POLARIZATION IN eRHIC ELECTRON STORAGE RING, AN ERGODIC APPROACH

F. Méot, Brookhaven National Laboratory, Upton, New York 11973, U.S.A*

Abstract

Based on considerations of ergodicity of the dynamical system of an electron bunch at equilibrium, the preservation of polarization in an electron storage ring is estimated from the tracking of a very limited number of electrons. This has a substantial impact on required High Power Computing resources, in noticeable contrast with the method generally used that tracks tens of electron bunches, each comprised of thousands of particles, for several thousands of turns. It is also shown that a minimum number of tracking turns is required in order to ensure the numerical convergence of the linear regressions that yield depolarizing time constant values from tracking, in both methods.

INTRODUCTION

The 18 GeV eRHIC electron storage lattice used in the spin polarization simulations discussed here has been provided by S. Tepikian (BNL), detailed optical properties can be found in [1] and in references therein. The eRHIC lattice includes a double rotator system encompassing the interaction region (IR), comprised of strong solenoids and series of bends, which allows to locally move the stable spin precession direction $\vec{n}_0$, from vertical in the arcs to longitudinal at the IP. In a defect-free ring, this region of off-vertical $\vec{n}_0$ and bends is potentially a major contribution to stochastic spin diffusion.

Bunches are injected in the storage ring with both up and down polarizations, and replaced every 6 minutes in order to ensure an average polarization of about 70% over the hundreds of bunches stored [1]. A proper lattice should maintain bunch depolarization below 20% (absolute) over the 6 minute storage. The evolution of the polarization, from $P_0 = \pm 0.85$ at injection to P_{eq} at equilibrium (an asymptotic quantity to be determined), is given by

$$P(t) = P_{eq}(1 - e^{-t/\tau_{eq}}) + P_0 e^{-t/\tau_{eq}} \qquad (1)$$

This results from antagonic (i) synchrotron-radiation (SR) self-polarization (Sokolov-Ternov (S-T) effect in a flat ring) with asymptotic value P_{SP} (92.4% in S-T hypotheses, taken instead 90% here to account for $\vec{n}_0$ defects around eRHIC ring) and time constant τ_{SP} ($\tau_{SP[sec.]} \approx 99 \rho^2_{[m]} R/E^5_{[GeV]}$ in S-T hypotheses, this is about 30 min. at eRHIC at 18 GeV, 10 hrs at 10 GeV), and (ii) polarization loss by diffusion, with time constant τ_D, such that $1/\tau_{eq} = 1/\tau_{SP} + 1/\tau_D$.
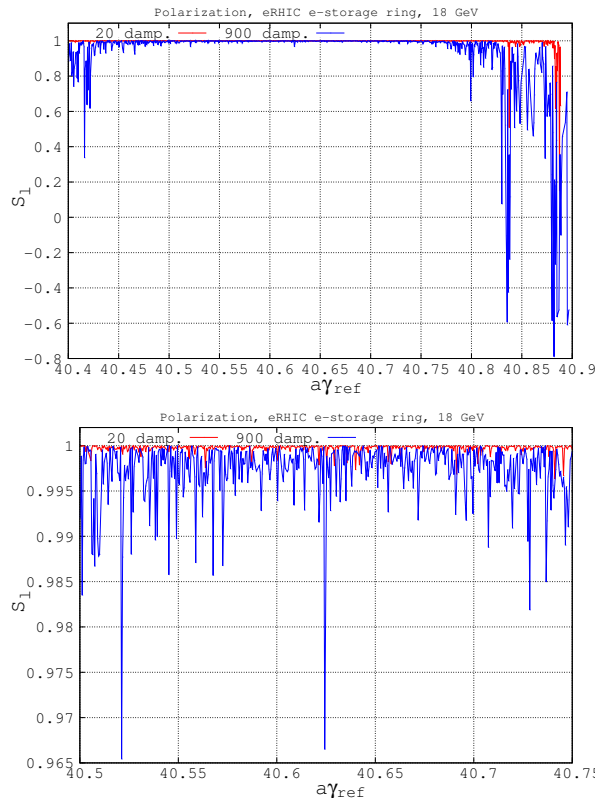
Figure 1: A "single-particle" depolarization landscape $P(a\gamma_{ref})|_{time=t}$, resulting from the tracking of one parti-cle in each one of 1024 rings with energy settings evenly distributed over $40.4 < a\gamma_{ref} < 40.9$ (one ring is one bin, here). Top plot: longitudinal polarization at IP af-ter $20 \times \tau_{SR}$ (red) and after $900 \times \tau_{SR}$ (blue). Bottom plot: a zoom-out on the local topology of the distribution $P(a\gamma_{ref})|_{time=t}$, over a reduced $40.5 < a\gamma_{ref} < 40.75$ in-terval; the topology appears to be similar at these two dif-ferent timings (sort of "fractal"), and it is similar as well in any reduced interval taken in $40.45 < a\gamma_{ref} < 40.75$.

Essentially, the goal of tracking simulations is (i) to validate a ring design including spin matching and other compensation methods, and (ii) to determine an optimal working point $a\gamma_{ref}$ for that ring, amongst a polarization landscape $P(a\gamma_{ref})|_{time=t}$ which may typically look as displayed in Fig. 1. It results from what precedes that, given τ_{SP} (which can be estimated separately from polarization integrals proper to the lattice, and P_{SP} together

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with it [2]), the tracking does not need to include the self-polarization process and in that case $P_{eq} = 0$ and $\tau_D = \tau_{eq}$ ($\tau_{SP} = \infty$). τ_D has to be long to ensure the required polarization survival; this eliminates, for instance, the regions surrounding $a\gamma_{ref} = 40.4$ or 40.85, Fig. 1-top. With $\tau_D \gg$ tracking duration t (see next sections), one can simply use

$$P(t)/P_0 = 1 - t/\tau_D \qquad (2)$$

A fit of this equation with the tracking data provides τ_D and thus $\tau_{eq} = (1/\tau_{SP} + 1/\tau_D)^{-1}$, yielding in turn $P_{eq} = P_{SP} \times \tau_{eq}/\tau_D$ [2] and as well $P(t = 6\,\text{min.})$ (Eq. 1).

The simulations discussed here were performed on NERSC [3], with the tracking code Zgoubi [4]. Electron dynamics and spin diffusion in the presence of Monte Carlo SR is a long installed and, needless to say, thoroughly benchmarked feature of the code [5].

"HPC-HUNGRY": MANY BUNCHES

In order to allow comparisons in the next section, Fig. 2 first summarizes this very HPC-hungry (High Power Computing) method, namely, many particles in a bunch, each bunch tracked for many τ_{SR} (SR damping times) and this repeated for a number of $a\gamma_{ref}$ values for appropriate resolution in determining the optimum ring energy setting [6].

The diffusion time constant τ_D (the quantity of concern in this discussion) is obtained from a linear regression using Eq. 2, its value out of the regression is a function of the number of turns accounted for. It can be observed that about $160 \times \tau_{SR}$, 80,000 turns, are needed in the present hypotheses for τ_D to be converged to better than 10% accuracy, Fig. 2-bottom. Converged values are, respectively $\tau_D \approx 48$, 63, 46 minute for $a\gamma_{ref} = 40.615$, 40.66, 40.705. Apparent as well from the topology in the single-particle landscape, Fig. 1, τ_D changes quickly with $a\gamma_{ref}$.

Note that spin tracking in these simulations only starts when the particles have reached the equilibrium regime, namely after a few damping times. Initial spins are aligned, parallel to the ideal $\vec{n}_0$ (longitudinal at the IP).

"ERGODIC": SINGLE PARTICLE

The dynamical system of a high energy stored electron bunch at equilibrium is ergodic: over a long time interval, trajectories in the system cover all parts of the 6D phase space. Time averages over a trajectory are equivalent to phase space averages,

$$\int f(\vec{X})\rho(\vec{X})d^N\vec{X} = \lim_{T\to\infty} \int_0^T f(\vec{X}(t))dt$$

In particular, and essential in the matter of spin diffusion, over a sufficiently long time (practically, many τ_{SR}), any electron will have explored all possible $\delta E/E$ values within the Gaussian equilibrium momentum distribution, and a sufficiently long time that possible depolarizing effects can manifest themselves.
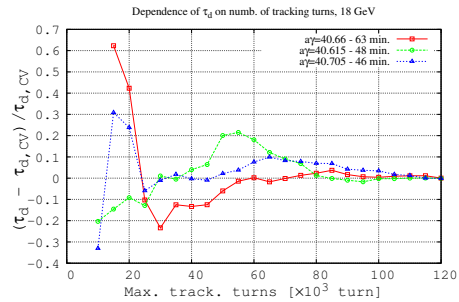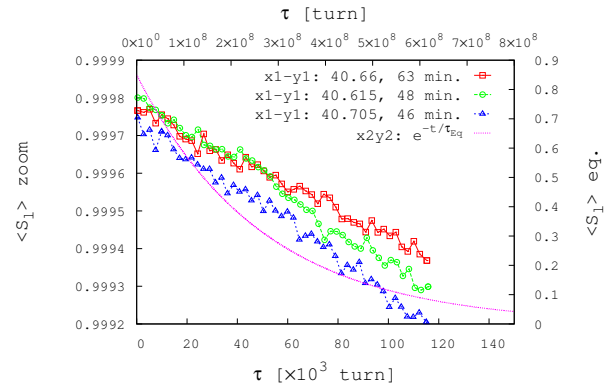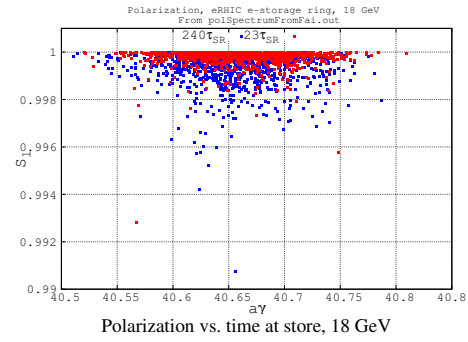
Figure 2: A 10^3-particle bunch is launched in a ring set on some $a\gamma_{ref}$ value, for 240 SR damping time ($\approx$120,000 turns in the 18 GeV region). Top graph: value of the longitudinal component S_l of the 10^3 particles at turn 11540 (red) and turn 119281 (blue). Note: on the horizontal ($a\gamma$) scale, the bunch spans the expected equilibrium $\sigma_{\delta E/E_{ref}} \approx 1.2 \times 10^{-3}$. Middle: evolution of the polarization as a function of turn number, $P(t)$, in three different rings, set respectively at $a\gamma_{ref} = 40.615$, 40.66, 40.705; markers are from the tracking (x1-y1 scales); the solid line represents Eq. 1 on a long time scale for $\tau_D = 63$ minute (x2-y2 scales). Bottom: convergence towards τ_D as a function of the number of turns (the time interval $t : 0 \to t_{max}$) accounted for in the regression.

Spin diffusion has to be a slow process for a lattice to be viable, in particular this cannot be the case if the energy excursion of the particle neighbors (is within a few resonance strengths from) a depolarizing spin resonance ($\nu_s \pm l\nu_x \pm m\nu_y \pm n\nu_l = \text{integer}$). In such case, the depolarization *is not* slow (the orientation of the spin vector

changes substantially during the tracking), thus the working point of concern, $a\gamma_{\text{ref}}$, is not optimal. *A contrario*, observing only slow change in the spin vector means absence of harmful resonance in the energy interval that the particle spans due to SR, and potentially a viable working point.

Tracking results are shows in Fig. 3. The top plot is similar to the middle one in the HPC-Hungry case, Fig. 2, and similar τ_D values come out of both methods. the convergence towards τ_D is within 10% within about $400 \times \tau_R$ at 18 GeV, about 2.5 times the number of turns needed to converge in the multi-particle bunch technique (*cf.* Fig. 2-bottom).
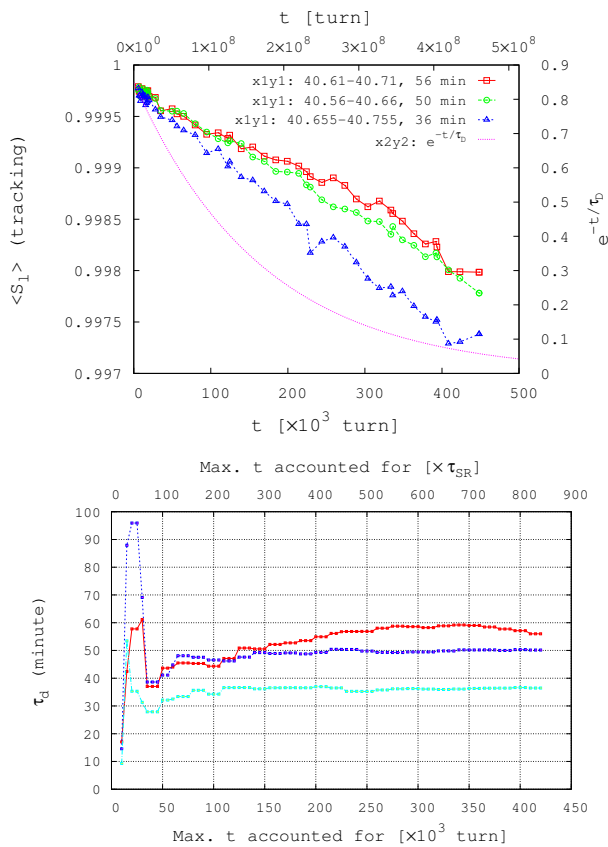

Figure 3: Top plot: (x1-y1 scales) as a function of turn, average polarization of a set of single particles launched evenly spread over three different $a\gamma_{\text{ref}}$ intervals (this is subsets of the interval displayed in Fig. 1-bottom), for $900\tau_{\text{SR}}$, $\approx 450,000$ turns; the solid line represents Eq. 1 on a long time scale (x2-y2 scales). Bottom: convergence towards τ_D (obtained from a linear regression using Eq. 2) as a function of the number of turns accounted for the regression, for the three different $a\Delta\gamma_{\text{ref}}$ cases; converged values are, respectively $\tau_D \approx 55$, 49, 37 minute, all reached within 10% accuracy in about 200,000 turns, $400 \times \tau_{\text{SR}}$.

Similar simulations, HPC-Hungry and "Ergodic" estimates, have been performed at 10 GeV and yield the same observations: similar topology of the polarization density

in the $(S_l, a\gamma_{\text{ref}})$ space, similar diffusion time in various $a\Delta\gamma_{\text{ref}}$ ranges, etc. [7].

SUMMARY

Assume similar resolution using both methods, "HPC-Hungry" and "Ergodic", namely, the same number of reference rings, nRings, in a given interval $a\Delta\gamma$.

In the present hypotheses (eRHIC lattice, energy, etc.):
- first method: the HPC volume is nRings $\times$ 10^3[particles/bunch] $\times 160\,\tau_{\text{SR}}$,
- second method: the HPC volume is nRings $\times 400\,\tau_{\text{SR}}$.

This is a ratio of $10^3 \times 160/400 \approx 400$. Larger HPC volume translates in one or the other of, longer queues, longer computing time, more processors, greater volume of I/Os, larger data analysis HPC volume,...

It remains to determine how close the single-particle method can get to the accuracy of the bunch method (an ongoing work). However it already appears an efficient first approach (in revealing fast depolarizing situation for instance), for qualifying a new lattice design, or error correction schemes. Because it is faster it allows a substantially greater reach (for instance in terms of parameter space exploration) in machine simulations and design optimizations.

CONCLUSION

Obviously these results are very preliminary, they are essentially indications that the HPC volume could be reduced. More simulations are required, for further inspection, comparisons between the two methods, etc. Mathematical background and support is in order.

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