ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-MOPML035

BETATRON FREQUENCIES IN COTANGENTIAL TRAJECTORY ACCELERATOR FOR PROTON BEAM THERAPY

T. Aoki†, F. Ebina, T. Hae, C. Hori, Y. Nakashima, T. Seki, Hitachi, Ltd., Hitachi, Japan

Abstract

In order to downsize accelerator of particle beam therapy, we suggested variable energy accelerator which have cotangential trajectories. This new type accelerator is expected to realize variability of beam energy with static main magnetic field. One of technological subjects of this new type accelerator is stability of betatron oscillation. We plan to utilize weak focusing field as main magnetic field, which is decreasing on the radial direction outward and uniform in longitudinal direction. We found two cases of the trajectory alignment and main magnetic field which realizes stable betatron oscillations as the result of numerical calculation. The betatron tune is limited to below of resonance line of Walkinshaw resonance ($\nu_r - 2\,\nu_z = 0$).

BUCKGROUND AND PURPOSE

It is required downsizing of accelerator to spreading of particle beam therapy (PBT) system. Synchrotron [1], cyclotron or synchrocyclotron [2] is adopted as an accelerator of PBT system. A new type of accelerator, which have cotangential trajectory, is proposed for a PBT system [3]. This accelerator have merits of both synchrotron and synchrocyclotron; one is variability of energy of extracted beam, the other is compactness of accelerator. One of main subject of realize new type accelerator is stability of transverse motion.

A purpose of this study is to understand the betatron frequencies (horizontal and vertical tune) in cotangential trajectory accelerator, especially, to analyse dependencies of tunes on trajectory alignment.

NEW TYPE ACCELERATOR CONCEPT

The new type accelerator have discriminative trajectory alignment (see Fig. 1). Magnetic field along the trajectory of single energy beam is uniform. Thus the designed trajectory form a circle such as in a synchrocyclotron. However, in new type accelerator, the trajectories does not form concentric circles. The centre point of trajectory move along a certain direction in accelerating plane with beam acceleration. Under this trajectory alignment, a point where trajectories of different energies are close to each other is formed. This point is named as trajectory concentration point. A beam channel is placed on outside of trajectory concentration point.

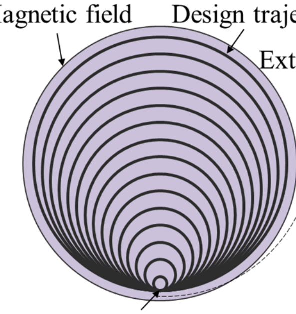

Figure 1: Concept of trajectory alignment of new accelerator.

A main magnetic field of this accelerator is weak focusing field which decreases outward of radius direction. As the result of this field, the circulation frequency is decreases with acceleration and it is required the frequency-modulated accelerating cavity which provides stable synchrotron oscillation based on phase stability principle. Therefore, this accelerator extracts the beam pulse by pulse in analogy with synchrocyclotron. This pulse repetition time will be determined by structure of frequency modulation.

The new type accelerator can extract variety of energy beam by kicker and/or peeler-regenerator field exciting unstableness of horizontal betatron oscillation under resonance condition ($2\nu_r = 2$). The beam having large amplitude of horizontal betatron oscillation will reach external of accelerating region from trajectory concentration point. This principle enables controlling the energy of extracted beam by excitation timing of kicker and/or peeler-regenerator field.

METHOD

We introduce method to estimate and determine the trajectory alignment and required magnetic field which realize stable transverse motion. This method has three steps. The first step is trajectory alignment. The position of trajectory is determined by analytical function, as result the trajectory concentration point is formed. The second step is formulization of required magnetic field by function of potion of the trajectory concentration point. The last step is solving equation of transverse motion. As result, we can obtain betatron frequencies.

Trajectory Alignment

We show the analytical method to determine positon of the trajectories. The trajectories are assumed to exist on single plane (XY plane), and to form circles as result of

† takamichi.aoki.my@hitachi.com

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azimuthally uniform magnetic field. The centre of the circle is placed on Y-axis, and the Y coordinate value of centre defined as Y_C. We can describe Y_C as a function of a radius of the circular trajectory ρ. We can derive degree of concentration κ as

$$\kappa(\rho) \stackrel{\text{def}}{=} \frac{dY_C}{d\rho}. \tag{1}$$

The degree of concentration $\kappa(\rho)$ should satisfy

$$\begin{cases} \kappa(0) = 0, \\ \lim_{\rho\to\infty} \kappa(\rho) \approx 1. \end{cases} \tag{2}$$

The first condition corresponds to stability of transverse motion in low energy. The second one corresponds to forming trajectory concentration point. If the degree of concentration κ is 1, the two trajectories whose energy is close to each other have tangency with each other. We assume the function Y_C and κ as

$$\begin{cases} Y_C(\rho) = \kappa_\infty \rho_Y \left(\sqrt{1 + \left(\frac{\rho}{\rho_Y}\right)^2} - 1 \right), \\ \kappa(\rho) = \frac{\kappa_\infty \left(\frac{\rho}{\rho_Y}\right)}{\sqrt{1 + \left(\frac{\rho}{\rho_Y}\right)^2}}, \end{cases} \tag{3}$$

where ρ_Y and κ_∞ are design parameter each other. The ρ_Y is a parameter which have a dimension of length corresponding to radius of trajectory whose centre begins moving. The κ_∞ is dimensionless parameter determining limit value of infinity radius.

Main Magnetic Field

The main magnetic field is assumed as quadratic function as Y coordinate value of trajectory concentration point Y_0, thus

$$B_z(\rho) = B_0 + \frac{1}{2} S Y_0(\rho)^2, \tag{4}$$

where the B_0 is magnetic field on original point, where acceleration starts, and S is sextupole magnetic field on trajectory concentration point. Under these condition, the magnetic field, trajectory radius, and ion energy of trajectory on arbitrary point in XY plane is able to be determined.

Equation of Transverse Motion

We can calculate betatron frequency by solving linear equation of motion,

$$\begin{cases} \frac{d^2 r}{d\theta^2} = -\left(1 - n(\theta)\right) r, \\ \frac{d^2 z}{d\theta^2} = -n(\theta) z, \end{cases} \tag{5}$$

where r, z and θ are displacement in radius direction and vertical direction of particle from design trajectory, and bended angle from trajectory concentration point respectively, and n is the normalize gradient of field defined as

$$n(\theta) = -\frac{\rho}{B} \cdot \frac{\partial B}{\partial r}. \tag{6}$$

RESULT

The calculated betatron frequency is derived from solving equation of transverse motion (Eq. (5)). The normalized gradient of field n is derived as

$$n(\theta) = \frac{n_0}{1 - \kappa \cos\theta} \tag{7}$$

from magnetic field distribution on XY plane derived from conditions of Eq. (3) and (4), where

$$n_0 = \frac{\rho}{B} \cdot \frac{dB}{d\rho}. \tag{8}$$

This parameter n_0 is called as "gradient scale" here. Therefore, the betatron frequencies depend only two parameters of n_0 and κ. As the result of solving Eq. (5) numerically, cosine of phase advances (μ_r, μ_z) per one turn are obtained. A boundary between stable region and unstable region is found as the contour of $\cos\mu = \pm 1$. The found boundaries in κ- n_0 plane are shown in Fig. 2.

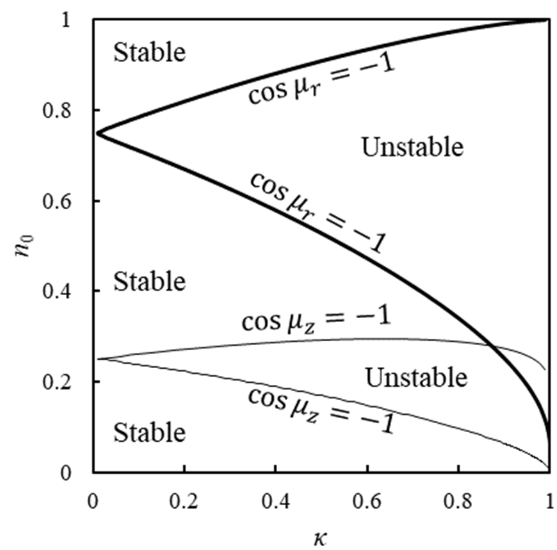

Figure 2: Boundary between stable and unstable region in κ- n_0 plane.

The four nontrivial boundaries are found in κ- n_0 plane. The bold curve respond to boundaries of horizontal motion stability ($\cos\mu_r = -1$), the thin curve respond to boundaries of vertical motion stability ($\cos\mu_z = -1$) respectively. In regions surrounded by boundaries, which noted as "Unstable" in Fig. 2, the transvers motion is unstable in the horizontal and/or vertical direction.

ACCLERATOR DESIGN

It is not applicable for accelerator design that parameters of n_0 and κ corresponding unstable regions indicated in Fig. 2. To realize the cotangential trajectory indicated in Fig. 1, the degree of concentration κ must be close to 1. This means that we must choose parameter where the gradient scale n_0 is below the vertical stability boundary passing through the point of $(\kappa, n_0) = (1, 0)$. These parameter should be $(\kappa, n_0) = (0, 0)$ in low energy. Therefore, these the degree of concentration κ must increase and gradient scale n_0 should keep relatively low value, approximately

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0.1 during acceleration. We show the two example sets of design parameters satisfying these constraints in Table 1. We design proton accelerator for PBT system. Here each parameter set can realize the cotangential trajectory in kinetic energy range larger than 70 MeV, and stable transverse motion in kinetic energy range smaller than 235 MeV. The one, which is called "case A", is relatively similar to flat field condition because of smaller parameter S. The other one, which is called "case B", has the larger gradient field. It should result in higher vertical betatron frequency.

Table 1: Parameter Examples for Proton Accelerator

Parameter	Value (case A)	Value (case B)
ρ_Y [m]	0.086	0.09
κ_∞ [1]	0.99	0.9999
B_0 [T]	-5	-5
S [T/m^2]	17	130

Behavior of Parameters

In the parameter settings, the gradient scale n_0 and the degree of concentration κ varies during acceleration. Behaviors of these parameter in the κ- n_0 plane are shown in Fig. 3. This result indicate that the parameter avoid boundary of vertical motion stability ($\cos\mu_z = -1$) in each case.

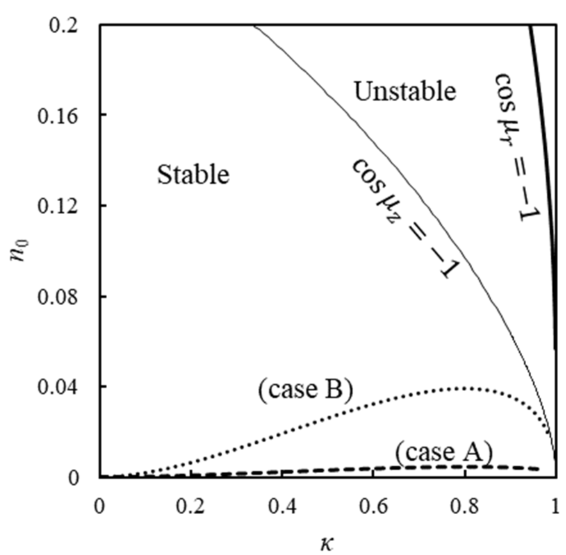

Figure 3: Behavior of parameters in κ- n_0 plane.

Betatron Frequencies

In these case, the behavior of betatron frequencies are different from each other. The result of numerical calculation of equation of motion (Eq. (5)) is shown in Fig. 4 and 5. These diagrams show the energy dependency of betatron tunes. In both case A and case B, the horizontal tunes and vertical tunes start from 1 and 0, respectively. To avoid the resonances causing beam loss, which is half resonance of vertical tune and Walkinshaw resonance ($\nu_r - 2\,\nu_z = 0$), the vertical tune is limited less than 0.5. The horizontal tune is typically 0.996 in the case A, or 0.96 in the case B. As a future task, we will decide which design should be adopted from the viewpoint of realizing of variable-energy extraction.

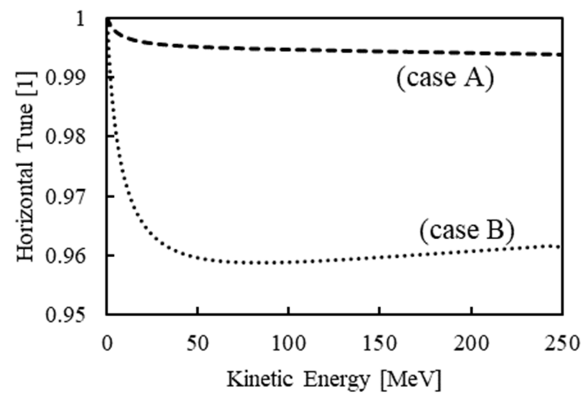

Figure 4: Energy dependency of horizontal tune.

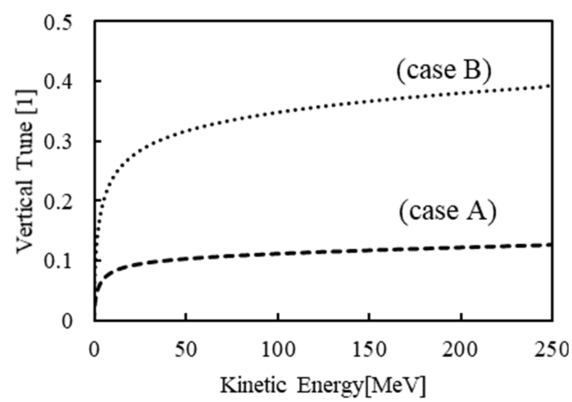

Figure 5: Energy dependency of vertical tune.

CONCLUSION

We proposed the new type accelerator for PBT system. This accelerator have cotangential trajectory alignment to realize variable-energy extraction. The vertical tune is limited low value to avoid the resonance conditions. This can be realized by the parameter indicated in Table 1. Design of magnet and establish of extraction process are future tasks.

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