THE INFLUENCE OF CHROMATICITY ON TRANSVERSE SINGLE-BUNCH INSTABILITY IN THE BOOSTER OF HEPS*

Haisheng Xu†, Na Wang
Key Laboratory of Particle Acceleration Physics and Technology,
Institute of High Energy Physics, Chinese Academy of Sciences, Beijing, China

Abstract

The study of the transverse single-bunch instability has been carried out for the HEPS booster to double check whether the required single-bunch charge can be achieved. The chromaticity has been varied in our study to see how the threshold changes accordingly. Usually, the slightly positive chromaticity is expected for stabilizing the beam. However, our simulations show that the single-bunch threshold current drops significantly when the chromaticity becomes non-zero. We present the simulation methods and results in details in this paper. The analysis of the simulation results is also presented.

INTRODUCTION

HEPS is a quasi- diffraction limited ring based synchrotron light source [1]. The on-axis injection is required because of the milimeter scale dynamic aperture. The proposed injection scheme of the HEPS storage ring [2] requires the injection of full-bunch charge at one time. However, the requirement of the single-bunch charge in the ramping process can be reduced by using the booster as a full-energy accumulator. Nevertheless, the requirement at the injection energy of the booster still strongly motivates the study of the single-bunch instability in the HEPS booster.

Our previous studies show that the transverse single-bunch instability is the limiting factor of the single-bunch charge in the booster. Therefore, a lot of efforts [3] have been made to increase the threshold current in the present design of the HEPS booster, such as, increasing the booster injection energy from 300 MeV to 500 MeV, increasing the momentum compaction factor, optimizing the energy ramping curve, and setting up the proper value of chromaticity, etc.

In our studies, we find that the influence of chromaticity on the threshold current in the HEPS booster is not exactly the same as what we expected. The chromaticity is usually set up as a slightly positive value to stabilize the beam in the rings above transition energy. However, we've found in the simulations that the threshold current drops remarkably when the nonzero chromaticity is used at the injection energy of the booster. To understand the simulation results, both analytic analysis and the more systematic simulations are needed.

The key lattice parameters used in this study are shown in Table 1. At 300 MeV beam energy, the peak RF voltage is set as 800 kV, which can provide the half bucket height $\delta_{sx} = 2.3\%$. This RF voltage is from the preliminary optimization of the energy ramping curve.

Table 1: Key Lattice Parameters of the Booster Used Here

Parameters	Symbols	Values and Units
Circumference	C	453.47 m
Beam Energy	E_0	300 MeV / 6 GeV
Betatron Tunes	ν_x/ν_y	16.40 / 10.73
Momentum Compaction Factor	α_c	4.2e-3
Horizontal Radiation Damping Time	τ_x	36.12 s / 4.56 ms
Vertical Radiation Damping Time	τ_y	36.12 s / 4.51 ms
Longitudinal Radiation Damping Time	τ_δ	18.06 s / 2.24 ms
Radiation Energy Loss per Turn	U_0	25.12 eV / 4.02 MeV
Repetition Rate	f_{rep}	1 Hz
RF Frequency	f_0	499.8 MHz
Harmonic Number	h	756

ANALYTIC ANALYSIS OF THE TRANSVERSE SINGLE-BUNCH INSTABILITY

The Transverse Mode-Coupling Instability (TMCI) and head-tail instability are two important transverse single-bunch instabilities. Two-particle model can be used to illustrate the mechanism of TMCI. The stability criteria of TMCI can be given by [4]

$$\Upsilon = \frac{\pi N r_0 W_0 c^2}{4\gamma C \omega_\beta \omega_s} \leq 2 \tag{1}$$

with the constant wake assumption. We can find that the strong betatron focusing, the faster synchrotron oscillation, and the high energy can all help stabilize the beam, as indicated by the fact that Υ is inversely proportional to ω_β, ω_s, and γ. No explicit dependence of the Υ on the chromaticity ξ can be found in Eq.(1).

For the head-tail instability, two-particle model can also be used to illustrate the mechanism. The growth rate of the betatron oscillations due to the head-tail instability can be therefore expressed by [4]

$$\tau_\pm^{-1} = \mp \frac{N r_0 W_0 c \xi \hat{z}}{2\pi\gamma C \eta} \tag{2}$$

---

[*] Work supported by National Natural Science Foundation of China (No. 11775239), NKPSTRD (2016YFA0402001), and Key Research Program of Frontier Sciences CAS (QYZDJ-SSW-SLH001)

[†] haisheng.xu@ihep.ac.cn

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPAF012

Eq.(2) indicates explicitly that the growth rates of both "+" mode and "-" mode are proportional to the chromaticity. This means that the head-tail instability won't happen at zero chromaticity. When the chromaticity is nonzero, the Eq.(2) shows that there is always one of the two modes unstable. However, it has been pointed out in [4] that the two-particle model has overestimated the growth rate of the "-" mode. Furthermore, if the other damping mechanisms, such as Landau damping and radiation damping, are considered, the slightly positive chromaticity ξ can lead to stable beam motion for the situation above transition.

The above mentioned analytic formulae show that there might two mechanisms to determine the threshod. At zero chromaticity, TMCI is the only mechanism to determine the threshold current. However, it's more complicated in the situations with nonzero chromaticities because either TMCI or the head-tail instability can be dominant. In the cases when head-tail instability dominates, the bunch collective motions will be sensitive to the damping mechanisms, e.g. Landau damping. By analyzing the bunch's unstable motion, one may be able to find out the dominant effect.

The above mentioned analytic models show clear physical pictures of the beam dynamics. However, it's not trivial to deal with the more complicated situations, such as the nonzero chromaticity cases and the influences of the longitudinal motion, etc. We therefore choose to do multi-particle tracking using the `elegant` code [5] and its parallel version `Pelegant` [6] in the detailed study. The detailed studies will be reported as follows.

THE ANALYSIS OF THE TRANSVERSE SINGLE-BUNCH INSTABILITY AT ZERO CHROMATICITY

In this section, we present the detailed procedure used to analyze the simulation data. There are 1,000,000 macro-particles per bunch used in the simulations. The linear one-turn map, represented by the `ILMATRIX` element, is used for tracking. Each bunch is tracked for 5×10^4 turns at 300 MeV beam energy.

The vertical centroids of the bunches with different bunch charges are plotted versus the number of turns in Figure 1. It's easy to judge from the figure that the bunch with 3.5 nC charge is still vertically stable. However, the bunch is already unstable at 4.0 nC/bunch. By carrying out the Fourier analysis of the vertical centorid motion of the bunches, the corresponding betatron frequencies can be obtained. Then, the head-tail mode analysis can be carried out, which is shown in Figure 2. It's clear to see that the mode 0 shifts downwards when the single-bunch charge increases. The mode 0 and mode -1 are coupled at about 4 nC/bunch, indicating that the threshold is about this value, which agrees well with the value obtained from the vertical centroid motion shown in Figure 1.

The Figure 3 is used to illustrate how an unstable vertical centroid oscillation looks like. By fitting the amplitude of the vertical centorid versus number of turns exponentionally, the growth rate can be obtained.

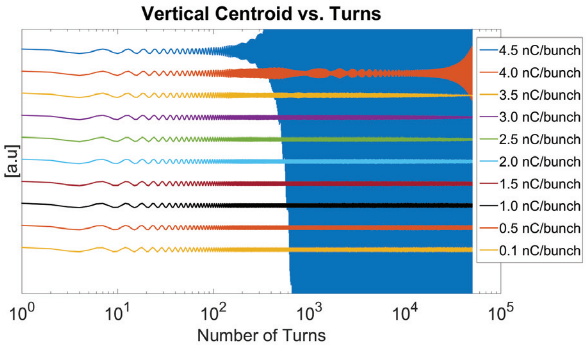

Figure 1: The vertical centroid of the single-bunches with different charges vs number of turns. To make the plot clear, the curves are vertically separated.

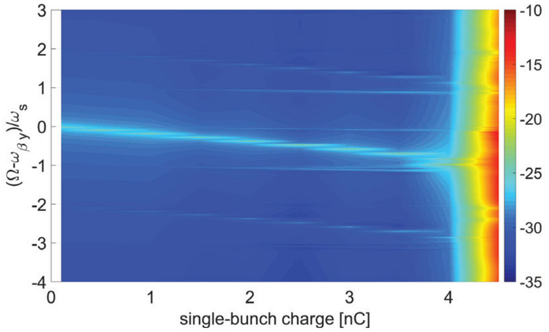

Figure 2: The vertical head-tail modes vs. the single-bunch charges at 300 MeV beam energy when $\xi_y = 0$. The colorbar shows the logarithm of the the amplitudes of the head-tail modes.

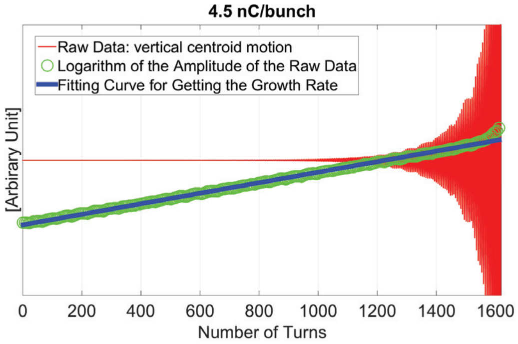

Figure 3: An example to show the method of calculate the growth rate by fitting the amplitude of the vertical centroid motion.

THE ANALYSIS OF THE TRANSVERSE SINGLE-BUNCH INSTABILITY AT NONZERO CHROMATICITY

Similarly as the analysis carried out at zero chromaticity, we carry out particle tracking at +1 chromaticity under same other settings. The vertical centroid of the bunches with different charge are plotted together in Figure 4. The threshold should be between 0.6 nC/bunch and 0.8 nC/bunch.

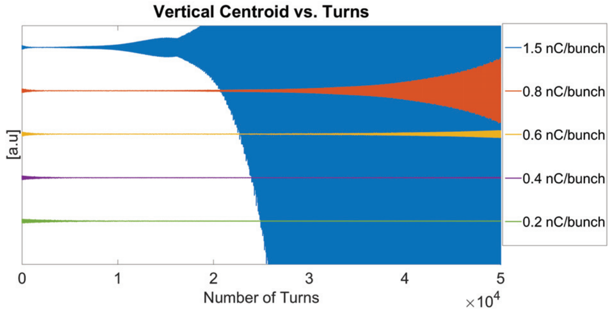

Figure 4: The vertical centroid of the single-bunches with different charges vs number of turns. To make the plot clear, the curves are vertically separated.

The similar head-tail mode analysis is shown in Figure 5. Comparing with Figure 2, the case with +1 chromaticity doesn't show any coupling between two modes. The growth of the +1 mode and -1 mode (hard to distinguish between the two modes that which is the reason of the instability) can be seen. In this case, we believe that the head-tail instability is responsable for the unstable motion of the bunch.

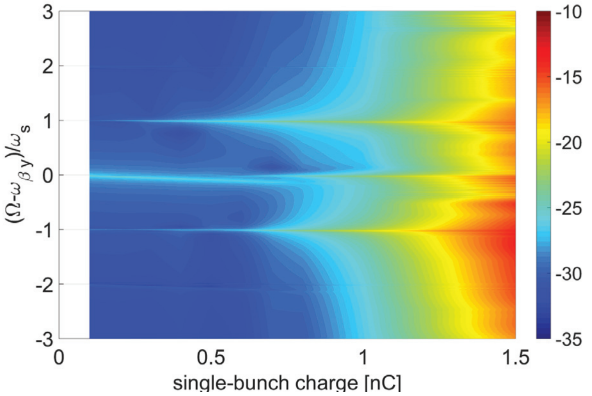

Figure 5: The vertical head-tail modes vs. the single-bunch charges at 300 MeV beam energy when $\xi_y = 1$. The colorbar shows the logarithm of the the amplitudes of the head-tail modes.

DISCUSSION AND CONCLUSIONS

In the above mentioned case with zero chromaticity, the mode coupling is shown clearly in the simulation data. The phenomenon can be well predicted by the analytic method (e.g., by Eq.(1)), which indicates that correctness of the simulation method as well as the physical understanding.

In the other case with +1 chromaticity, the growth of the individual mode instead of the mode coupling is shown by the simulation results, which means the head-tail instability dominates. This situation is able to be predicted by the theory of the TMCI and head-tail instability.

As mentioned above, both the simulation results at zero chromaticity and nonzero chromaticity can be explained by the theory. This fact means that the simulation results are trustable. So we can trust that the threshold at +1 chromaticity (between 0.6 nC/bunch and 0.8 nC/bunch) is much lower than the threshold at zero chromaticity (between 3.5 nC/bunch and 4 nC/bunch). This conclusion is different from the common sense that the slightly positive chromaticity can help stabilize the beam. However, it is most probably because the weak radiation damping at a relatively low energy of 300 MeV and the Landau damping hasn't been carefully included in the simulations.

It's also interesting to mention the simulation results at 6 GeV using the same lattice in [7]. At 6 GeV energy, the threshold at +1 chromaticity is a bit higher than that at zero chromaticity. The mode analysis shows that both the cases with the chromaticity of zero and +1 are dominated by the mode coupling instability. This phenomenon can be explained by the very strong suppression of the head-tail instability by the much stronger synchrotron damping at 6 GeV than at 300 MeV.

ACKNOWLEDGMENT

The authors would like to thank the colleagues in the Accelerator Physics Group of IHEP for the fruitful discussions. The author Haisheng Xu would like to thank the '100 Talents Program of Chinese Academy of Sciences' for the support.

REFERENCES

[1] G. Xu *et al.*, "Progress of Lattice Design and Physics Studies on the High Energy Photon Source", presented at IPAC'18, Vancouver, Canada, April 2018, TUPMF052, this conference.

[2] Z. Duan *et al.*, "The Swap-Out Injection Scheme for the High Energy Photon Source", presented at IPAC'18, Vancouver, Canada, April 2018, THPMF052, this conference.

[3] J. Y. Li *et al.*, "The Considerations of Improving TMCI Threshold in HEPS Booster", presented at IPAC'18, Vancouver, Canada, April 2018, TUPMF063, this conference.

[4] Alexander Wu Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators*, John Wiley & Sons, Inc. 1993.

[5] M. Borland, "Elegant: A Flexible SDDS-Compliant Code for Accelerator Simulation", Advanced Photon Source LS-287, September 2000.

[6] Y. Wang and M. Borland, "Pelegant: A Parallel Accelerator Simulation Code for Electron Generation and Tracking", in *Proceedings of the 12th Advanced Accelerator Concepts Workshop*, AIP Conf. Proc. 877, 241 (2006).

[7] Haisheng Xu *et al.*, "Studies of the Single-Bunch Instabilities in the Booster of HEPS", presented at IPAC'18, Vancouver, Canada, April 2018, THPAF014, this conference.