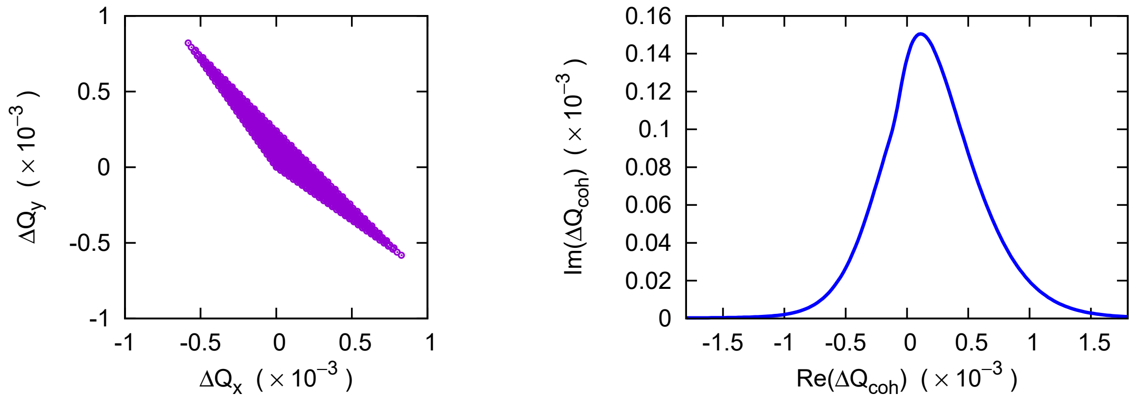

Figure 1: The tune footprint (left plot) and the stability diagram (right plot) due to octupoles for the FCC beam at the collision energy with N_{oct}=3554 of the LHC-type magnets, $I^F_{oct} = I^D_{oct} = +500$ A. For the advanced-technology octupoles, it corresponds to N_{oct}=508 magnets.

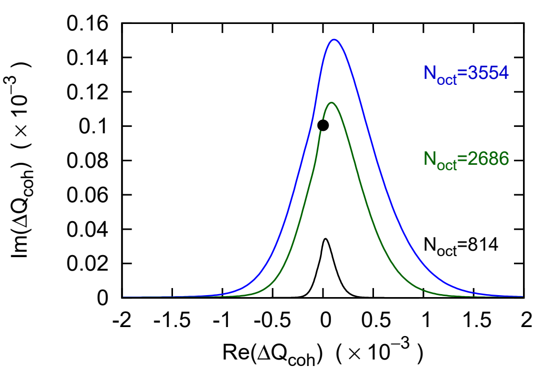

Figure 2: Stability diagrams for the FCC beam at the collision energy for the three scenarios of the octupole number in the ring. The blue line (N_{oct}=3554) is for the same stability as in LHC. The green, black lines are explained in the text.

this case, the horizontal and the vertical stability diagrams coincide.

Here we compare different scenario of the octupole magnets in FCC. Figure 2 shows the stability diagrams for the nominal FCC beam at the collision energy, the same beam and machine parameters are assumed as for Fig. 1. The black line corresponds to $N_{oct} = 814$ (the both families in a sum), which is equal to the number of the main arc quadrupoles. The green line corresponds to $N_{oct} = 2686$, which is just enough to ensure the stability for $Im(\Delta Q_{coh}) = 10^{-4}$ (black dot). Finally, the blue line corresponds to $N_{oct} = 3554$, which provides the same stability due to Landau octupoles as in the LHC case (identical to Fig. 1).

In the case of an advanced design of the octupole magnets in FCC, N_{oct} can be reduced. For example, the octupole strength $O_3 = (220 \times 10^3 \times I_{oct}[A]/550)$ should be technologically possible. This means a factor of 3.5 in comparison to the LHC magnets. In addition, it may possible to double the magnet length L_m. This implies a factor of 7 in comparison to the LHC magnets. N_{oct}=508 would then be enough for the LHC–type stability.

LANDAU DAMPING VERIFICATION

For the particle tracking simulations we use the code PATRIC [7, 10, 11]. The code has been validated using exact analytical prediction [8]. In order to study the head-tail modes we consider a Gaussian bunch and apply the resistive-wall wake $W(z) = w_0/\sqrt{z}$. For the chosen phase shift along the bunch [12] $\chi_b = -1.15$ the most unstable mode is the $k = 0$ head-tail mode. The amplitude increases exponentially (Fig. 3, the top plot) with the growth rate

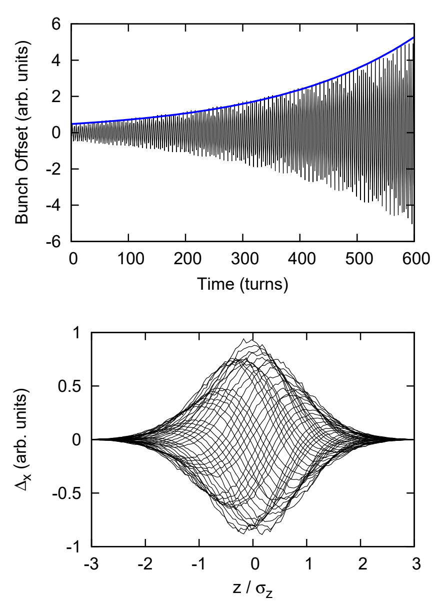

Figure 3: An unstable $k = 0$ head-tail mode from a simulation without octupoles. Top: time evolution of the bunch offset (black line), the blue line is an exponential. Bottom: related offset traces along the bunch.

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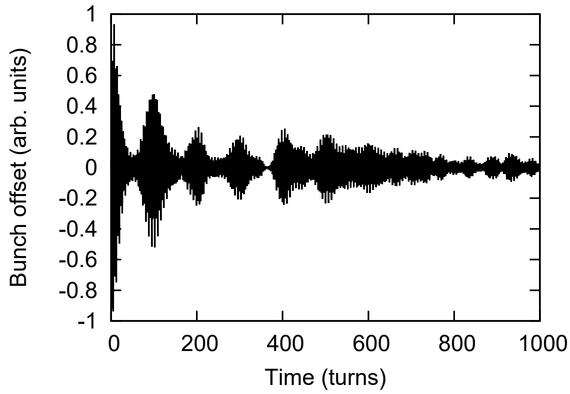

Figure 4: Outcome of a simulation with the same settings as in Fig. 3, but applying octupole magnets. The head-tail mode is stabilized.

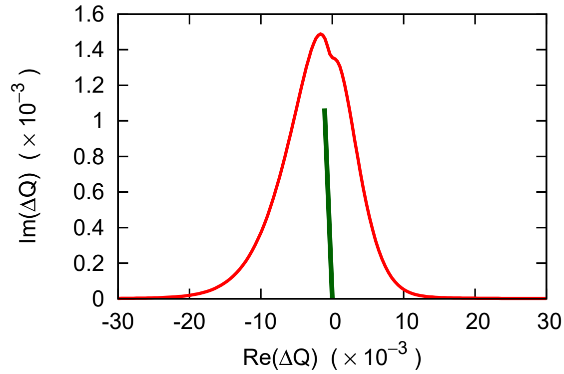

Figure 5: Comparison of the dispersion relation Eq. (1) predictions with the results of the particle tracking simulations. The red line: stability diagram from the dispersion relation for the beam parameters and the octupole magnets of the simulations. The green line: stabilized $k = 0$ head-tail modes due to the octupole magnets as obtained from the simulation series.

$\mathrm{Im}(\Delta Q) = 0.64 \times 10^{-3}$. The bottom plot in Fig. 3 demonstrates a typical trace overlap pattern of the $k = 0$ head-tail mode. Note that in the case of long bunches there are wiggles of the transverse offset along the bunch for the $k = 0$ mode, see Fig. 3. For long bunches (in terms of a large ξ_b) the $k = 0$ head-tail mode is not a rigid-bunch mode.

After inclusion of octupole magnets into the simulations, stabilization of the $k = 0$ head-tail instability has been observed, Fig. 4. The strength of the octupole magnets corresponds to the horizontal tune shift $\Delta Q_{\mathrm{oct}} = 0.517{\times}10^{-3}$ for a particle with $J_x = \varepsilon/2$, $J_y = 0$. The related rms tune spread in Q_x due to these octupoles is $(\Delta Q_{\mathrm{oct}})_{\mathrm{rms}} = 2.53 \times 10^{-3}$. We observe a fast decay of the initial perturbation without transverse emittance blowup.

In a series of simulations we increased the strength of the wake field and kept the octupole magnet strength constant. The results of the simulation scans for the $k = 0$ head-tail mode are summarized and compared to the predictions of the dispersion relation in Fig. 5.

In order to study the higher-order head-tail modes we have shifted the chromaticity in the particle tracking simulations. Simulation scans for the $k = 1$ head-tail modes and for the $k = 2$ head-tail modes have been performed. The octupole magnet configuration has been kept constant, and the resistive-wall wake strength has been gradually increased until the stability thresholds have been found. The results of the simulation series are summarized in Fig. 6 by black dots. The red line indicates the prediction of the dispersion relation Eq. (1), while the blue line shows the rms tune spread due to octupoles for reference.

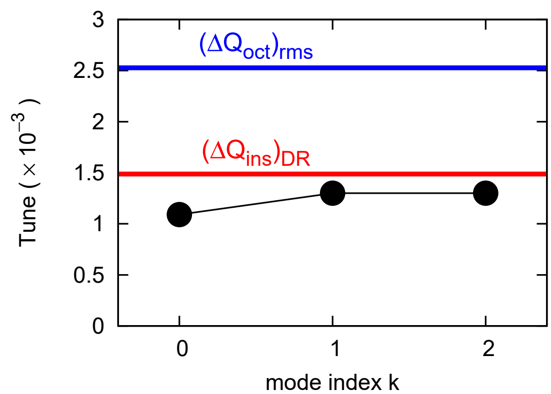

Figure 6: Summary of the simulation scans for Landau damping due to octupoles for head-tail modes: black dots show the stability thresholds (growth rates $\mathrm{Im}(\Delta Q)$ of the stabilized modes with maximal impedance). The prediction of the dispersion relation Eq. (1) is shown by the red line. The blue line indicates the rms tune spread due to the octupole magnets.

CONCLUSIONS

A configuration of octupole magnets for Landau damping in FCC-hh has been considered. Similar to the experience of LHC [2, 3, 5], we solve the dispersion relation Eq. (1) in order to determine the stability properties of the octupole magnets. An assumption of equal impedances per length in FCC and LHC provides a similar ΔQ_{coh} for the both cases. The required number of LHC-type octupole to ensure this transverse stability in FCC-hh is $N_{\mathrm{oct}} = 3554$. For octupole magnets of a higher strength, this number can be reduced to N_{oct}=508. In the case of more realistic impedance estimations, N_{oct} should be changed linearly according to ΔQ_{coh} of the expected instabilities.

Landau damping predictions from the dispersion relation Eq. (1) have been quantitatively verified using particle tracking simulations. Starting from non-rigid $k = 0$ head-tail modes, a fairly satisfactory agreement has been observed. Applying different chromaticities, higher-order head-tail modes have been excited. We find that the damping boundaries for the higher-order head-tail modes are close to that of the $k = 0$ mode. Furthermore, all these stability thresholds are fairly close to the prediction of the rigid-bunch dispersion relation Eq. (1).

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