# Direct digitization and ADC parameter trade-off for bunch-by-bunch signal processing

Irene Degl’Innocenti
Università di Pisa, CERN

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# Outline

- Introduction
  - Direct digitization of bunch signals in beam instrumentation
  - ADC parameter trade-off

# Outline

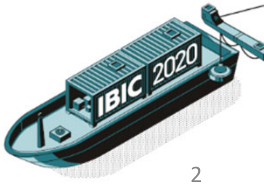
- Introduction
- Analysis of the error in the energy measurement of a direct digitally acquired pulse signal
    - The effect of a limited sampling rate
    - The effect of a limited sampling resolution
    - A combined SNR expression

# Outline

- Introduction
- Analysis of the error in the energy measurement of a direct digitally acquired pulse
- Application example
  - A proposed architecture for the LHC BPM read-out electronics
  - Expected position resolution with commercial ADCs

# Outline

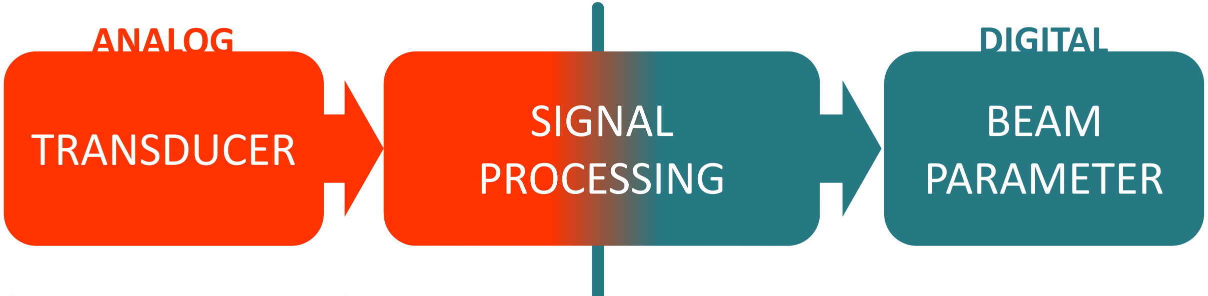
- Introduction
- Analysis of the error in the energy measurement of a direct digitally acquired pulse
- Application example
- Summary

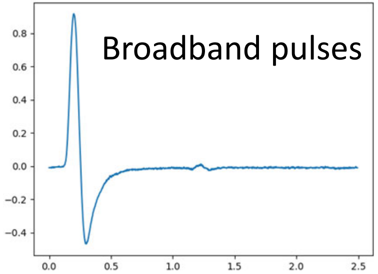


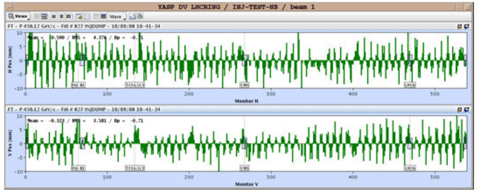
# Introduction

# Introduction

## Direct digitization in Beam Instrumentation







YASP DV LHCRING / INJ-TEST-HB / beam 1

FT - P 450.12 GeV/c - Fill # 827 INJDUMP - 10/09/08 10-41-34

Mean = -0.590 / RMS = 4.376 / Dp = -0.71

FT - P 450.12 GeV/c - Fill # 827 INJDUMP - 10/09/08 10-41-34

Mean = -0.323 / RMS = 3.581 / Dp = -0.71

# Introduction

## Direct digitization in Beam Instrumentation

ANALOG

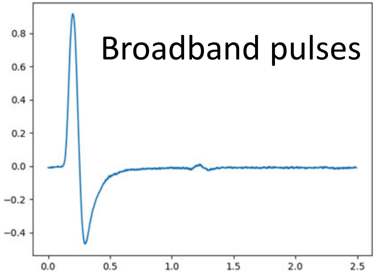
TRANSDUCER

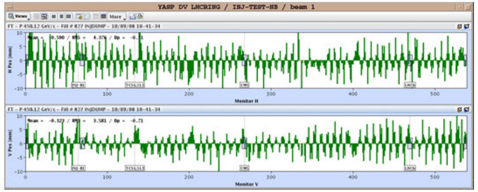
SIGNAL PROCESSING

ADC

DIGITAL

BEAM PARAMETER





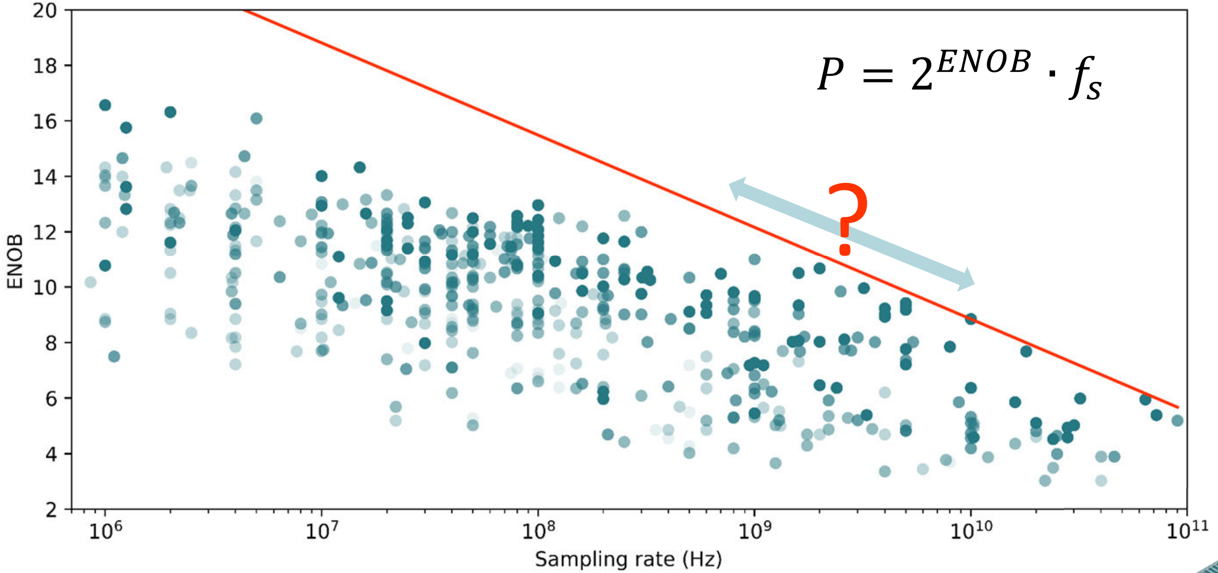
# Introduction

## Direct digitization in Beam Instrumentation

- Less analogue components
  - Less parameter spread
  - Less parameter drifts effects
- Reprogrammable algorithms
- BUT demanding requirements in terms of resolution and sampling rate on the digitization stage

# Introduction

## Analog to Digital Converter trade-off



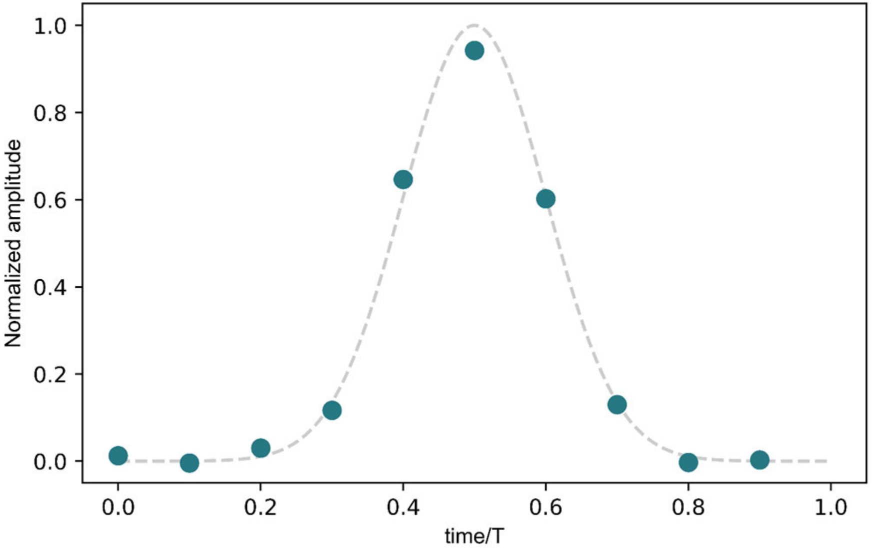
$$P = 2^{ENOB} \cdot f_s$$

Data source: B. Murmann, "ADC Performance Survey 1997-2020," [Online].
Available: http://web.stanford.edu/~murmann/adcsurvey.html

# Analysis of the error in the power measurement of a direct digitally acquired pulse signal

# Problem Definition

## Power measurement of a pulsed signal



$$P_T = \frac{1}{T}\int_0^T |x(t)|^2 dt$$

$$\overline{P_T} = \frac{1}{N}\sum_{n=0}^{N-1}\left(\hat{x}_{n,\tau} + \nu_n\right)^2$$

# Problem Definition

## Power measurement of a pulsed signal

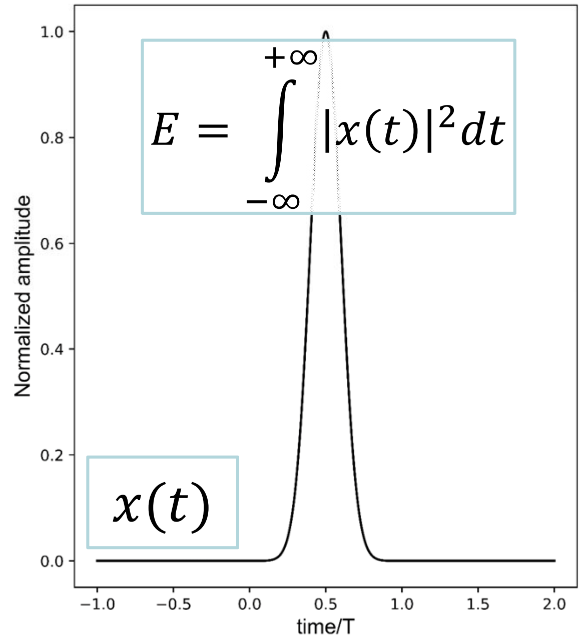
$$P_T = \frac{1}{T}\int_0^T |x(t)|^2 dt$$

$$\overline{P_T} = \frac{1}{N}\sum_{n=0}^{N-1}\left(\hat{x}_{n,\tau} + \nu_n\right)^2$$

- Is $\overline{P_T}$ a good estimation of $P_T$?
  - What is the effect of a limited unsynchronised sampling rate?
  - What is the effect of the finite resolution of the converter?

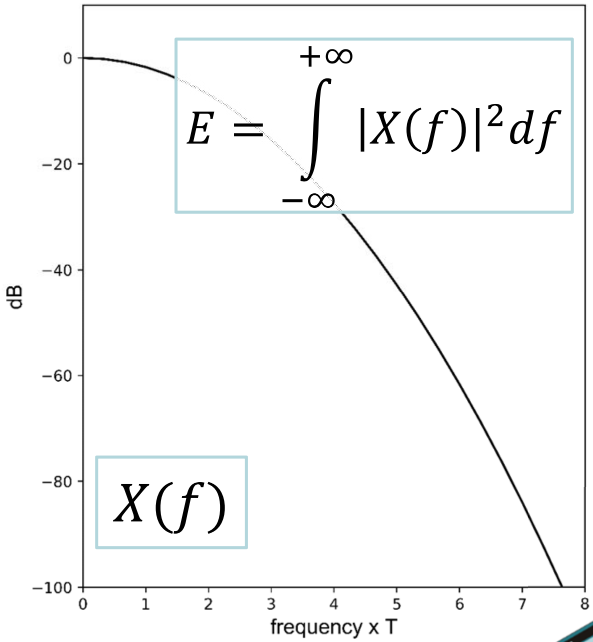
# Limited Sampling Rate

## Energy and power in time and frequency domain



$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt$$

$$x(t)$$



$$E = \int_{-\infty}^{+\infty} |X(f)|^2 df$$

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# Limited Sampling Rate

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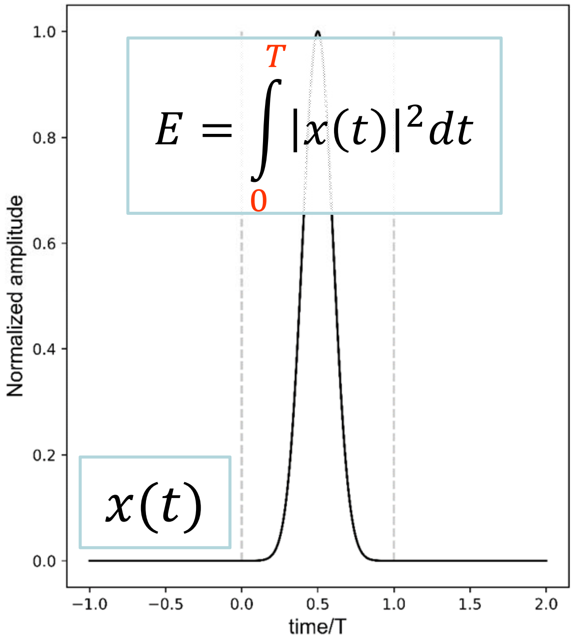
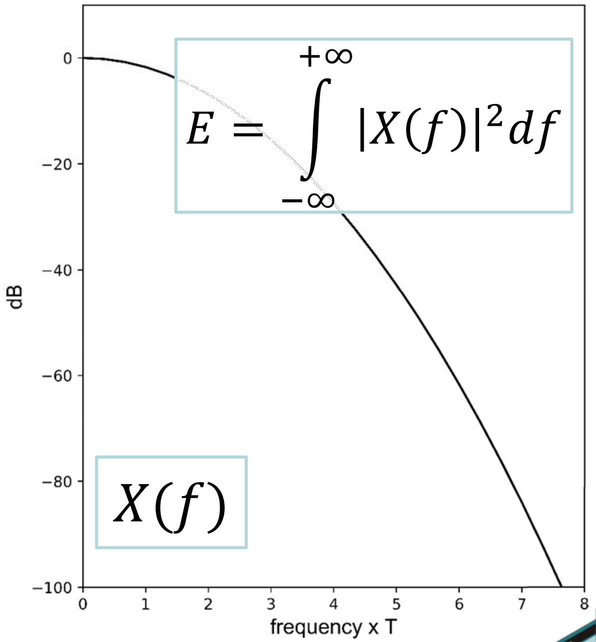


$$E = \int_{0}^{T} |x(t)|^2 dt$$

$x(t)$



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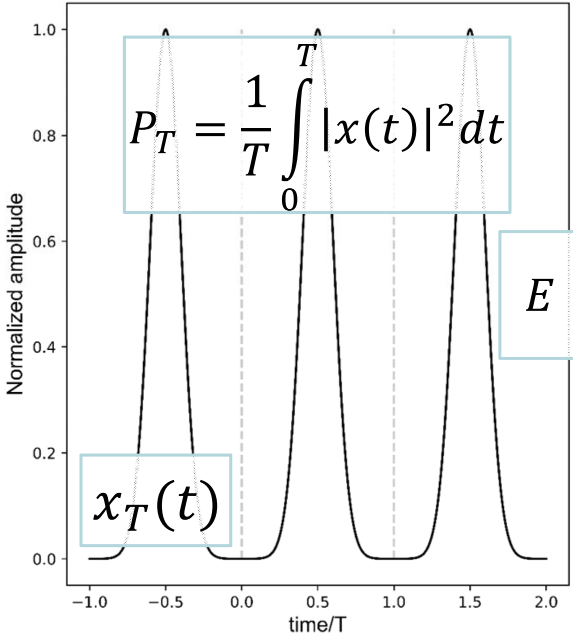
$X(f)$

# Limited Sampling Rate

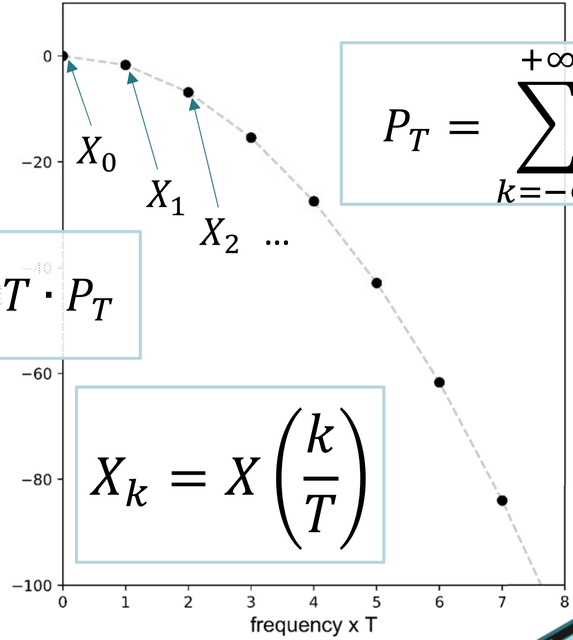
## Energy and power in time and frequency domain



$$P_T = \frac{1}{T}\int_0^T |x(t)|^2 dt$$

$$x_T(t)$$

$$E = T \cdot P_T$$

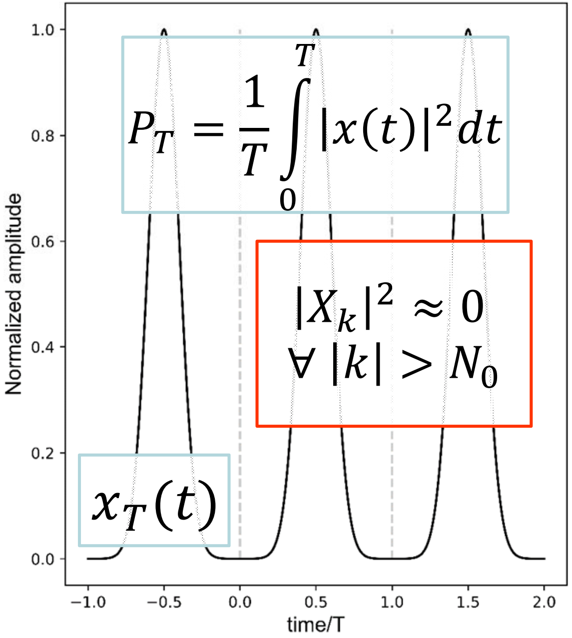


$$P_T = \sum_{k=-\infty}^{+\infty} |X_k|^2$$

$$X_k = X\left(\frac{k}{T}\right)$$

# Limited Sampling Rate

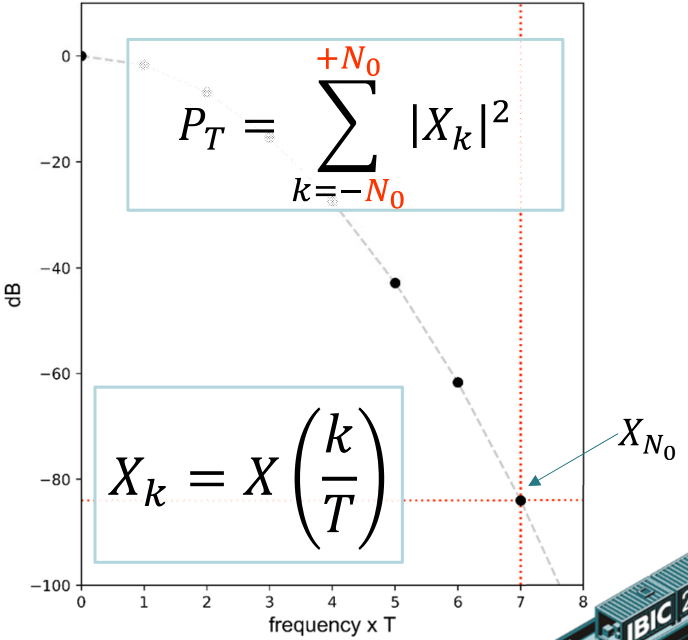
## Energy and power in time and frequency domain



$$P_T = \frac{1}{T}\int_0^T |x(t)|^2 dt$$

$$|X_k|^2 \approx 0$$
$$\forall\ |k| > N_0$$

$$x_T(t)$$



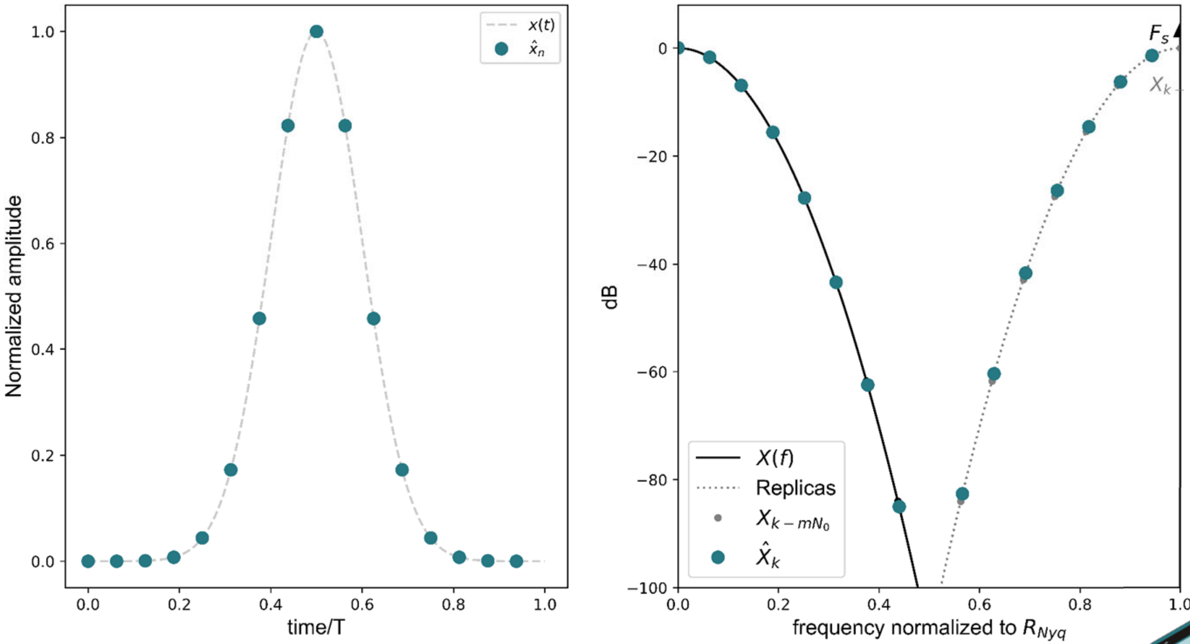
$$P_T = \sum_{k=-N_0}^{+N_0} |X_k|^2$$

$$X_k = X\left(\frac{k}{T}\right)$$

$$X_{N_0}$$

# Limited Sampling Rate

## What happens when we sample the pulse



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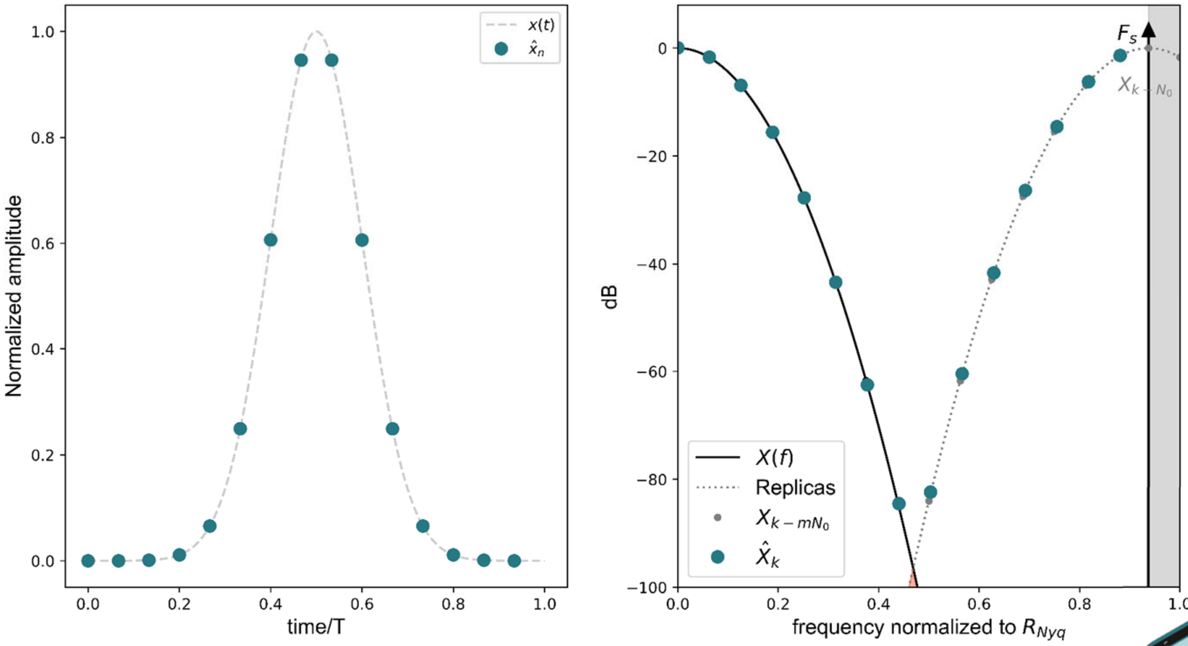

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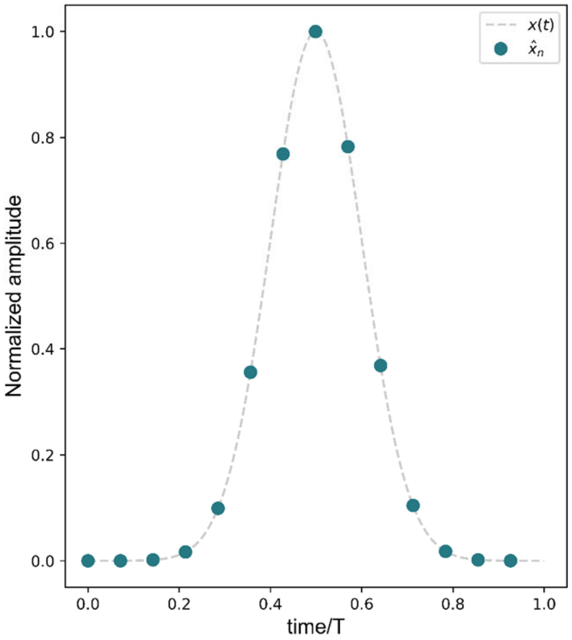

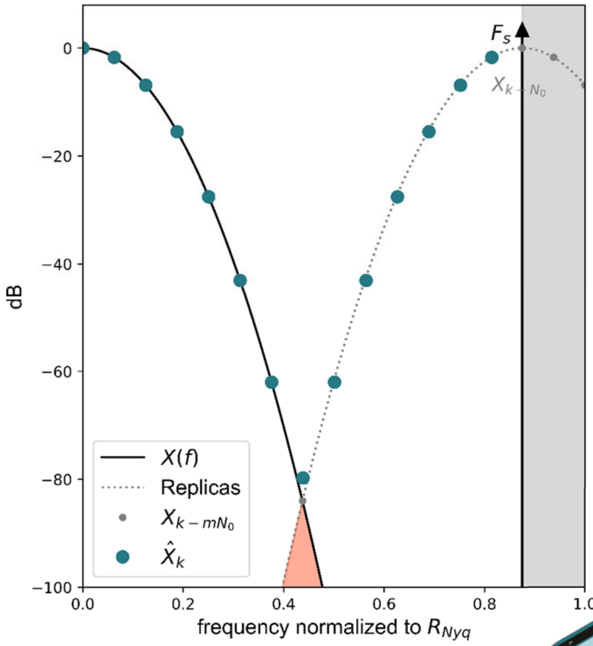

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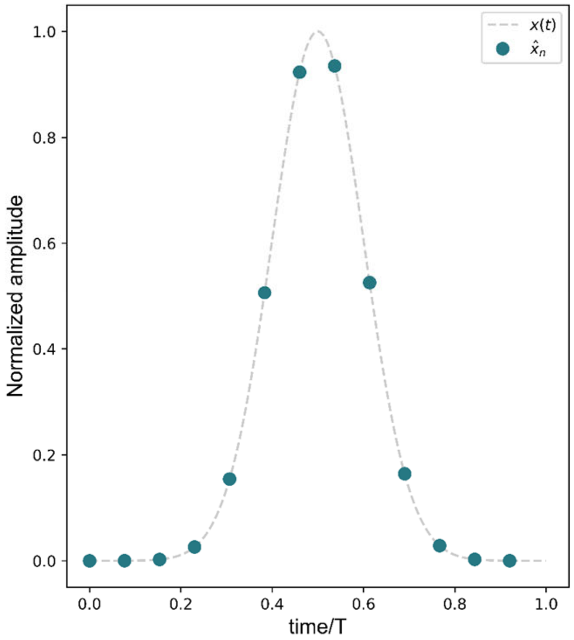

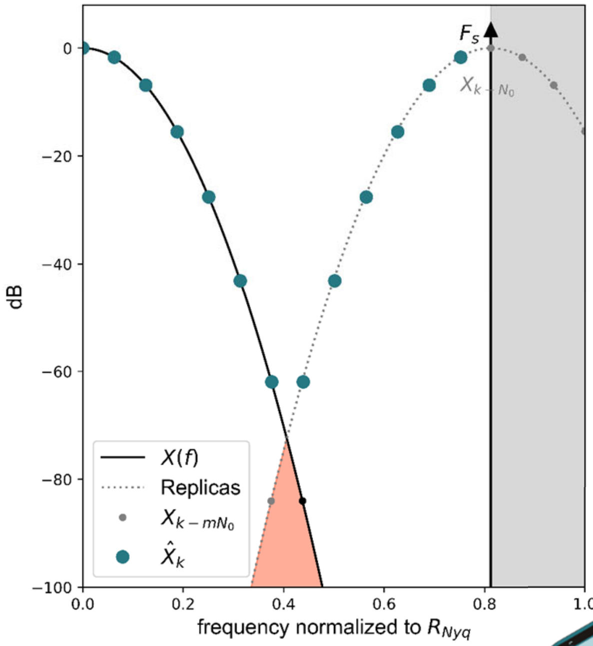

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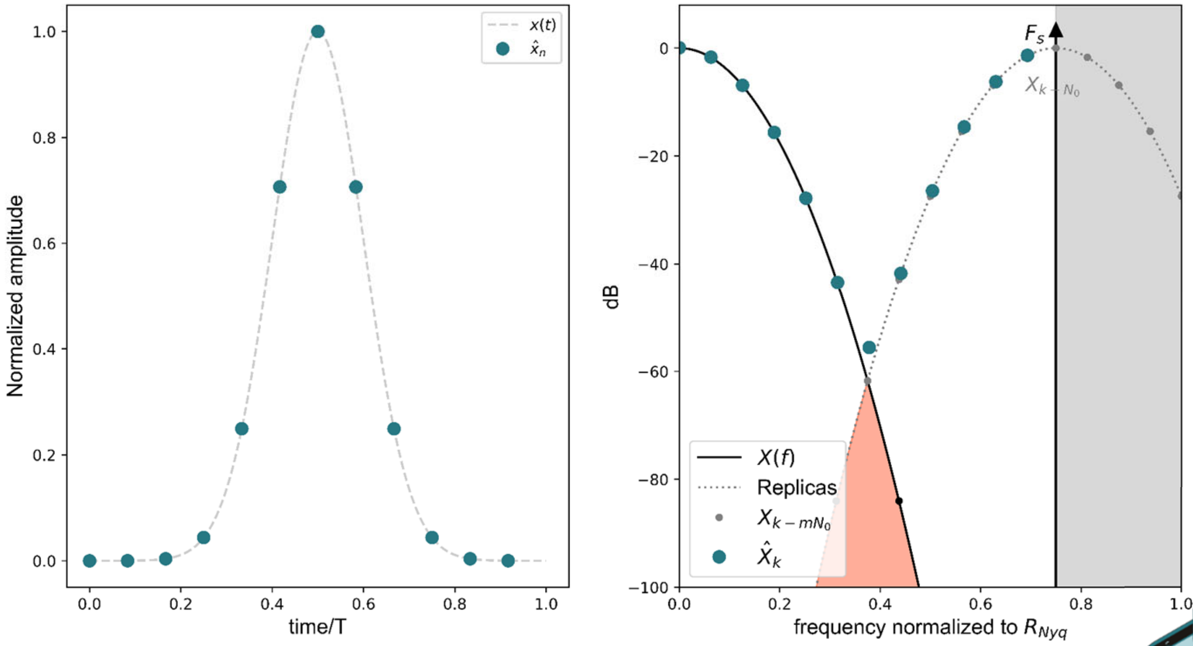

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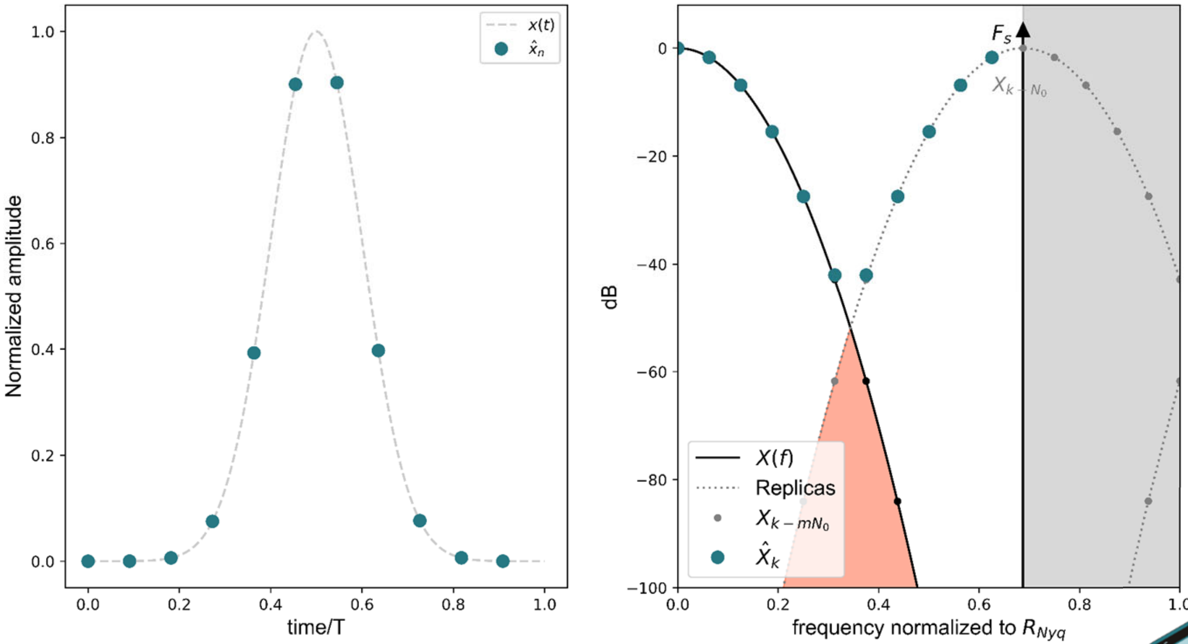

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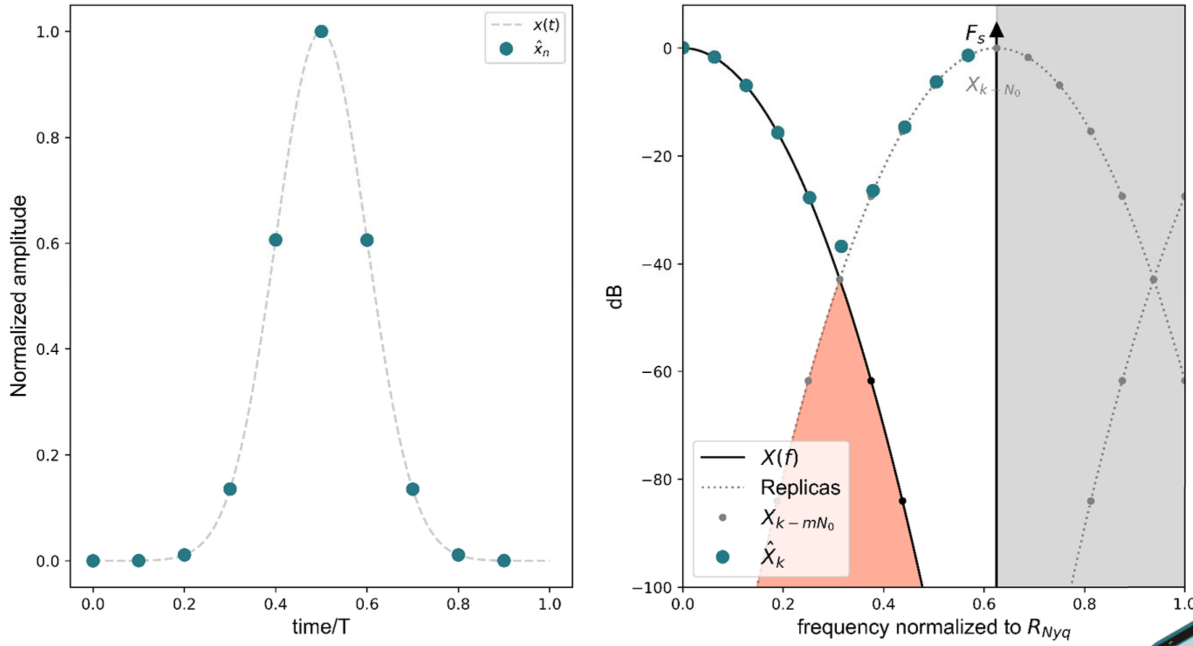

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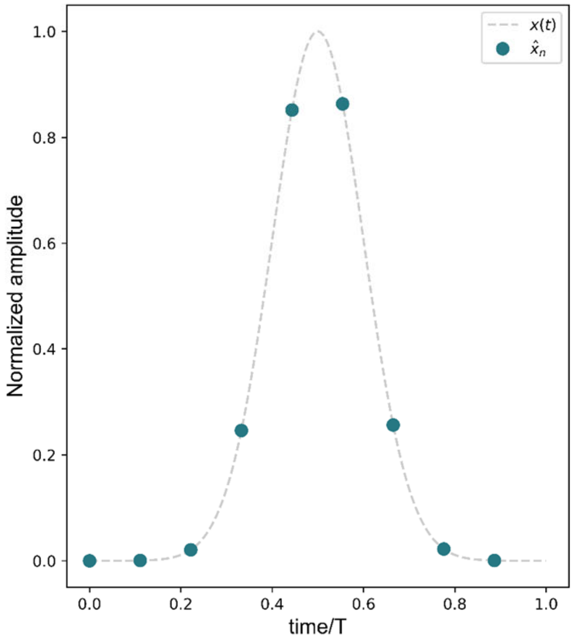

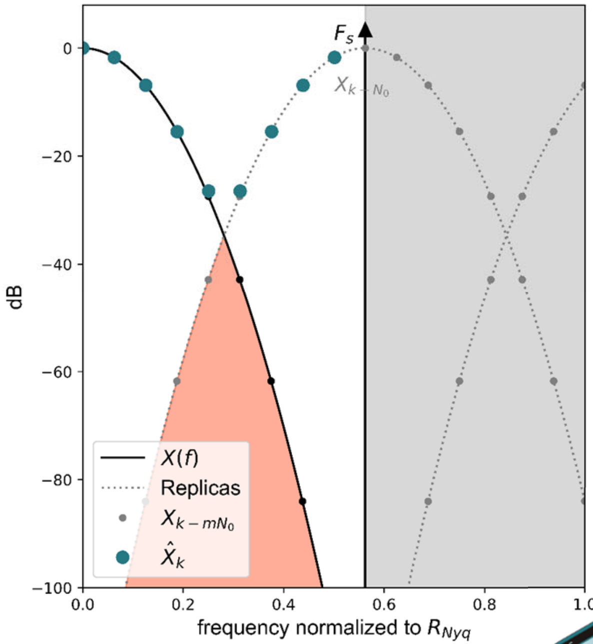

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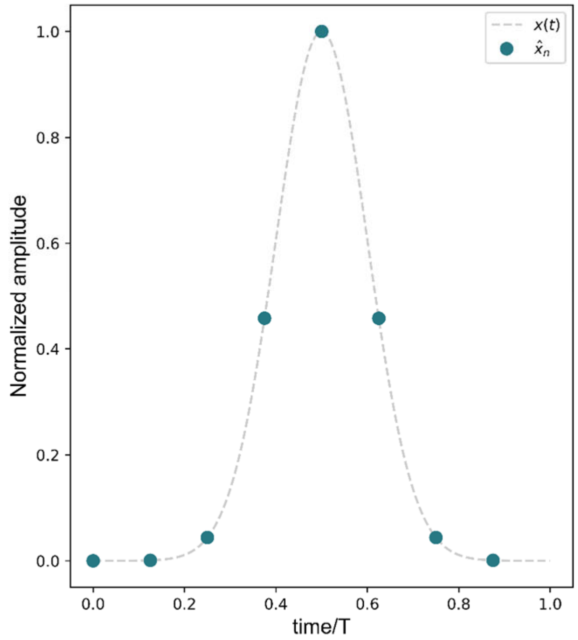

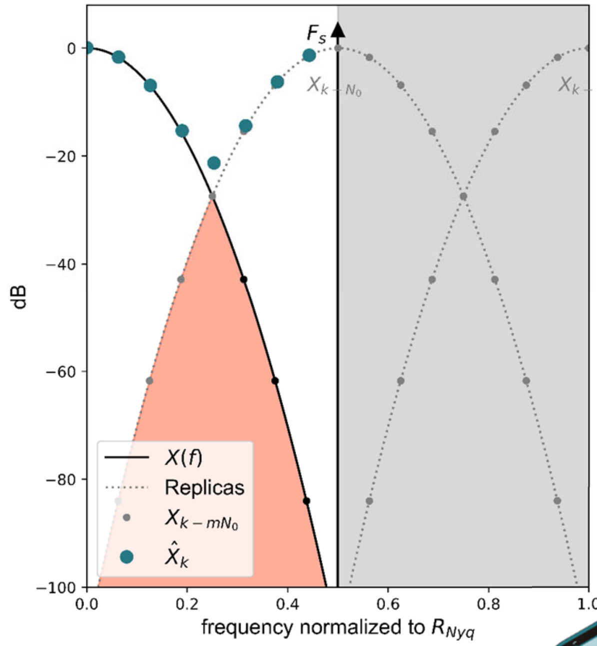

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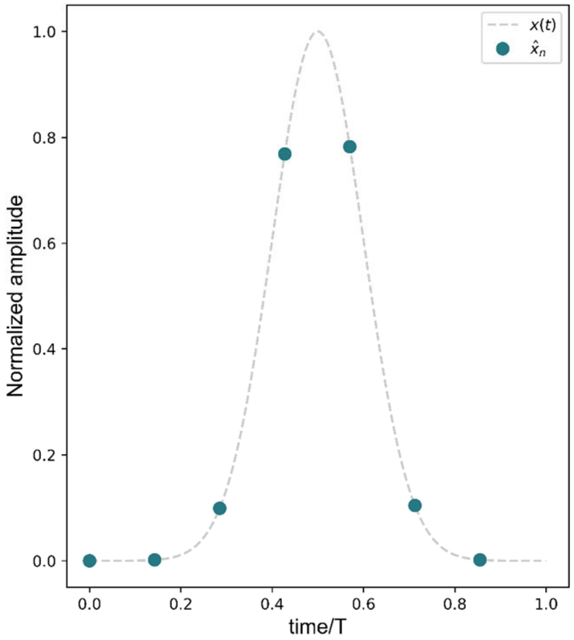

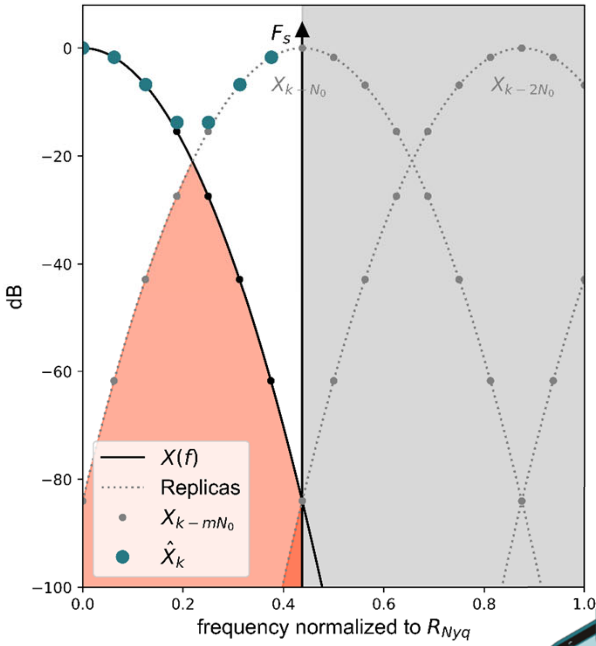

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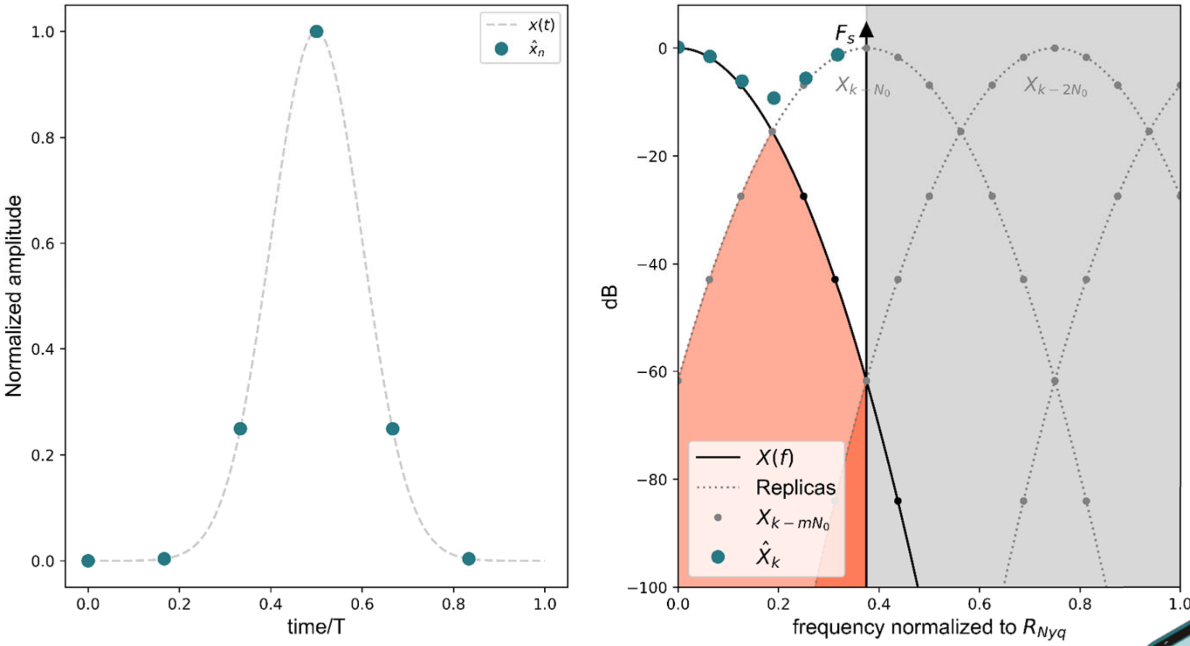
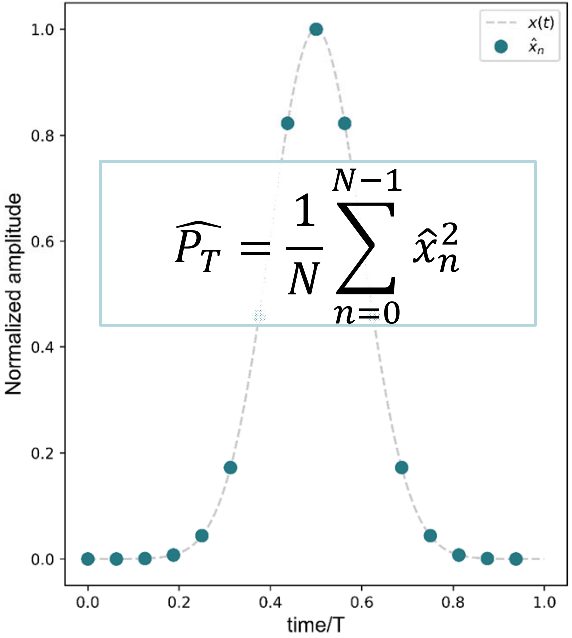
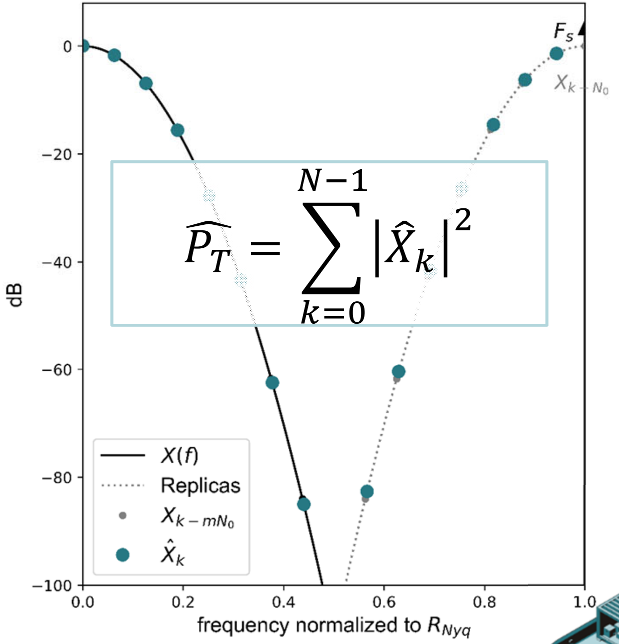


# Limited Sampling Rate

## And the energy estimation?



$$\widehat{P_T} = \frac{1}{N}\sum_{n=0}^{N-1} \hat{x}_n^2$$



$$\widehat{P_T} = \sum_{k=0}^{N-1} \left|\hat{X}_k\right|^2$$

# Limited Sampling Rate

## And the energy estimation?

- If the *Nyquist-Shannon* criterion is met ($F_s > R_{Nyq}$)

$$\Rightarrow \quad \widehat{P_T} = P_T = {}^{E}/_{T}$$

- BUT what if $F_s < R_{Nyq}$?

# Limited Sampling Rate

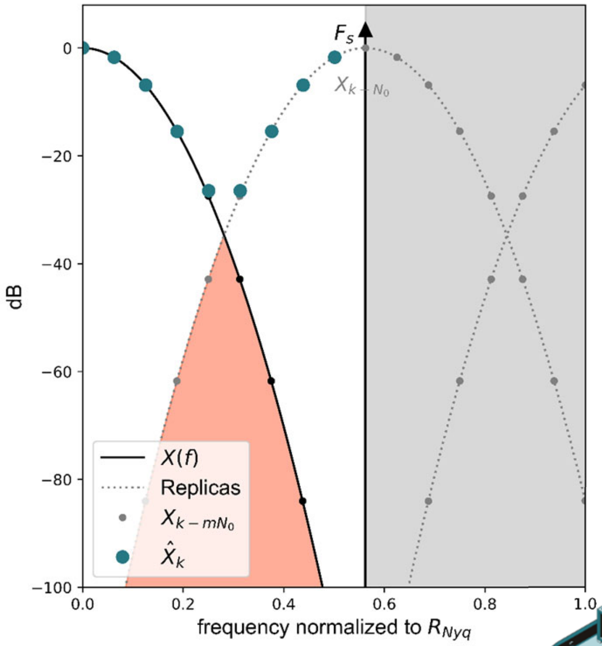
## And the energy estimation?

$$\frac{R_{Nyq}}{2} < F_s < R_{Nyq}$$

$$\widehat{X_k} = X_{k-N} + X_k$$

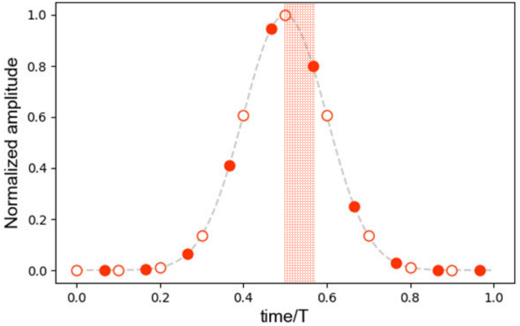
$$\Downarrow$$

$$\boxed{\widehat{P_T} = \sum_{k=0}^{N-1} \left|\hat{X}_k\right|^2}$$



# Limited Sampling Rate

## And the energy estimation?



Sampling rate → Hyp: $\frac{R_{Nyq}}{2} < F_s < R_{Nyq} \leftrightarrow N_0 < N < 2N_0$

Input signal spectrum → $X_k = |X_k| e^{-i\phi_k}$

Sampling delay → $\tau$

$$\widehat{P_T} = \boxed{P_T} + \boxed{2 \sum_{k=0}^{N-1} |X_k||X_{k-N}| \cdot \cos(\phi_k - \phi_{k-N} - 2\pi F_s \tau)}$$

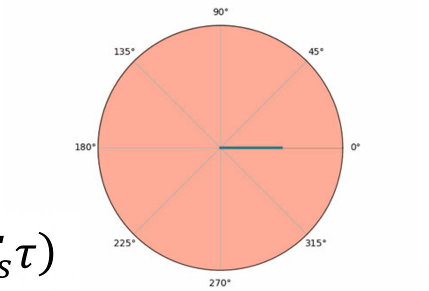
Quantity to estimate

Error term

# Limited Sampling Rate

## The estimation error

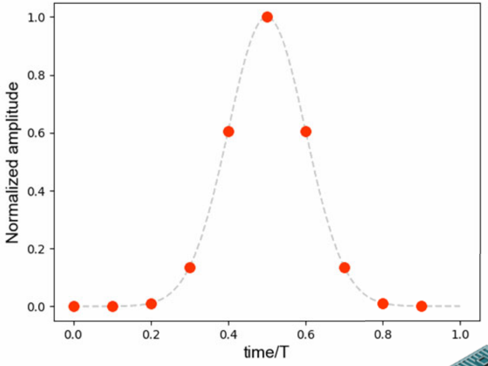
$$\boxed{\epsilon(X_k, F_s, \tau) \triangleq \widehat{P_T} - P_T}$$



$$\epsilon(X_k, F_s, \tau) = A_{X_{k,N}} \cdot 2\cos(2\pi F_s \tau) + B_{X_{k,N}} \cdot 2\sin(2\pi F_s \tau)$$

$$A_{X_{k,N}} \triangleq \sum_{k=0}^{N-1} |X_k||X_{k-N}| \cdot \cos(\varphi_k - \varphi_{k-N})$$

$$B_{X_{k,N}} \triangleq \sum_{k=0}^{N-1} |X_k||X_{k-N}| \cdot \sin(\varphi_k - \varphi_{k-N})$$

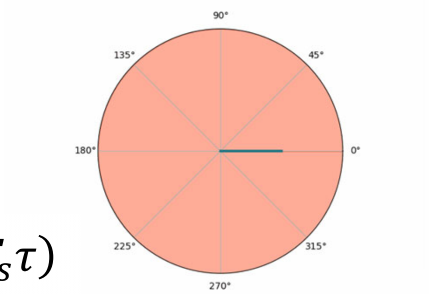


# Limited Sampling Rate

## The estimation error

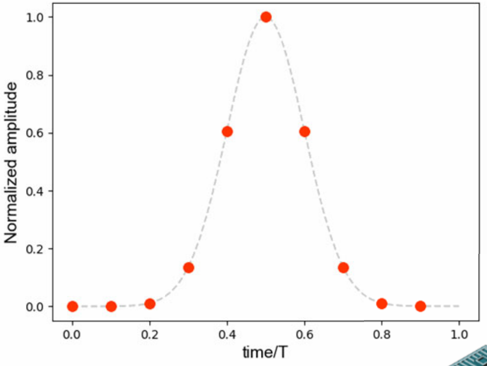
$$\epsilon(X_k, F_s, \tau) \triangleq \widehat{P_T} - P_T$$

$$\epsilon(X_k, F_s, \tau) = A_{X_{k,N}} \cdot 2\cos(2\pi F_s \tau) + B_{X_{k,N}} \cdot 2\sin(2\pi F_s \tau)$$



Hyp: $\tau = \mathrm{U}\left[0, \frac{1}{F_s}\right]$

- $\mu_\epsilon = 0;$
- $\sigma_\epsilon^2 = 2\left(\left(A_{X_{k,N}}\right)^2 + \left(B_{X_{k,N}}\right)^2\right)$



# Limited Sampling Resolution

## The introduction of the converter noise $\nu$

- We now take into account the limited resolution of the ADC
- How does it propagate in the estimator?

$$\boxed{\widehat{P_T} = \frac{1}{N}\sum_{n=0}^{N-1} \hat{x}_n^2} \Rightarrow \boxed{\overline{P_T} = \frac{1}{N}\sum_{n=0}^{N-1} (\hat{x}_n + \nu_n)^2}$$

- Zero-mean Gaussian variable, with $\sigma_\nu^2$ variance

# Limited Sampling Resolution

## The introduction of the converter noise ν

- We now take into account the limited resolution of the ADC
- How does it propagate in the estimator?

$$\overline{P_T} = \widehat{P_T} + \boxed{\frac{1}{N}\sum_{n=0}^{N-1} \nu^2 + \frac{1}{N}\sum_{n=0}^{N-1} 2\hat{x}_n \nu_n} \Rightarrow \boxed{\eta \triangleq \overline{P_T} - \widehat{P_T}}$$

- Zero-mean Gaussian variable, with $\sigma_\nu^2$ variance

# Limited Sampling Resolution

## The introduction of the converter noise ν

$$\boxed{\eta \triangleq \overline{P_T} - \widehat{P_T}}$$

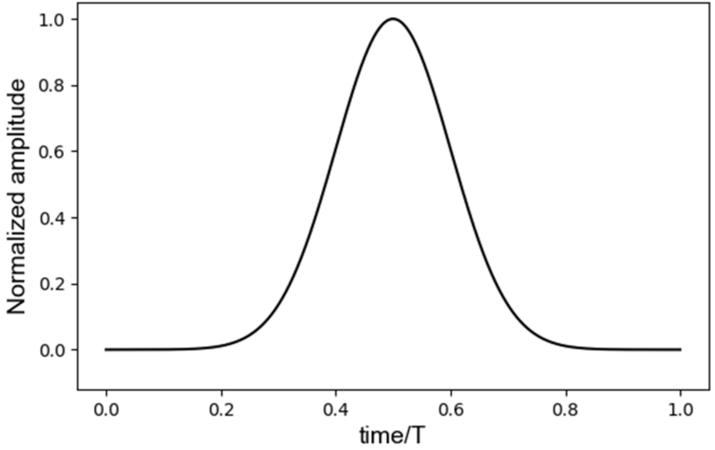
$$\mu_\eta = \sigma_\nu^2$$

$$\sigma_\eta^2 = \frac{\sigma_\nu^4}{N} + 4P_T\frac{\sigma_\nu^2}{N}$$

# A combined SNR expression

## Total error

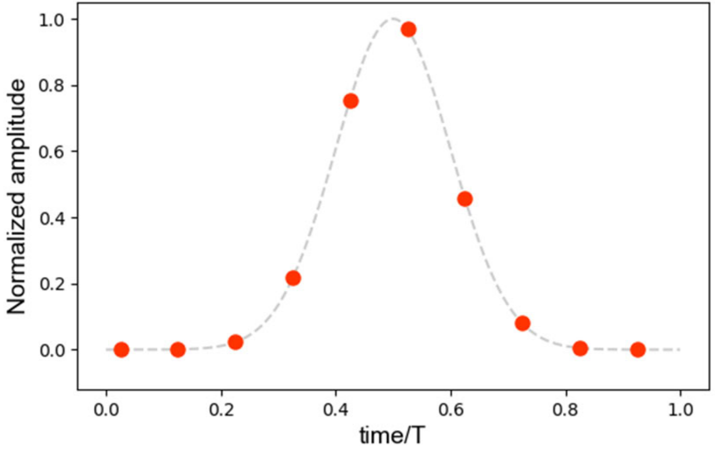
$$\overline{P_T} = \frac{1}{N}\sum_{n=0}^{N-1}(\hat{x}_n + v_n)^2 = P_T$$



# A combined SNR expression

## Total error

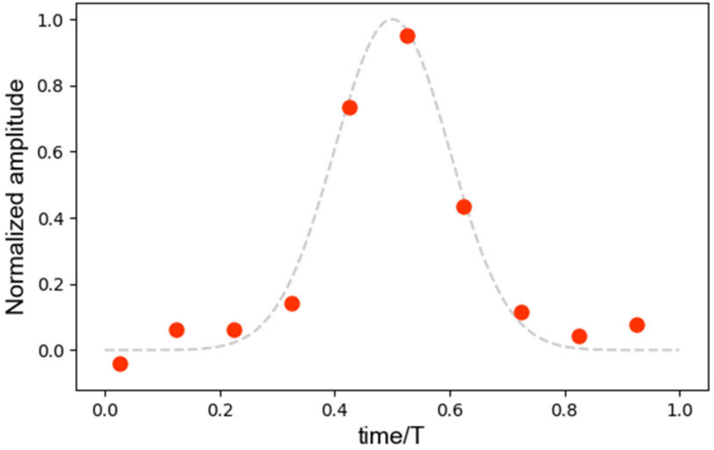
$$\overline{P_T} = \frac{1}{N}\sum_{n=0}^{N-1}(\hat{x}_n + v_n)^2 = P_T + \epsilon(X_k, F_s, \tau)$$



# A combined SNR expression

## Total error

$$\overline{P_T} = \frac{1}{N}\sum_{n=0}^{N-1}(\hat{x}_n + \nu_n)^2 = P_T + \epsilon(X_k, F_s, \tau) + \eta(P_T, \sigma_\nu^2, F_s)$$



# A combined SNR expression

## Total error

$$\overline{P_T} = \frac{1}{N}\sum_{n=0}^{N-1}(\hat{x}_n + v_n)^2 = P_T + \epsilon(X_k, F_s, \tau) + \eta(P_T, \sigma_v^2, F_s)$$

- Mean value:

$$\sigma_v^2$$

- Variance:

$$\boxed{2\left(\left(A_{X_{k,N}}\right)^2 + \left(B_{X_{k,N}}\right)^2\right)} + \boxed{\frac{\sigma_v^4}{N} + 4P_T\frac{\sigma_v^2}{N}}$$

# A combined SNR expression

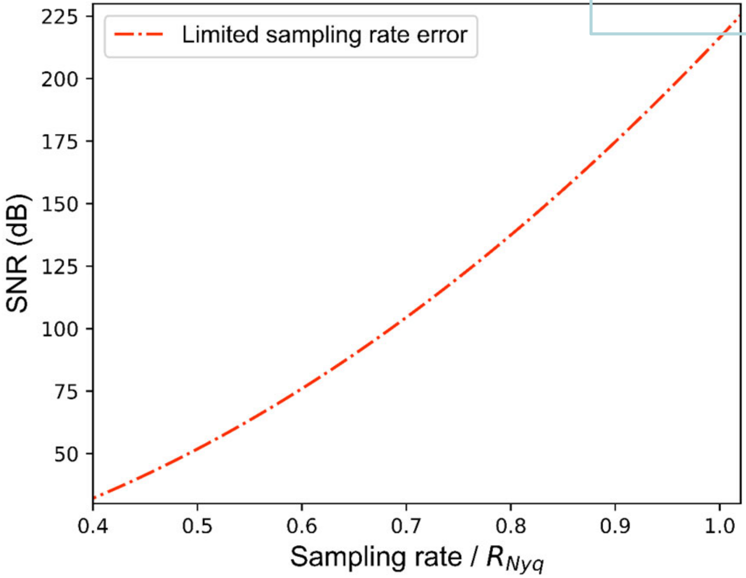
## SNR expression of the averaged power measurement

$$\boxed{SNR_{dB} = 10\log_{10}\left(\frac{P_T^2}{2\left(\left(A_{X_{k,N}}\right)^2 + \left(B_{X_{k,N}}\right)^2\right) + \frac{\sigma_\nu^4}{N} + 4P_T\,\frac{\sigma_\nu^2}{N}}\right)}$$

# A combined SNR expression

## SNR expression of the averaged power measurement

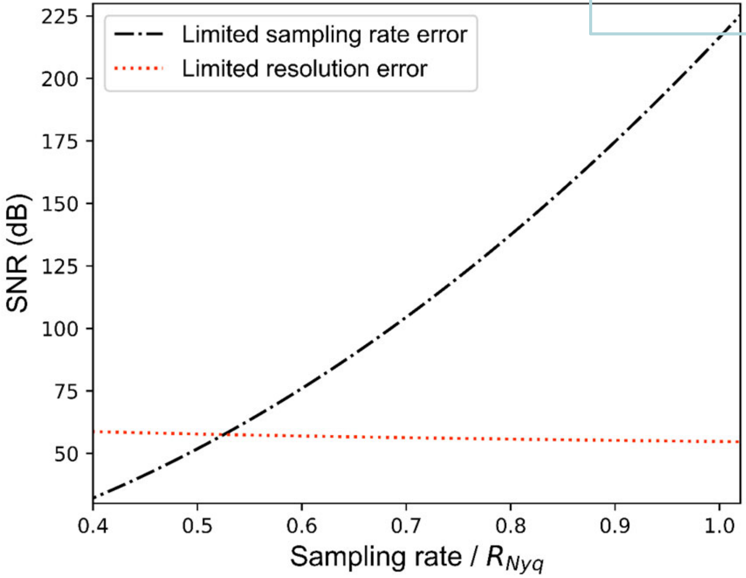
$$SNR_{dB} = 10\log_{10}\left(\frac{P_T^2}{\boxed{2\left(\left(A_{X_k,N}\right)^2+\left(B_{X_k,N}\right)^2\right)}+\frac{\sigma_v^4}{N}+4P_T\frac{\sigma_v^2}{N}}\right)$$



# A combined SNR expression

## SNR expression of the averaged power measurement

$$SNR_{dB} = 10\log_{10}\left(\frac{P_T^2}{2\left(\left(A_{X_k,N}\right)^2 + \left(B_{X_k,N}\right)^2\right) + \boxed{\frac{\sigma_\nu^4}{N} + 4P_T\frac{\sigma_\nu^2}{N}}}\right)$$



- ADC resolution in function of the sampling rate

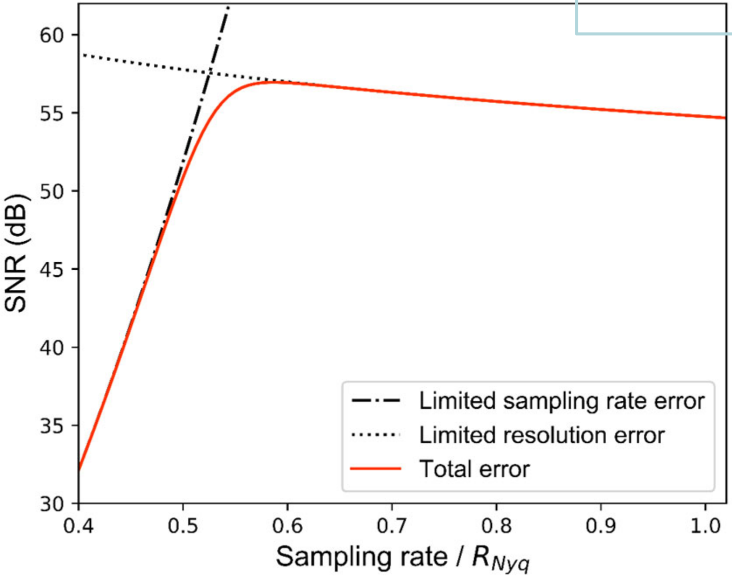
$$\sigma_\nu = \frac{V_{DD}}{\sqrt{12}}2^{-(ENOB-1)}$$

$$P = 2^{ENOB} \cdot f_s$$

# A combined SNR expression

## SNR expression of the averaged power measurement

$$SNR_{dB} = 10\log_{10}\left(\frac{P_T^2}{\boxed{2\left(\left(A_{X_k,N}\right)^2 + \left(B_{X_k,N}\right)^2\right) + \frac{\sigma_v^4}{N} + 4P_T\frac{\sigma_v^2}{N}}}\right)$$



- ADC resolution in function of the sampling rate

$$\sigma_v = \frac{V_{DD}}{\sqrt{12}} 2^{-(ENOB-1)}$$

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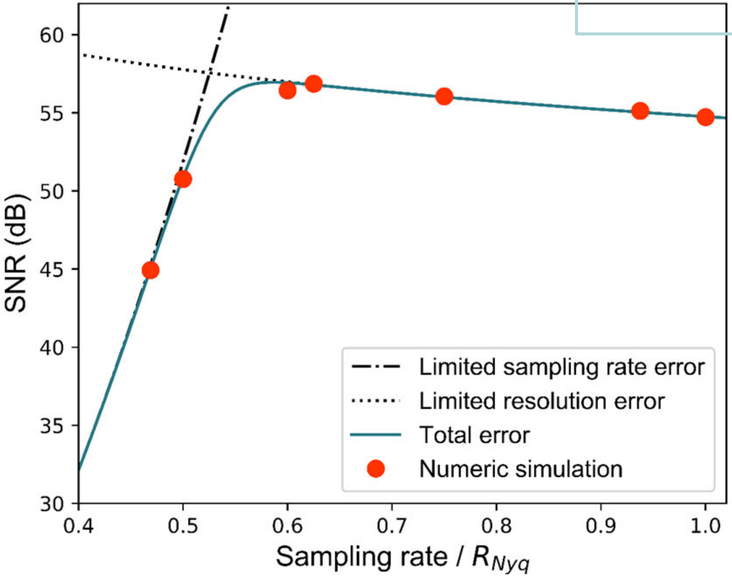


- ADC resolution in function of the sampling rate

$$\sigma_\nu = \frac{V_{DD}}{\sqrt{12}}2^{-(ENOB-1)}$$

$$P = 2^{ENOB} \cdot f_s$$

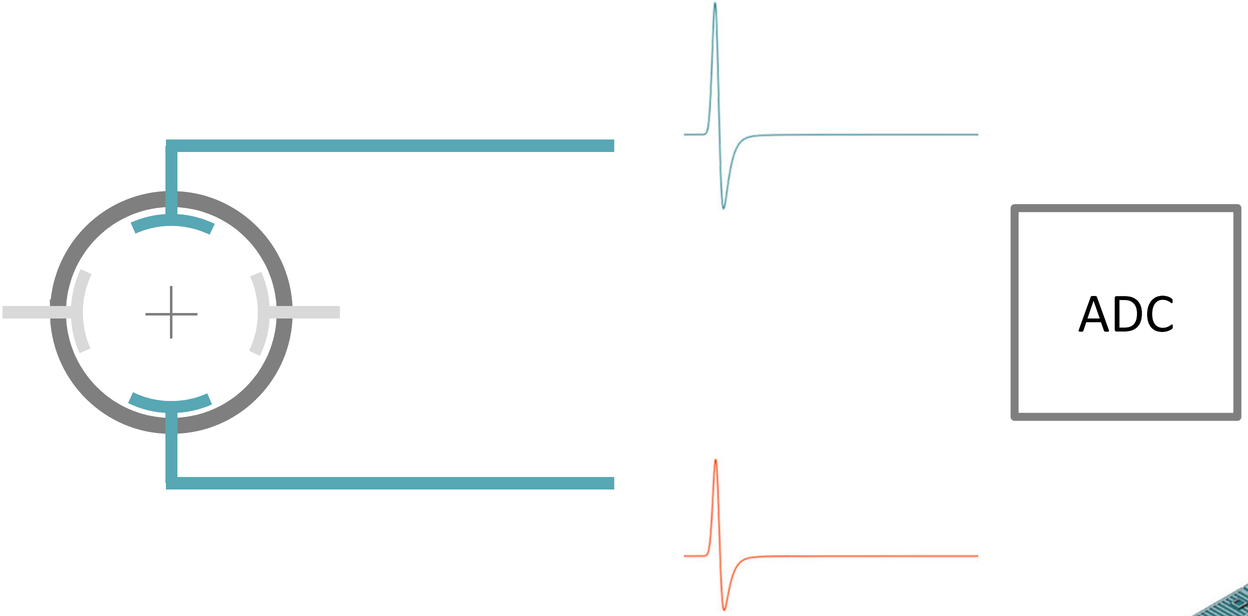
- Results verified with numeric simulations

# Application example

*A proposed new bunch-by-bunch read-out system for the LHC BPM*

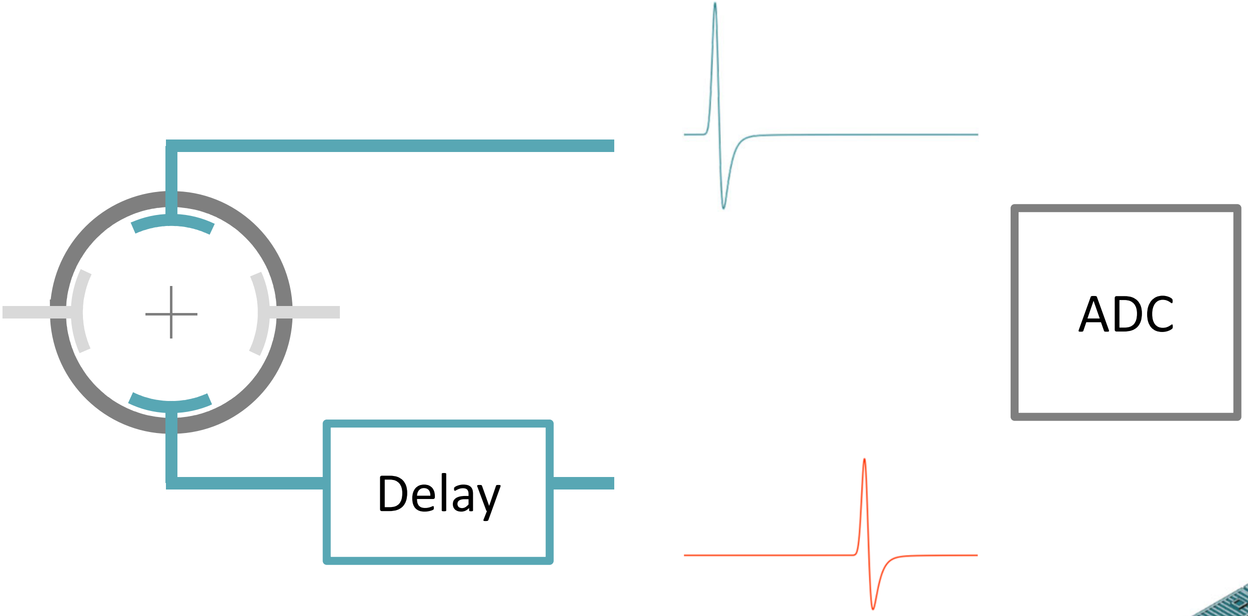
# A possible architecture

## Before the ADC: electrodes combination



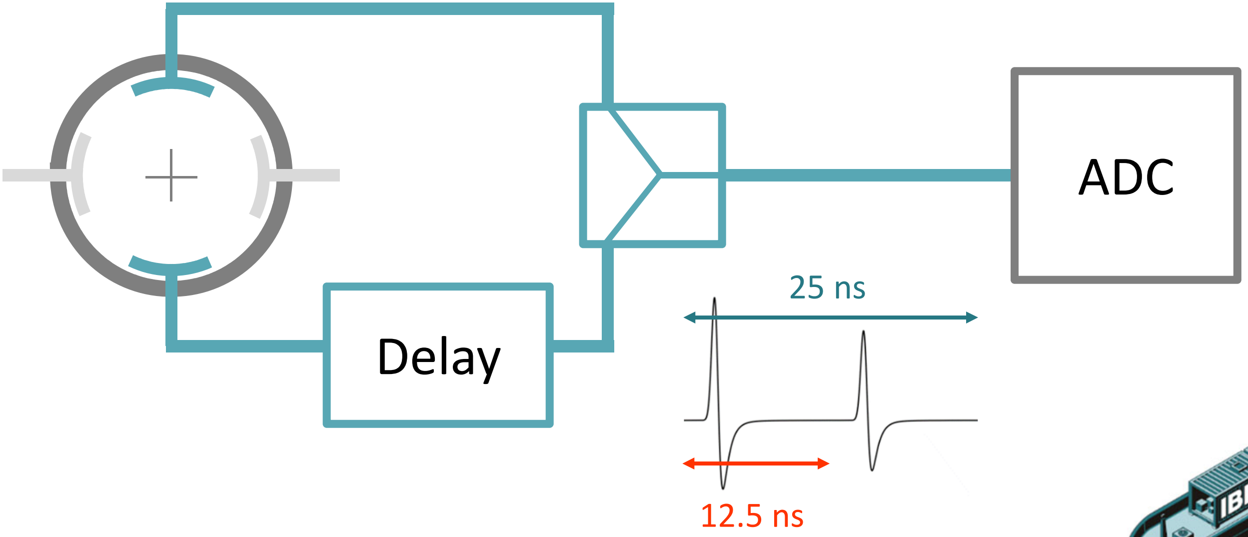
# A possible architecture

## Before the ADC: electrodes combination



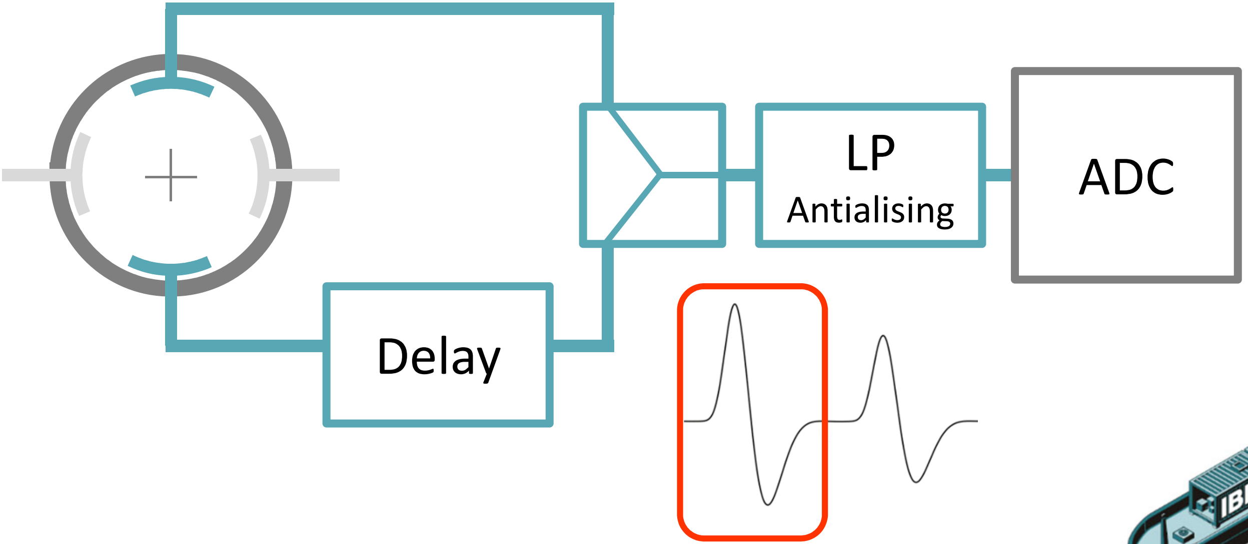
# A possible architecture

## Before the ADC: electrodes combination



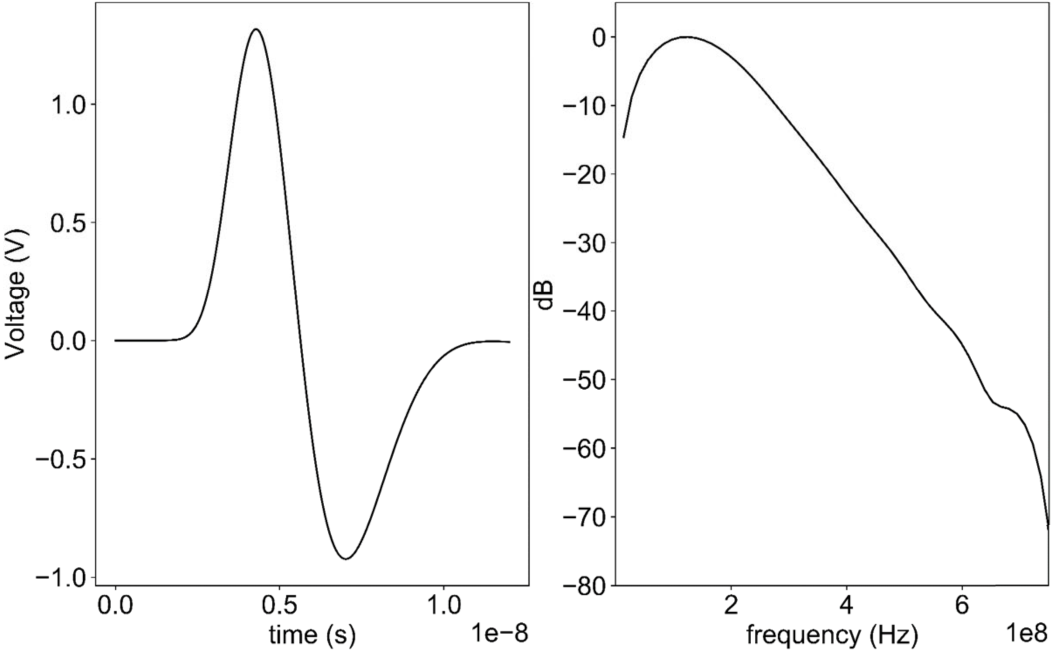
# A possible architecture

## Before the ADC: signal "stretching" with LP filter



# Single bunch pulse signal

- LP Filter
  - 200 MHz
  - N=4
- Antialiasing
  - 600 MHz
  - N=8



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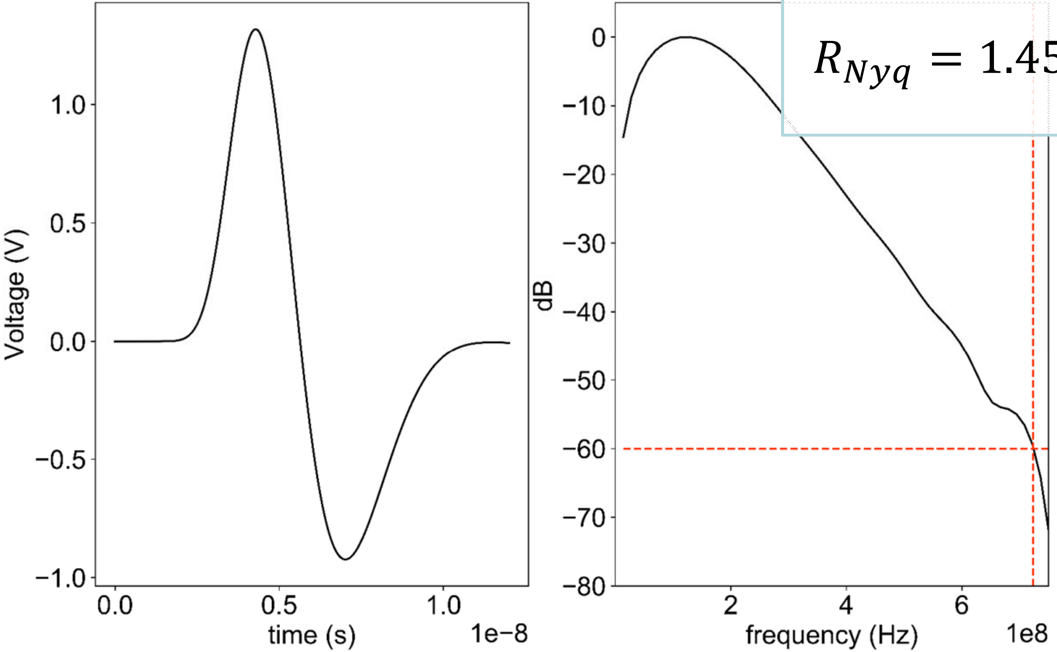
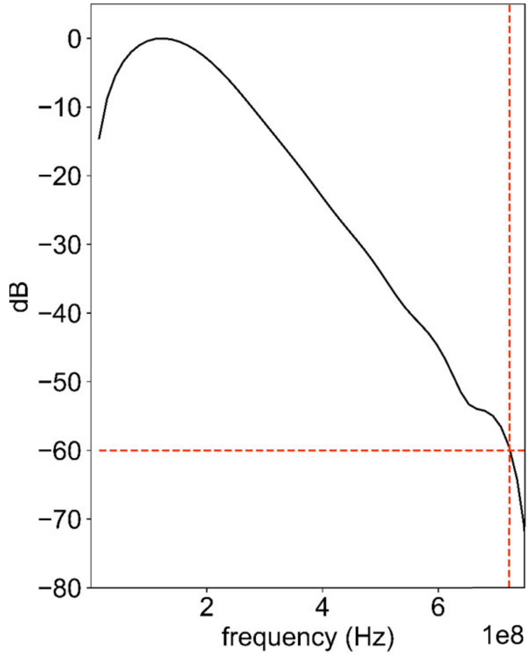


$R_{Nyq} = 1.45$ Gsps

# Power measurement SNR Analysis

$$SNR_{dB} = 10\log_{10}\left(\frac{P_T^2}{\boxed{2\left(\left(A_{X_{k},N}\right)^2+\left(B_{X_{k},N}\right)^2\right)}+\frac{\sigma_v^4}{N}+4P_T\frac{\sigma_v^2}{N}}\right)$$



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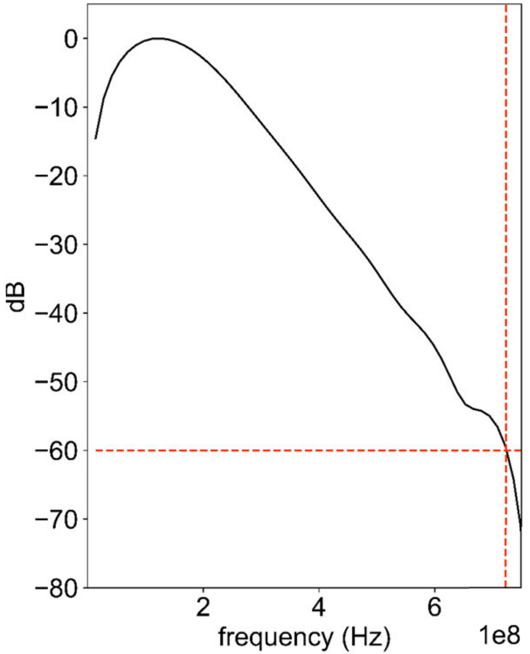

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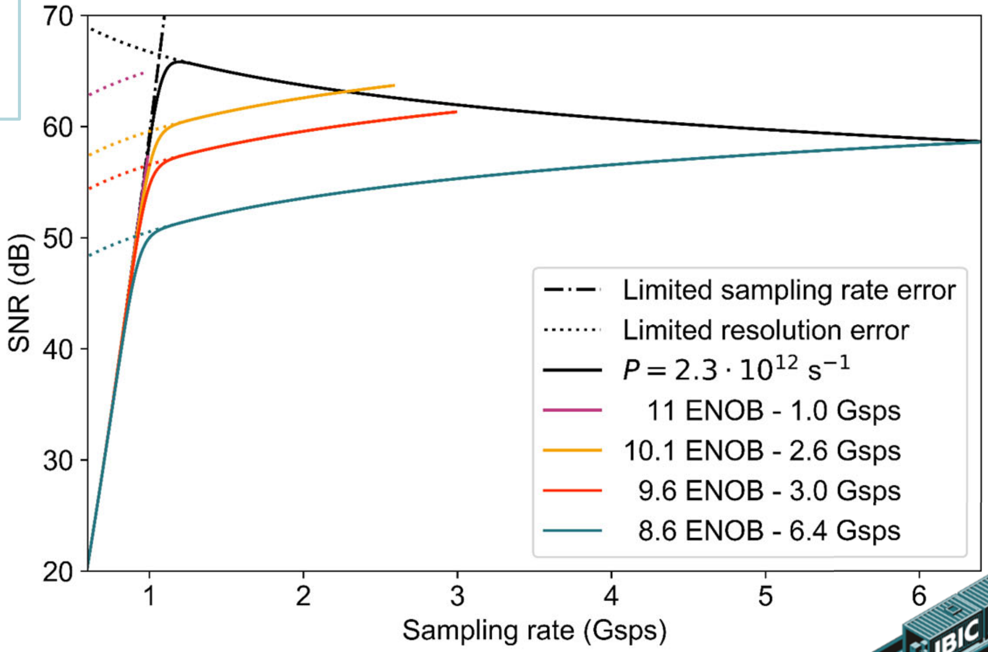
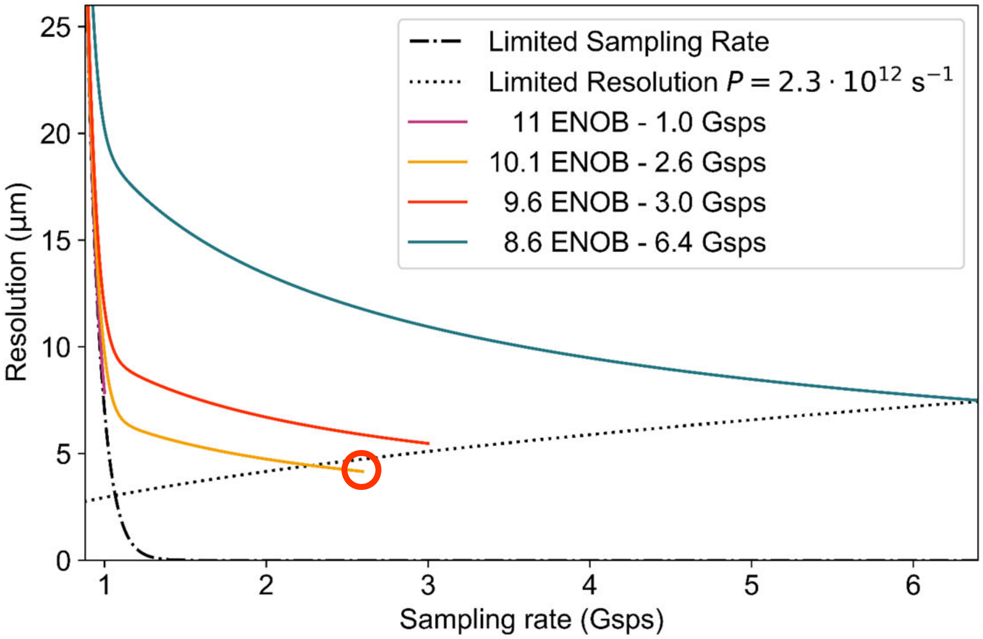


# Position Resolution Analysis

$$\partial y \approx \boxed{k_{\frac{\text{mm}}{\text{dB}}}} \cdot 20 \log_{10}\left(\sqrt{1+\frac{\sqrt{\sigma_\epsilon^2+\sigma_\eta^2}}{P_T}}\right)$$

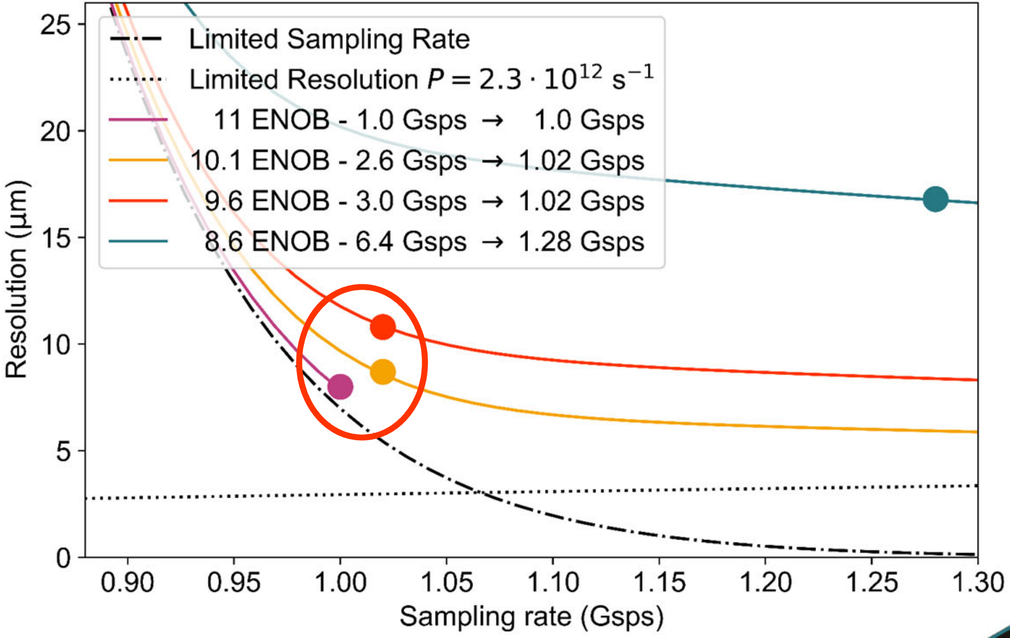


# Position Resolution Analysis

## With transmission bandwidth limit

$$\partial y \approx k_{\frac{\mathrm{mm}}{\mathrm{dB}}} \cdot 20 \log_{10}\left(\sqrt{1+\frac{\sqrt{\sigma_\epsilon^2+\sigma_\eta^2}}{P_T}}\right)$$

2 serial links x 10.24 Gbps



# Summary

# Summary

- **Direct digitization** based systems are of growing interest for Beam Instrumentation applications.
- ADC state-of-the-art imposes a **trade-off** between sampling rate and resolution.
- It is possible to **estimate the error** introduced in the energy estimation of a digitized pulse as a function of the sampling rate and resolution.
- This analytic tool can facilitate the **analysis** of the performance of a system, but also assist in the **design** of a new system, especially in the selection of the ADC.