SIMULATIONS OF MODULATOR FOR COHERENT ELECTRON COOLING

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Abstract

Highly resolved numerical simulations have been performed using the code SPACE for the modulator, the first section of the Coherent electron cooling (CeC) device installed in the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL). Simulation results have been benchmarked with analytical solution using uniform electron beam with realistic thermal velocities. Electron bunches with Gaussian distribution and quadrupole field with realistic settings have been applied in the simulations to predict the modulation process and final bunching factors induced by ions with reference and off-reference energies in the CeC experiment at BNL RHIC.

INTRODUCTION

A coherent electron cooling (CeC) [1,2] device consists of three sections: a modulator, where the ion beam imprints a density wake on the electron distribution, an amplifier, where the signal induced by the ions is amplified, and a kicker, where the electron beam carrying the amplified signal interacts with the ions, resulting in cooling for the ion beam. A general schematic of CeC using FEL as amplifier is shown in Figure 1.

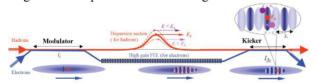
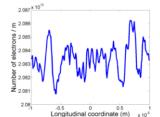


Figure 1: Schematic of coherent electron cooling concept.

Our Simulation tool is the code SPACE [3], a parallel, relativistic, 3D electromagnetic Particle-in-Cell (PIC) code, which has been used for the study of plasma dynamics in a dense gas filled RF cavity [4], the study of mitigation effect by beam induced plasma [5], and the study of plasma-cascade micro-bunching amplifier [6]. The electrostatic module in SPACE has been used in this work as simulations were performed in beam frame. This electrostatic module contains two different approaches. One is PIC method for Poisson-Vlasov equation, which is effective for uniform beam and periodic boundary condition, and numerical convergence has been achieved for this method [7]. The other is adaptive Particle-in-Cloud (AP-Cloud) method [8], which uses an adaptively chosen set of computational particles as the mesh. AP-Cloud method is beneficial for non-uniform distribution particles, geometrically irregular computational domains and mixed type boundary conditions.

Modulation process is basically the Coulomb interactions between ions and surrounding electrons. We have used single ion in all modulator simulations, because the relative density modulation induced by ions is orders of magnitudes smaller than unity and therefore we can treat each ion individually. Analytical solution of modulation process exists for a moving ion in a uniform electron beam [9]. Direct macro-particle simulations are needed for spatially non-uniform electrons in a realistic quadrupole beam-line.

One major difficulty in macro-particle simulations is the detection of the modulation signal, because the shot noise in electron distribution is orders of magnitudes larger than the perturbation induced by the ion. Our method of extracting modulation signal from shot noise is to perform two simulations with identical initial electron distribution, one simulation without ion while the other simulation contains an ion. The modulation induced by the ion can be obtained by taking difference in the final electron distributions of these two simulations. Figure 2 shows the typical noise-to-signal ratio in our simulations. Beam parameters for numerical simulations are relevant to RHIC at BNL. Similar approach of signal extraction has been successfully applied to simulate the FEL amplification process [10].



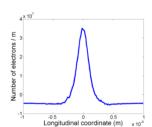


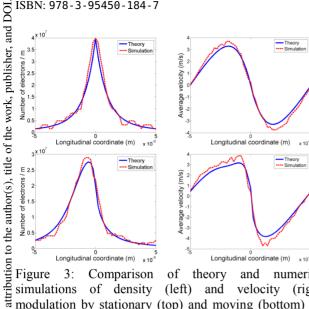
Figure 2: Comparison of magnitudes between shot noise (left) and modulation signal (right) in density distribution of electrons.

UNIFORM BEAM

Spatial uniform electron beam has been used to compare simulation results with theory, as analytical solution of density and velocity modulation exists for the case of a moving ion in uniform electron clouds [11,12], with Kappa-2 probability density function [9,11] modelling thermal velocity. We used a computational domain with 3D periodic boundary condition filled with uniform electron beam to approximate the infinite beam, which is assumed in the analytical solution.

A good agreement of density and velocity modulation between theory and simulations is shown in Figure 3.

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3: Comparison of theory and numerical simulations of density (left) and velocity (right) modulation by stationary (top) and moving (bottom) ion maintain with respect to uniform electron cloud.

NON-UNIFORM BEAM

A realistic finite electron beam with Gaussian distribution has been used in numerical simulations to predict the modulation results in CeC experiments. The effect on modulation process purely from the finite beam size has been studied by applying an artificial linear focusing field [13]. In this section we present simulation results using a Gaussian electron beam in a realistic quadrupole beam-line.

We have compared beam envelope in modulator section using MAD-X, SPACE and Impact-T [14]. SPACE and Impact-T have the capability of turning on and off space charge, while MAD-X doesn't include space charge effect. Figure 4 shows a good agreement between the three codes when space charge is turned off, and reasonably close results between SPACE and Impact-T when space charge is turned on. The discrepancy e between SPACE and Impact-T can be explained by different resolutions. SPACE with AP-Cloud method always uses optimal resolution to balance discretization error and Monte-Carlo error and therefore achieve minimum total error [8], while Impact-T uses uniform mesh, which may introduce larger error calculating space be used under the terms charge forces by a non-uniform beam.

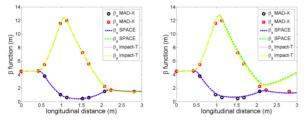


Figure 4: Comparison of transverse β function evolution in quadrupole beam-line using MAD-X, SPACE and Impact-T, with space charge turned off (left) and on from (right).

Figure 5 shows the growth of longitudinal density modulations by a stationary ion and a moving ion at the center of the Gaussian electron beam. A stationary ion has reference energy and stands still in the beam frame. The density modulation induced by a stationary ion maintains a symmetry shape. A moving ion with off-reference energy has different velocity compared with the electrons' group velocity, and results in slippage during the propagation in modulator section. The slippage between the ion and the electron beam causes the asymmetry in the density modulation and reduction in the magnitude of modulation signal.

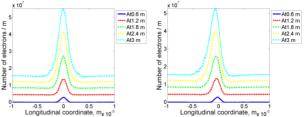


Figure 5: Longitudinal density modulation by stationary (left) and moving (right) ion at the center of the Gaussian electron beam in quadrupole beam-line.

Understanding modulation results by ions with various energies and locations is of essential importance in the simulation studies of CeC, because we need these information and use super position principle to calculate the modulation signal from an ion beam, and therefore predict the cooling time. Modulation process using ions with reference and off-reference energies and on-axis and off-axis positions has been studied [13,15].

BUNCHING FACTOR

Bunching factor is an important parameter in freeelectron laser (FEL) simulations using code GENESIS [16]. FEL acts as the amplifier, the second section of CeC, in the CeC proof-of-principle experiment at BNL RHIC.

The bunching factor is defined as [16] in Equation (1),

$$b \equiv \frac{1}{N_{\lambda}} \sum_{k=1}^{N_{\lambda}} e^{i\frac{2\pi}{\lambda_{opt}} z_k}, \quad -\frac{\lambda_{opt}}{2} \le z_k \le \frac{\lambda_{opt}}{2} \quad (1)$$

where λ_{opt} is the FEL optical wavelength, the summation is over a slice of λ_{opt} wide, centered at the ion's location, and N_{λ} is the total number of electrons within that slice.

Theoretical value of the bunching factor has been derived [17] from the analytical solution of modulation process [11,12], with the assumption that the infinite electron beam has spatial uniform distribution with Kappa-2 velocity distribution [9,11].

Figure 6 shows the theoretical values of the bunching factor amplitudes for different ion velocities. According to this plot, increased ion velocity causes reduction in bunching factor, which is also observed in numerical simulations shown in Figure 5.

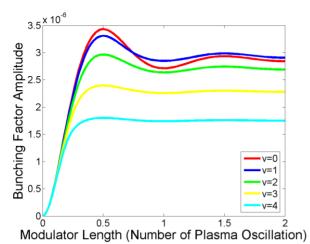


Figure 6: Amplitude of bunching factor for various longitudinal velocities of ion as a function of modulator length. The ion velocity is in unit of electron longitudinal velocity spread [9].

We have compared the bunching factor amplitude between theory and simulations at the exit of 3-meterlong modulator section, which corresponds to a quarter plasma oscillation, and present the comparison in Figure 7. The blue line is calculated from analytical solution [17] and red dots are obtained from numerical simulations shown in Figure 5. The discrepancy between theory and simulation in Figure 7 is due to different distributions of electrons. Uniform distribution is assumed in theoretical solution, while numerical simulations use Gaussian distribution for electrons and a quadrupole beam-line.

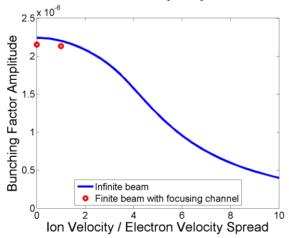


Figure 7: Comparison of amplitude of bunching factor between theory and simulation for various longitudinal velocities of ion at the exit of modulator section (a quarter plasma oscillation). The ion velocity is in unit of electron longitudinal velocity spread [9].

CONCLUSION

We have used an efficient method extracting modulation signal from shot noise. Modulator simulations have achieved a good agreement with analytical solution for a stationary ion and a moving ion in electron clouds with spatial uniform distribution. Modulation process has

been predicted by simulations using a realistic Gaussian beam in a quadrupole beam-line, and bunching factor amplitude from numerical simulation is reasonably close to theoretical value at the exit of modulator section.

With the output of modulator simulations, we are able to perform additional simulations of amplifier and kicker, the second and third section of CeC [18]. From such start-to-end simulations, we will be able to predict the cooling time of the CeC system, which is a help for the CeC experiment in RHIC at BNL.

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