

# DIPOLE FIELD QUALITY AND DYNAMIC APERTURE FOR FCC-hh

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## Abstract

The Nb<sub>3</sub>Sn dipole design for the hadron machine option of the Future Circular Colliders enters in an intense and long R&D phase. As a result, more realistic dipole field quality evaluations are available for beam dynamics studies. This paper discusses the impact of the main dipole field quality on the first and second order design of the hadron machine and on its dynamic aperture.

## MAIN DIPOLE FIELD QUALITY

For each harmonic  $b_n$  ( $a_n$ ), the field error can be written as the sum of three components, as in the LHC [1]:

$$b_n = b_{nS} + \frac{\xi_U}{1.5} b_{nU} + \xi_R b_{nR}$$

where  $\xi_U$  and  $\xi_R$  denote the random numbers with Gaussian distribution truncated at 1.5 and  $3\sigma$ , respectively. Each of the three components is the sum of three effects: geometric, persistent and ramp induced errors. The systematic values  $S$  are determined with ROXIE [2] for a given design. The random component is determined by Monte-Carlo simulations considering a random displacement with root-mean square amplitude of 50  $\mu\text{m}$ , equally shared by the magnet sub-element degrees of freedom. The field quality tables are given for the cos-theta design option, which is the baseline option [3–5]. The values of the uncertainty  $U$  are set equal to the random values. At this stage a uniform production is assumed. At a later stage the uncertainty values may be different for each manufacturer. The field quality at injection (3.3 TeV) is dominated mainly by the persistent current effects. These are proportional to the critical current density and the effective filament size. Two values for the variation of the critical current density are considered ( $\pm 10\%$  and  $\pm 5\%$ ) which impact the random part components of table v2 and v3, as defined in Ref. [3]. Also, two different effective filament sizes (50  $\mu\text{m}$  and 20  $\mu\text{m}$ ) are considered, which produce the slightly different values for the multipoles of order higher than 3 at injection (respectively table v2 and v3), while no difference is expected at collision. The two dipole field quality tables show a systematic  $b_2$  value of  $\pm 50$  units at collision due to saturation from the iron. Its impact on the arc optics is shown and discussed in Ref. [6].

## $b_3$ CORRECTORS AND ALIGNMENT

Early dynamic aperture simulations have shown that the local correction of the systematic  $b_3$  error of the main dipole is mandatory at collision for FCC-hh [7]. With the current systematic value of  $b_3$  (about 60 units), its correction is

also required at injection energy. One spool-piece corrector (MCS) is placed at each dipole of the arc. Its length is 0.11 m (as in LHC) and the strength of 3000 T/m<sup>2</sup> is required to cancel about 4 units of  $b_3$  at collision and 60 units at injection. The 60 units of systematic  $b_3$  at injection are also a concern for their impact on beta-beating in presence of MCS misalignment. The rms value of the beta-beating due to a random  $b_2$  of the main dipoles can be computed as:

$$\left(\frac{\Delta\beta}{\beta}\right)_{rms} = \frac{\sqrt{Nmb}}{2\sqrt{2}\sin(2\pi Q)} \sqrt{\frac{1}{Nmb} \sum \beta^2} \frac{\alpha}{R_{ref}} \sigma_{b_2}, \quad (1)$$

where  $Q$  is the tune in the plane considered,  $Nmb$  is the number of dipoles,  $\alpha$  is the dipole angle and  $R_{ref}$  is the reference radius of harmonics components [1]. At injection, the residual beta-beating due to the random component of  $b_2$  of the main FCC dipoles is about 5%. There is an additional contribution to the beta-beating, coming from feed-down of  $b_3$  to  $b_2$ , in presence of dipole and MCS misalignments. The  $\sigma_{b_2}^{\text{feed-down}}$  can be computed using the following formulas:

$$\begin{aligned} b_2^{\text{feed-down},mb} &= \frac{2}{R_{ref}} b_3 (x_{mb} - x_{co}) \\ &= \frac{2}{R_{ref}} (\langle b_3 \rangle + \sigma_{b_3}) (\langle x_{mb} \rangle \\ &\quad + \sigma_{mb} - \langle x_{co} \rangle - \sigma_{co}) \end{aligned}$$

for the misalignment of the dipoles, and

$$\begin{aligned} b_2^{\text{feed-down},sp} &= -\frac{2}{R_{ref}} b_3 (x_{sp} - x_{co}) \\ &= -\frac{2}{R_{ref}} \left[ \langle b_3 \rangle (\langle x_{mb} \rangle + \sigma_{mb} + \langle x_{sp} \rangle + \sigma_{sp} - \right. \\ &\quad \left. \langle x_{co} \rangle - \sigma_{co} - \langle x_{co,sp-mb} \rangle - \sigma_{co,sp-mb}) \right] \end{aligned}$$

for the misalignment of the MCS, where  $x_{mb}$  is the alignment of the magnetic axis of the dipole. In LHC its systematic value  $\langle x_{mb} \rangle$  is  $\pm 0.1$  mm, and its standard deviation  $\sigma_{mb}$  is  $\pm 0.5$  mm. We consider  $\langle x_{co} \rangle = \langle x_{co,sp-mb} \rangle = 0$  and we have  $\sigma_{co} \sim 0.4$  mm at injection. The axis of the MCS is supposed perfectly aligned to the dipole (i.e.  $\langle x_{sp} \rangle = 0$ ) while  $\sigma_{sp}$  is the random misalignment of the spool piece relative to the dipole. The contribution due to the small orbit difference between the dipole and its attached spool piece is considered to be random and uncorrelated,  $\sigma_{co,sp-mb} = 0.06$  mm [8]. Adding also a systematic error  $\Delta$  in the spool piece setting,

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we get a total  $b_2$  from feed-down of:

$$b_2^{\text{feed-down}} = -\frac{2}{R_{\text{ref}}} \left[ \Delta \langle b_3 \rangle (\langle x_{mb} \rangle + \sigma_{mb} - \sigma_{co}) + (1 + \Delta) \langle b_3 \rangle (\sigma_{co^{sp-mb}} - \sigma_{sp}) - \sigma_{b_3} (\langle x_{mb} \rangle + \sigma_{mb} - \sigma_{co}) \right]$$

The standard deviation of the feed-down component is then [1]:

$$\sigma_{b_2}^{\text{feed-down}} = \frac{2}{R_{\text{ref}}} \left[ \Delta^2 \langle b_3 \rangle^2 (\sigma_{mb}^2 + \sigma_{co}^2) + (1 + \Delta)^2 \langle b_3 \rangle^2 (\sigma_{sp}^2 + \sigma_{co^{sp-mb}}^2) + \sigma_{b_3}^2 (\sigma_{mb}^2 + \sigma_{co}^2 + \langle x_{mb} \rangle^2) \right]^{1/2}. \quad (2)$$

At injection, for FCC-hh, we have  $\langle b_3 \rangle = -60 \cdot 10^{-4}$ ,  $\sigma_{b_3} = 4 \cdot 10^{-4}$ , and using  $\sigma_{sp}$  equal to  $\pm 0.5$  mm gives  $\sigma_{b_2}^{\text{feed-down}} = 3.6 \cdot 10^{-4}$ . Inserting this value in Eq. 1 results in  $\sim 22\%$  rms beta-beating, which is much higher than the beta-beating coming from the dipoles random  $b_2$  component.

Fixing the maximum  $\sigma_{b_2}^{\text{feed-down}}$  to be  $\leq \sigma_{b_2}$ , and inverting Eq. 2 for  $\sigma_{sp}$  gives:

$$\sigma_{sp}^2 = \left[ \frac{R_{\text{ref}}^2 \sigma_{b_2}^{\text{feed-down}^2}}{4(1 + \Delta)^2 \langle b_3 \rangle^2} - \frac{\Delta^2 \langle b_3 \rangle^2 (\sigma_{mb}^2 + \sigma_{co}^2) + (1 + \Delta)^2 \langle b_3 \rangle^2 \sigma_{co^{sp-mb}}^2}{(1 + \Delta)^2 \langle b_3 \rangle^2} + \frac{\sigma_{b_3}^2 (\sigma_{co}^2 + \langle x_{mb} \rangle^2 + \sigma_{mb}^2)}{(1 + \Delta)^2 \langle b_3 \rangle^2} \right]^{1/2}.$$

The random relative alignment of the MCS with respect to the dipoles as a function of the systematic  $b_3$  value, is shown in Fig. 1. The relative MCS alignment should be between  $\sim 0.070$  and  $\sim 0.1$  mm for a systematic  $b_3$  value of about  $60 \cdot 10^{-4}$ . Keeping the LHC value for the MCS alignment ( $\pm 0.5$  mm,  $1 \sigma$ ) requires a systematic value of  $b_3$  between  $\sim 13$  and  $\sim 14.3 \cdot 10^{-4}$ .

## DYNAMIC APERTURE

The dynamic aperture (DA) is computed using Six-Track [9], for the same initial conditions and parameters described in [7, 10]. At collision, since the systematic component of  $b_3$  is corrected, the main source of DA reduction is the random  $b_3$  component, as shown in Fig. 2. In fact, when the field errors  $b_3$  only are considered in main dipoles, the minimum DA is above  $25 \sigma$ . It reduces to  $20 \sigma$  when all other multipoles are included. It is worth noting that as shown in Fig. 2 the different seeds considered in the simulation produce a very different values of DA between  $20 \sigma$  (minimum) up to almost  $70 \sigma$  (maximum). As will be shown for the injection case, the mean value is highly influenced by the maximum position value scanned in the DA study. Therefore, in the following the minimum DA only is quoted, which is also the worst case. For the CDR

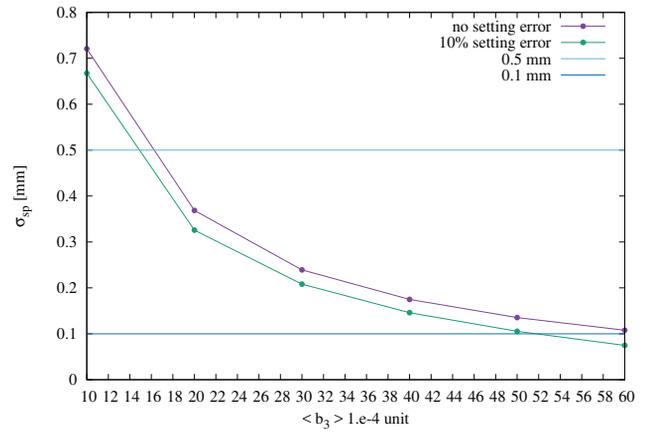


Figure 1: MCS random alignment to have  $\sigma_{b_2}^{\text{feed-down}} = 0.9 \cdot 10^{-4}$  as a function of the systematic  $b_3$  value at injection.

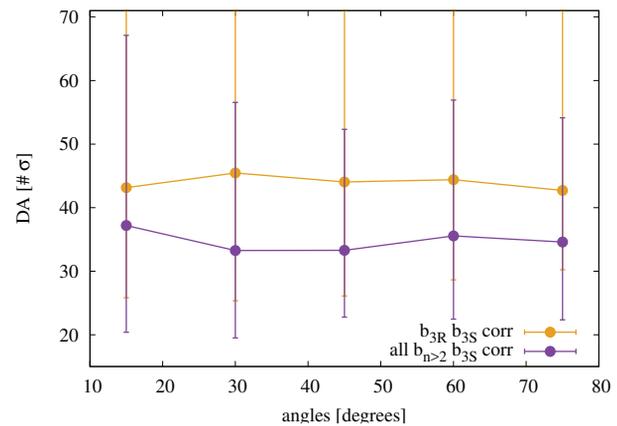


Figure 2:  $10^5$  turns dynamic aperture for 5 directions  $\phi = \arctan(\sqrt{\epsilon_y}/\epsilon_x)$  of the space and 60 seeds. FCC collision optics without systematic  $b_2$  in the dipole and nominal tunes 110.31/108.32.

optics [6, 11], with systematic  $b_2$  component of main dipoles and with the difference in phase advances between IPA and IPG (required to minimize the impact on DA due to the main IR non-linearities [12], called in the following IPA-G collision  $\mu$ ), the DA is greatly improved. A minimum value above  $50 \sigma$  is found due to main dipole errors only.

At injection, the DA is the result of the combination of random dipole errors, as shown in Fig. 3. Contrary to the collision case, the DA significantly changes when the random  $b_3$  alone, or with the random  $b_5$  or when all multipoles are considered. As for the collision case, there is a large spread of DA values. Figure 4 shows the DA histogram of the different seeds for each angle considered: no regular shape can be identified. Table 1 summarizes the minimum DA due to main dipole field quality, considering different optics and including the triplet, separation dipole errors and octupoles for Landau Damping [13, 14]. The reduction of all multipoles with order higher than 3 going from table v2 to table v3 strongly increases DA for a fixed optics. The IPA-G collision  $\mu$  can also strongly change the DA. The minimum

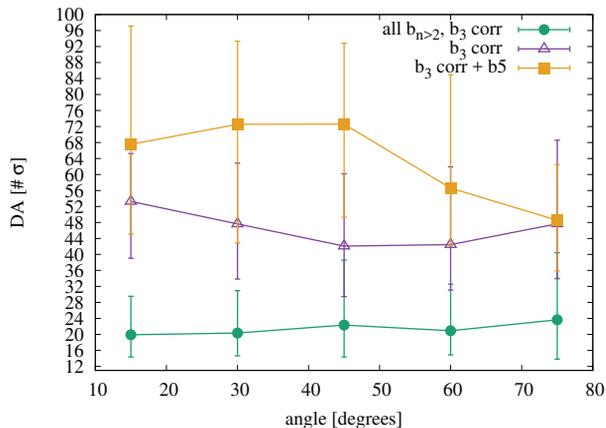


Figure 3:  $10^5$  turns dynamic aperture for 5 directions  $\phi = \arctan(\sqrt{\epsilon_y/\epsilon_x})$  of the space and 60 seeds. FCC injection optics without systematic  $b_2$  in the dipole and with nominal tunes 110.28/108.31. Dipole errors table v2.

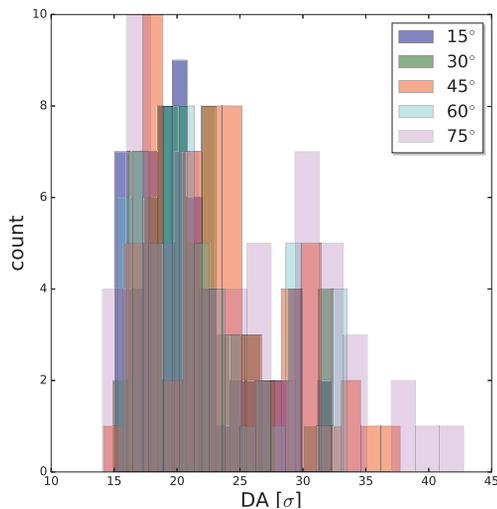


Figure 4: Distribution of  $10^5$  turns dynamic aperture for 5 directions  $\phi = \arctan(\sqrt{\epsilon_y/\epsilon_x})$  of the space and 60 seeds. Data correspond to the green dot and error lines in Fig. 3.

DA due to the inner triplet and separation dipoles errors [12] only is above  $28\sigma$ . When the main dipole errors are also considered the minimum DA decreases: it remains above the target value without the IPA-G collision  $\mu$  while it goes below the target with the IPA-G collision  $\mu$ . Landau Damping Octupoles have a smaller impact on DA, increasing it a bit. In summary, DA is above the target of  $12\sigma$  when main dipole errors are considered only. Therefore, from DA point of view a local correction of  $b_4$  and  $b_5$  systematic dipole field errors are not needed. Interplay of main dipole errors with other magnet errors around the ring can reduce DA below the target value, according to the phase advance between symmetric points in the ring. Therefore, an optimal phase advance between IPA and IPG at injection has to be found taking into account as many errors/non-linear components as possible. In particular, linear imperfections, normal and skew quadrupole errors of both main dipoles

Table 1: Minimum DA at injection. When specified triplet and separation dipoles errors and octupoles are included.

15°	30°	45°	60°	75°	comments
14.4	14.7	14.3	14.9	13.8	$b_2=0$ units, table v2
33.3	31.1	39.2	29.4	30.2	$b_2=0$ units, table v3
16.8	18.4	18.7	20.3	19.9	$ b_2 =50$ units, table v3
17.9	15.3	15.9	15.6	16.3	$ b_2 =50$ units, table v3, triplet and separation dipoles errors
34.1	36	37.2	37.1	37.5	$ b_2 =50$ units, table v3, IPA-G collision $\mu$
5.2	5.2	5.6	3.7	3.8	$ b_2 =50$ units, table v3, IPA-G collision $\mu$ , triplet and separation dipoles errors
7.0	6.9	6.8	5.1	7.1	$ b_2 =50$ units, table v3, IPA-G collision $\mu$ , triplet and separation dipoles errors, octupoles 15/720 A

and quadrupoles [15], as well as non-linear errors of main quadrupoles, are not included in these simulations.

## INJECTION ENERGY CHOICE

In the FCC-hh study, two possible injection energies are considered: the baseline value at 3.3 TeV, using LHC or an HEB as injector (to which all previous results refer) and an alternative energy at 1.3 TeV, using a superconducting SPS as injector [16]. The Dipole Field Quality table is evaluated for the injection energy of 1.3 TeV, as well. The main difference with respect to the 3.3 TeV table is that the Uncertainty and Random components of the natural higher order harmonics are much bigger in the 1.3 TeV table. The corresponding minimum DA is about  $2\sigma$  and the maximum DA in the horizontal plane is about  $8\sigma$ , independently of the optics or the chosen IPA-G phase advance. Finally, it is worth noting that in the 1.3 TeV case the maximum persistent current will occur during the ramp of the magnets to higher field.

## CONCLUSION AND PERSPECTIVES

Present tables of main dipole field quality alone ensure a DA above  $12\sigma$  at the injection energy of 3.3 TeV and much more at collision energy. Injection energy at 1.3 TeV is presently excluded from DA point of view. Local correction of systematic  $b_4$  and  $b_5$  is not required from DA point of view. Instead, local correction of systematic  $b_3$  is mandatory, LHC correctors with a maximum gradient of  $3000\text{ T/m}^2$  allow to correct up to 4 units at collision and 60 units at the injection energy of 3.3 TeV. Corrector misalignments may be a concern at injection due to feed-down of systematic  $b_3$  to  $b_2$ . Finally, interplay of main dipole errors with other magnet errors around the ring can reduce DA below the target value at injection if phase advances between symmetric points in

the ring are not properly tuned. Main quadrupoles errors, as well as linear imperfections of main dipoles are planned to be included in simulations.

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