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THE BEAM OPTICS OF THE FFAG CELL OF THE CBETA ERL ACCELERATOR*

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Abstract

The CBETA project [1] is a prototype electron accelerator for the proposed eRHIC project [2]. The electron accelerator is based on the Energy Recovery Linac (ERL) and the Fixed Field Alternating Gradient (FFAG) principles. The FFAG arcs and the straight section of the accelerator are comprised of cells with one focusing quadrupole and one combined function defocusing quadrupole. Both cell magnets are Halbach type permanent magnets [3]. We will present the beam optics of the FFAG cell which is based on 3D field maps which are derived with the use of the OPERA computer code [4]. We will also present the electromagnetic design of the correctors of the FFAG's cell magnets.

INTRODUCTION

The Cornell Brookhaven Energy Recovery Linac Test Accelerator (CBETA) [1] is currently under construction at the Cornell Wilson Laboratory. CBETA is an electron accelerator designed to combine two innovative concepts, the Energy Recovery Linac (ERL) and the Fixed Field Alternating Gradient (FFAG) concepts. The accelerator consists of a 1.3 GHz 80 MeV Linear accelerator and one recirculating beam line. Figure 1 shows the layout of the CBETA with various labelled sections, including the Injector (IN), Linear accelerator (LA), Splitters (SX, RX), FFAG recirculating beamline (FA, TA, ZA, ZB, TB, FB) and the Beam Stop section (BS). The current design of CBETA aims to achieve

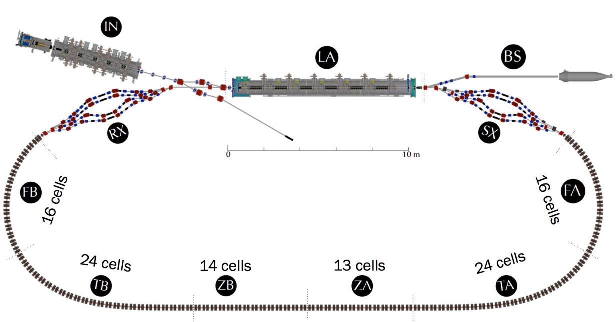

Figure 1: Layout of the CBETA accelerator. The section labeled (LA) is the ERL, The sections labeled (FA), (TA), (ZA), (ZB), (TB), and (FB) are the FFAG sections which will accommodate four recirculating orbits with an energy range from 42 MeV to 150 MeV.

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maximum electron beam energy at 150 MeV. This will be accomplished by injecting 6 MeV bunches into the LINAC and accelerating each bunch by 36 MeV each time they pass through the ERL via the single recirculating FFAG beamline. After the bunches reach the top energy of 150 MeV, they are recirculated back into the LINAC, each time delivering 36 MeV of energy back to the LINAC until they reach the 6 MeV energy and be dumped at the beam stop (see Fig. 1). The ERL concept allows us to accelerate electron bunches to a high energy with minimal power consumed in the LINAC since almost all of the LINAC energy required to accelerate the bunches is recycled from the decelerated electron bunches. On the other hand, the FFAG concept allows all recirculating bunches, with energy ranging from 42 MeV to 150 MeV, to be confined in one single beamline with the transverse dimension smaller than 46.6 mm. With great reduction in construction cost, this also allows all the bunches to lie within the "good field region" generated by the FFAG permanent magnets.

THE FFAG CELL OF THE CBETA ACCELERATOR

The FFAG arcs (FA, FB), transition (TA, TB), and FFAG straight sections (ZA, ZB) of the CBETA consists of doublet cells, and each cell comprises of one pure focusing quadrupole (QF) and one defocusing quadrupole with an additional dipole component (BD). Figure 2 shows the trajectories (orbits) in the FFAG arcs followed by the electron bunches of the four design energies. The maximum transverse displacement between these bunches is less than 46.6 mm. The FFAG remarkably allows bunches with a large energy range to coexist in one transversely small beamline.

Before we study the beam optics of the FFAG cells, we must first investigate the magnetic fields generated by them. In the next subsection we will present results from the 2D and 3D electromagnetic calculations of both QF and BD.

The 2D and 3D Electromagnetic Study of the QF and BD Elements of the FFAG Cell

To generate the final beam optics of the cell we start with 'hard edge' magnets which provide the required multipoles (dipole and quadrupole only for CBETA FFAG). Then a 2D model of the magnet is generated to provide these multipoles in the transverse plane. Then a 3D model based on the 2D model is made to produce 3D field maps for the beam optics calculations. Few iterations are required to finalize the 3D design of the magnet with desired beam optics. In the next

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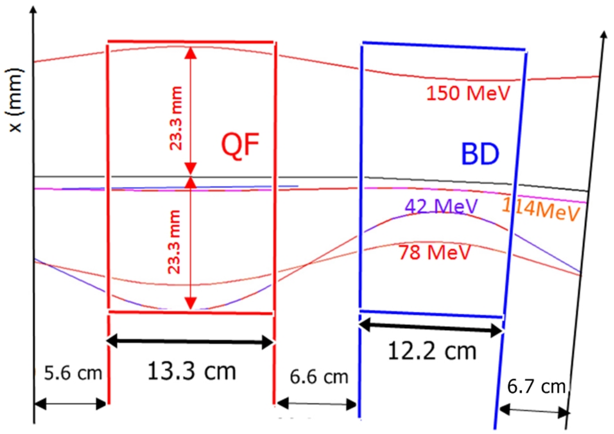

Figure 2: A schematic diagram of the FA doublet cell showing the transverse trajectories of the electron bunches with energy ranging from 42 MeV to 150 MeV. Both QF and BD are permanent Halbach type magnets.

two subsections we provide results from the 2D and 3D calculations.

The 2D design From the beam optics calculations which are based on 'hard edge' magnets we obtain the required multipoles of the magnets. The 2D electromagnetic calculations determine the cross section of the cell magnets. Figure 3 shows the design cross section of QF and BD. The black lines on the figures are the equipotential vector lines. The QF is pure quadrupole Halbach type magnet consisting of 16 magnetized wedges, and its quadrupole component is given by [5]

$$B(r_i, r_o) = 2B_{rm}\cos^2\left(\frac{\pi}{M}\right)\frac{sin\left(\frac{2\pi}{M}\right)}{\left(\frac{2\pi}{M}\right)}\left(\frac{1}{r_i} - \frac{1}{r_o}\right) \quad (1)$$

In Eq. (1) B_{rm} is the remnant magnetization, r_i and r_o the inner and outer radii of the annulus made by the wedges, and M is the number of wedges. The magnetization direction α of each of the wedges is given by $\alpha = 3\theta + \frac{\pi}{2}$, where θ is the azimuthal angle of each wedge.

The BD magnet which provides both the quadrupole and dipole field can be constructed as a superposition of two layers of Halbach magnets, each layer made of 16 wedges. In this paper we present an alternative 2D design of the BD magnet [6] which consists of a single layer of 16 wedges of varying transverse size. The 2D computer code which optimizes the design is available upon request [6].

The 3D design The 3D design of the magnets is essential as it provides an accurate description of the field profile. Unlike 2D models, 3D design accounts for the fringe fields of the cell magnets as well as the field contribution from the neighboring magnets. Table 1 contains the geometry of the QF and BD magnets which generate the required fields to satisfy the beam optics requirements. The beam optics

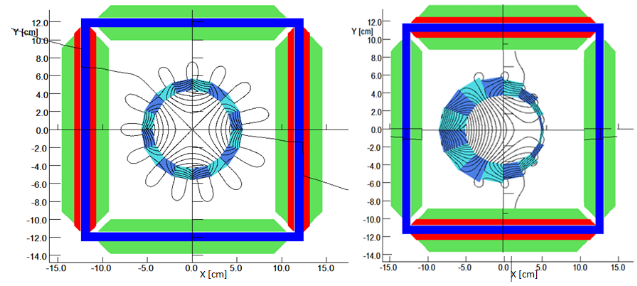

Figure 3: The cross sections of the QF, a pure focusing Halbach quadrupole magnet (left), and the BD, a defocusing quadrupole magnet with a dipole component (right). Both magnets have window frame electromagnets as correctors, which can provide a normal dipole (red areas) and a normal quadrupole (green areas).

Table 1: Cell Magnets Specifications: The inner and outer radii the length and the dipole and quadrupole multipole strengths generated by the QF and BD magnets of the cell.

Mag	Ri[mm]	Ro[mm]	L[mm]	D[T]	Q[T/m]
QF	44.05	56.51	122	0.0	11.43
BD	44.05	varies	133	-0.317	10.87

calculations provide the relative positions of the QF and BD magnets within the FFAG cell. This information is used in the OPERA code to generate the 3D model of the cell. Figure 4 shows an isometric view of a model with three consecutive cells. The field map of the middle cell is used in the beam optics calculations.

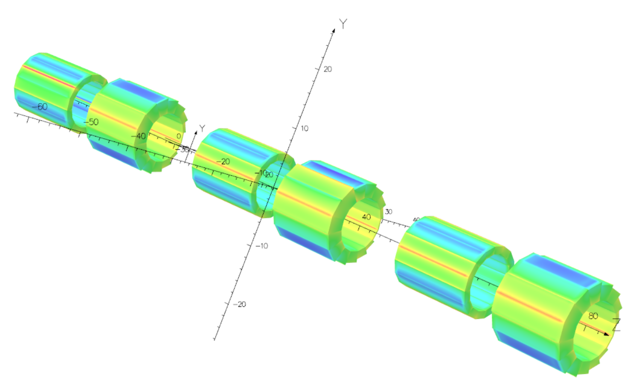

Figure 4: Isometric view of model with three consecutive FFAG cells. The field map of the middle cell is used in the beam optics calculations.

Arc Cell Design

The magnets described so far are used to produce a final arc design for FA (and FB). The final geometry is related to our beam pipe configuration and the desired path length. For each cell the beam pipe consists of two straight segments connected by a 42mm long BPM block, at which a 5 degree bend occurs (2.5 degree on either side of the block). Other parameters are chosen based on space constraints for

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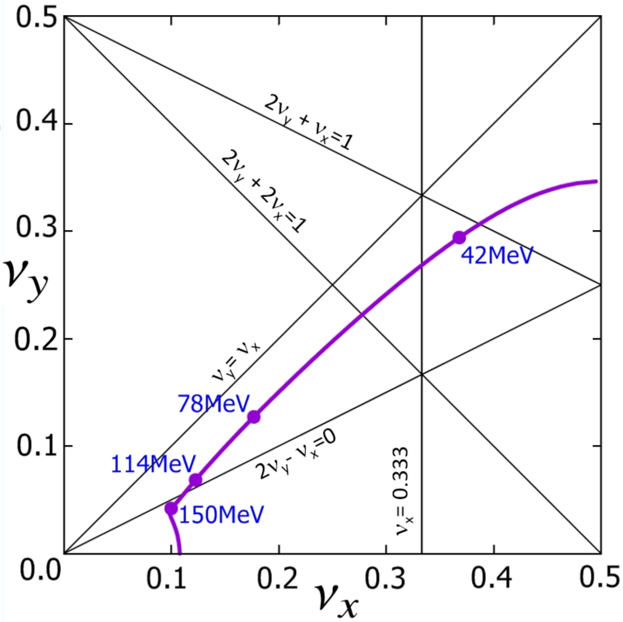

Figure 5: Tune per cell for the arc cell, treated as periodic. Design energies are shown with dots.

instrumentation. The overall length was adjusted to achieve a desired machine circumference considering the entire machine as a whole.

Once the longitudinal length is fixed, there are three free parameters: the gradient of QF and BD, and the dipole field of BD. They are used to center the beam pipe and to optimize the two transverse betatron tunes. To center the beam pipe means that the maximum orbit excursion at 150MeV and the minimum orbit excursion at 42MeV, relative to the line defining the coordinate system, are of approximate equal magnitude (23.3 mm in the final design).

For betatron tune optimization, we aim to keep the working points of all four design energies away from the problematic resonance lines on the tune plane. Figure 5 shows the tune plane and the dominant resonance lines [7]. Physically we are concerned with how much the gradients would need to change to reach these lines. The change is quantified by

$$\sqrt{(\Delta G_F/G_F)^2 + (\Delta G_D/G_D)^2}, \qquad (2)$$

where G_F (G_D) is the gradient of QF (BD). We define the parametric distance to be the minimum value of this quantity for the working point to reach the most nearby resonance line.

The final choice is shown by the purple dots on Fig. 5. The parametric distance from the 150MeV point to the $\nu_y = 0$ line is approximately equal to the parametric distance from the 42MeV point to the $\nu_x + 2\nu_y = 1$ line, and the parametric distances from the 150MeV and 114MeV points to the $\nu_x - 2\nu_y = 0$ line are about the same. The resulting working point is reasonably well-defined by the 42 MeV horizontal and 150 MeV vertical tunes, which are given in Table 2. The parametric distance to the $\nu_x + 2\nu_y = 1$ line is 3.0%, to the $\nu_y = 0$ line is 3.8%, and to the $\nu_x - 2\nu_y = 0$ line is 1.3% (114 MeV) and 1.2% (150 MeV).

Table 2: Parameters for the Arc Cell

BPM block length (mm)	42
Pipe length (mm)	402
Magnet offset from BPM block (mm)	12
Focusing quadrupole length (mm)	133
Defocusing magnet length (mm)	122
Single cell horizontal tune, 42 MeV	0.368
Single cell vertical tune, 150 MeV	0.042
Integrated focusing magnet strength (T)	−1.528
Integrated defocusing magnet strength (T)	+1.351
Integrated field on axis, defocusing (T m)	−0.03736

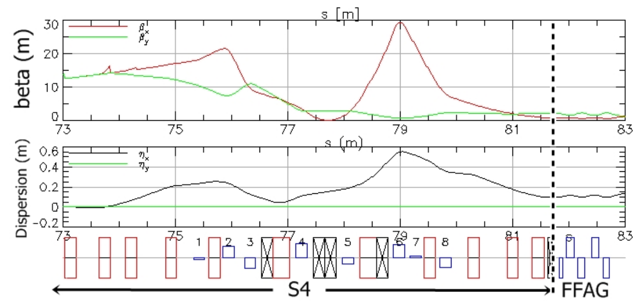

Figure 6: S4 splitter line with optics matched into the FFAG arc using 8 quadrupole magnets.

Beginning with the field maps described in the previous section, we scale and shift them to achieve the desired orbit centering and tune working point. Magnet designs are then modified to have the resulting integrated gradient and central field. Lastly, field maps are computed from those designs, and the results are found to be in good agreement. The final parameters chosen are shown in Table 2.

SPLITTER OPTICS DESIGN

One primary function of the splitter section SX (RX) is to match the optics into (from) the FFAG arc FA (FB). Each splitter line has 8 quadruple magnets, and for each of the four orbits, there are 6 optical parameters to be matched to: $\beta_x, \alpha_x, \beta_y, \alpha_y, \eta_x$, and η'_x (Twiss parameters and dispersion). Figure 6 shows an optical solutions found for the S4 line using a numerical optimizer in Bmad [8]. Another important quantity to be concerned is the r_{56} contribution from the FFAG beamline (from FA to FB) for each design orbit. For each recirculation pass to be isochronous (to the first order), we need the total r_{56} to be zero. This requires the r_{56} from SX and RX sub-beamlines to together cancel the FFAG contribution for each of the 4 design energies. For instance, the FFAG r_{56} contribution of the 42 MeV orbit is -19.9 mm, so S1 and and R1 together needs to contribute +19.9 mm. This adds one extra constraint on the optics matching for each splitter line, and the solution can be numerically difficult to find. A few quadrupole magnets with greater gradient (20 T/m) are required for the solutions eventually found.

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