

DESIGN OF A ONE-DIMENSIONAL SEXTUPOLE USING SEMI-ANALYTIC METHODS*

L. Gupta[†], S. Nagaitsev, S. S. Baturin, Y.-K. Kim University of Chicago, Chicago, IL, USA

Abstract

Sextupole magnets provide position-dependent momentum kicks and are tuned to provide the correct kicks to particles within a small acceptance region in phase space. Sextupoles are useful and even necessary in circular accelerators for chromaticity corrections. They are routinely used in most rings, i.e. CESR. Although sextupole magnets are necessary for particle energy corrections, they also have undesirable effects on dynamic aperture, especially because of their nonlinear coupling term in the momentum kick. Studies of integrable systems suggest that there is an analytic way to create transport lattices with specific transfer matrices that limit the momentum kick to one dimension. A one-dimension sextupole is needed for chromaticity corrections: a horizontal sextupole for horizontal bending magnets. We know how to make a “composite” horizontal sextupole using regular 2D sextupoles and linear transfer matrices in an ideal thin-lens approximation. Thus, one could create an accelerator lattice using linear elements, in series with sextupole magnets to create a “1D sextupole”. This paper describes progress towards realizing a realistic focusing lattice resulting in a 1D sextupole.

INTRODUCTION

Modern particle accelerators have become invaluable tools for research in many fields, including biology, chemistry, and material science. The field of accelerator physics therefore, is always working towards expanding the capabilities of light source accelerators. There are many different ways to create better light sources, and studying and improving the transport of the beam of electrons through the accelerator is one such method. In particular, research into nonlinear beam dynamics, and finding ways of creating effectively linear which function as non-linear accelerator elements such as sextupole and octopole magnets. The electron beam which circulates within a synchrotron consists of many particles, each of which ideally have almost the same energy. However, as the particles in the beam circulate, they radiate away momentum and can become “off-energy.” To maintain the quality of the beam for continued radiation production, sextupole magnets are used to correct off-energy particles. Sextupole magnets provide position-dependent momentum kicks and are tuned to provide the correct kicks to particles within a small acceptance region in phase space. Thus, particles outside of this regions could be kicked farther off-energy and eventually result in beam loss by physically hitting the beam pipe. This beam loss is detrimental to beam

lifetime and brightness [1-2]. Although sextupole magnets are necessary for particle energy corrections, the nonlinearities they introduce can cause nonlinear beam instabilities. The region in phase space in which the beam is stable, called the dynamic aperture, and is often limited by the effects of sextupole magnets, but can be extended in many ways. Studies of integrable systems for particle accelerators [2-3] by Sergei Nagaitsev suggest that there is an analytic way to create transfer matrices which limit the momentum kick to one dimension. Thus, one could create an accelerator lattice using linear elements (dipoles and quadrupoles), in series with sextupole magnets to create a “one-dimensional sextupole.” This method, which is a semi-analytic method of determining how sextupole magnets can be placed to limit nonlinearities in an accelerator, is discussed here.

THEORY

The one-dimensional sextupole was developed by considering a simple transfer matrix, which can be created using linear elements such as drift spaces and quadrupole magnets, and has the following properties. The matrix must be diagonal, and invertible, with diagonal elements as shown in Eq. 1.

$$D = \begin{bmatrix} a & & & \\ & \frac{1}{a} & & \\ & & b & \\ & & & \frac{1}{b} \end{bmatrix} \quad (1)$$

where both a and b are any real scalar numbers. For a transfer matrix of the form of Eq. 1 and the inverse the matrix of D , when applied to two consecutive sextupole magnets results in an entirely one-dimensional momentum kick:

$$DS_1 D^{-1} S_2 = \begin{bmatrix} x \\ p_x + \Delta p_x \\ y \\ p_y \end{bmatrix} \quad (2)$$

where S_1 and S_2 are non-linear transfer matrices, and are represented by:

$$S_i = \begin{bmatrix} x \\ p_x + q_i(x^2 - y^2) \\ y \\ p_y - 2q_i xy \end{bmatrix}, \quad (3)$$

for a current q_i , which is proportional to the field gradient in the sextupole.

* Work supported by NSF Award No. PHY-1549132 (Center for Bright Beams)

[†] lipigupta@uchicago.edu

By calculating the transformation $DS_1D^{-1}S_2$

$$DS_1D^{-1}S_2 = \begin{bmatrix} ax & & & \\ p_x + \frac{q_1(x^2-y^2)}{a} + q_2[(ax)^2 - (by)^2] & & & \\ & by & & \\ \frac{p_y - 2q_1xy}{b} - 2q_2abxy & & & \end{bmatrix} \quad (4)$$

the condition under which the sextupole kick will be decoupled can be identified and solve. The transformation in Eq. 2 is a composition of transformations, and is not completed by simple matrix multiplication due to the non-linear nature of sextupole transformation. In order to do this calculation, the sextupole kicks are added to the initial state vector, then the matrix multiplication of D on the state is carried out. The second sextupole kick is then added to the resulting state vector, and then the final matrix multiplication D^{-1} is carried out. Lastly, the condition that must be satisfied in order for the coupling term in the y-direction to vanish is:

$$2(q_1 + ab^2q_2)xy = 0, \quad (5)$$

$$q_2 = \frac{-q_1}{ab^2}. \quad (6)$$

By setting q_2 as shown in Eq.6, the transformation yields:

$$DS_1D^{-1}S_2 = \begin{bmatrix} x & & & \\ p_x + q_1x^2 - \frac{a^2q_1x^2}{b^2} & & & \\ y & & & \\ p_y & & & \end{bmatrix}. \quad (7)$$

Therefore, one can see from the final state vector, Eq. 7, the change in the momentum in the x-direction, due to the 1-d sextupole transfer matrices is:

$$\Delta p_x = q_1 \left(1 - \left(\frac{a}{b} \right)^2 \right) x^2. \quad (8)$$

From these calculations, we have shown that if the transfer matrices in Eq. 2 could be created by an accelerator lattice, then the momentum kick in the transverse direction would be given by Eq. 8 where the currents q_1 and q_2 are related by Eq. 6. This would eliminate the x-y coupling in the y-direction, though in practice we hope this would simply render the coupling negligible.

THIN LENS LATTICE SOLUTION

Generally, any lattice that satisfies the conditions shown above could produce a 1-D sextupole, but a short segment with minimal quadrupoles is preferable, as it could be easily inserted into an existing machine. The following lattice, which uses the thin lens approximation for quadrupole magnets with focusing gradient k_i , and drift lengths L_i . To calculate a general solution for a 1-D sextupole, a simple transformation of four quadrupoles (Q) and three drift lengths (O) therefore:

$$D_{lat} = Q(k_1)O(L_1)Q(k_2)O(L_2)Q(k_3)O(L_3)Q(k_4), \quad (9)$$

can be calculated, and by setting D_{lat} equal to Eq. 1, where the transfer matrices are define as:

$$Q(k_i) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ k_i & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -k_i & 1 \end{pmatrix} \quad (10)$$

$$O(L_i) = \begin{pmatrix} 1 & L_i & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_i \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (11)$$

a solution for the parameters in the system can be determined in terms of a and b such that the transformation D_{lat} satisfies Eq. 1. By fixing a length scale, L_1 , each of the other parameters, L_2, L_3 , and each k_i are function only of L_1, a , and b . Values for a and b can be set according to any necessary conditions, thus determining the necessary drift length and quadrupole focusing gradients. In doing this, now a thin lens approximation lattice can be determined, and optimized further to create a realistic lattice which would preserve the 1-D sextupole properties.

ENERGY SPREAD CONSIDERATIONS

In order to gauge if a 1-D sextupole is a practical option, it is prudent to determine how the system will change when we consider realistic beam parameters. Figure 1 demonstrates how the phase space coordinates for the centroid of a bunch would be transformed in a 1-D sextupole, and the effect of beam energy spread which results in a non-zero coupling between the horizontal and vertical motion. Thus it is important to understand the effect of beam energy spread and how it can be mitigated. The calculations shown earlier consider only on-axis ($\Delta x/x = 0$) and on-energy ($\Delta p/p = 0$) particles. In order to determine the magnitude of the effects energy spread has in a 1-D sextupole, we recalculated the theoretical values by introducing a detuning parameter, δ . This analysis and evaluation of a numerical examples using realistic beam and lattice parameters is presented here.

The transfer matrices representing a quadrupole are now:

$$Q_\delta(k_i) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ k_i(1-\delta) & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -k_i(1-\delta) & 1 \end{pmatrix} \quad (12)$$

From this, we can calculate a D_δ , which accounts for the effect of energy spread. In order to find parameters which minimize the effects from energy spread in a 1-D sextupole, a residual matrix $\Delta T = D_{lat} - D_\delta$ was calculated. Because the specific values of a and b in the diagonal transformation are not as important as maintaining the conditions for the diagonal transformation, they can be varied as needed. In order to minimize the contribution from beam energy spread, the parameters a and b were varied such that the elements in ΔT were minimized.

Table 1: Shown are the thin lens solution parameters which minimize the change in the diagonal 1D sextupole transformation matrix, for $\delta = 10^{-3}$.

a	b	L_1	L_2	L_3	k_1	k_2	k_3	k_4	$\Delta p_y / \Delta p_x (\%)$
-2.9	-0.2	1.00m	0.78m	0.78m	$0.90m^{-1}$	$-1.94m^{-1}$	$2.18m^{-1}$	$-1.17m^{-1}$	0.68%
-3.5	-0.3	0.75m	0.66m	0.76m	$1.21m^{-1}$	$-2.40m^{-1}$	$2.39m^{-1}$	$-1.19m^{-1}$	0.86%
-4.8	-0.4	0.50m	0.57m	0.72m	$1.77m^{-1}$	$-3.24m^{-1}$	$2.72m^{-1}$	$-1.23m^{-1}$	0.87%
-7.5	-0.4	0.25m	0.44m	0.55m	$3.24m^{-1}$	$-5.61m^{-1}$	$3.67m^{-1}$	$-1.53m^{-1}$	0.49%

By optimizing the system for various initial a and b settings, and for $L_1 = 1m, 0.75m, 0.5m, 0.25m$ and energy detuning $\delta = 10^{-3}$, which are reasonable drift lengths for a short portion of a long linear or circular machine, the following settings were determined to minimize the energy spread contribution to the diagonal transformation matrix. The merit of these solutions is demonstrated by the percent increase of Δp_y from 0, when the parameters are applied to the theory which considers beam energy spread. The percent change was calculated by considering the solutions shown in Table 1, for the following initial phase space parameters: $x_0 = 0.5mm, y_0 = 0.05mm, p_x = 0, p_y = 0$, and the sextupole currents scaled such that $q_1 = 1, q_2 = \frac{-1}{ab^2}$.

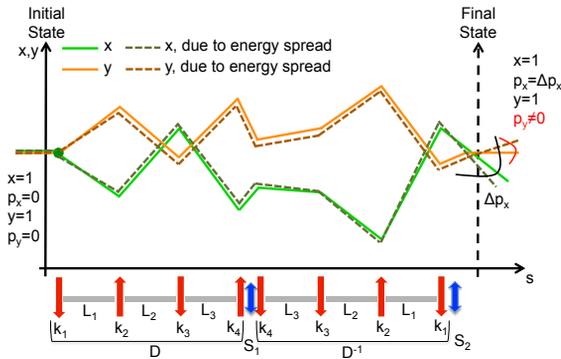


Figure 1: A simple example of how a 1-D sextupole transforms the phase space coordinates for the centroid of a bunch, with and without beam energy spread. A particle with $x_0 = y_0, p_x = p_y = 0$ entering the 1-D sextupole would exit with $x_0 = y_0, p_x = \Delta p_x, p_y = 0$, but particles that are off-energy would end up with $x_0 = y_0, p_x = \Delta p_x, p_y \neq 0$.

The parameters listed in Table 1 show that the drift lengths are reasonable, though the focusing gradients may not be. The ideal solution would be the one in which $\Delta p_y / \Delta p_x$ is minimal, such as the solution for $a = -7.5, b = -0.4$. However, the focusing gradients in the quadrupoles for this solution are very high and may not be realizable in a normal-conducting machine. Still, this solution is further ideal due the short drift lengths needed to construct it. A short 1-D sextupole would be desirable because it could be easily inserted

into an existing machine. However, further optimization could yield other similar solutions with physically realizable quadrupole strengths, that still minimize the momentum coupling due to beam energy spread. The solutions presented therefore display the broad solution space available for 1-D sextupoles parameters.

CONCLUSIONS AND FUTURE WORK

In this paper, we consider the theory for simple 1-D sextupole model, using the thin lens approximation to describe the action of quadrupole magnets on a particle, and the same theory when considering beam energy spread. An optimization of lattice parameters is also presented, in which the 1-D sextupole parameters are optimized to minimize the effects of beam energy spread. Further studies will focus on understanding the stability of 1-D sextupole solutions. These solutions will also need to be generalized, such that a realistic thick quadrupole lattice solution can be designed. The testing of such a lattice segment could take place by constructing the segment at a facility in which the machine can be altered with minimal issue. A facility such as the Integrable Optics Test Accelerator (IOTA) would be an ideal facility, and would further the mission of the project; pursuing methods of making non-linear accelerator elements integrable.

ACKNOWLEDGMENTS

This work is supported by NSF Award No. PHY-1549132 (Center for Bright Beams).

REFERENCES

- [1] S. Webb, D. Bruhwiler, A. Valishev, S. Nagaitsev, and V. Danilov, "Chromatic and Dispersive Effects in Nonlinear Integrable Optics", arXiv:1504.05981.
- [2] S.A. Antipov, S. Nagaitsev and A. Valishev, Single-particle dynamics in a nonlinear accelerator lattice: attaining a large tune spread with octupoles in IOTA, Journal of Instrumentation, Volume 12, April 2017.
- [3] S. Nagaitsev, Thin 1-D sextupole in X, Fermilab Note, Beamsdoc-6276-v1. (2017).

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2018). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI.