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APPLICATION OF WAVELET ALGORITHM IN TUNE MEASUREMENT*

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Abstract

Tune is a very important parameter for storage ring of advanced synchrotron radiation facilities. At present, fast Fourier transform (FFT) is the core algorithm of the beam spectrum analysis used in tune measurement. Taking into account the nonlinear effect in the accelerator, tune changes during the process of storage ring injection and booster energy upgrading. However, the Fourier method is used to analyse the global sampling point, and the ability to distinguish the local variation of the tune in the sampling time is poor. This paper leads wavelet analysis method as the core algorithm into beam spectrum analysis method, further analyses the change of the tune with beam amplitude in sampling time, and compares this new algorithm with the traditional Fourier method. New experimental results and corresponding analysis for the data from SSRF will be introduced in this paper.

INTRODUCTION

Tune measurement is very important for a storage ring. Some key accelerator parameters are calculated by measuring the tune value, including beta function, chromaticity, impedance, etc [1]. SSRF starts top-up operation since 2012. Injection introduced beam oscillation can be used to measure real-time tune value on storage ring during user operation [2, 3]. Fast Fourier transform (FFT) is the most commonly used algorithm in the tune calculation, which frequency resolution is determined by the number of sampling points N, that is 1/N. The tune accuracy and tune drift measurement in the injection process is limited by the short oscillation damping time (about 10,000 turns).

As for the booster ring, the tune drift during ramping reflects the real-time operating status of the booster. The short-time Fourier transform (STFT) is used to calculate the tune drift [4]. However, the short ramping time (about 20,000 turns) during booster top-up injection makes the accurate tune drift measurement with STFT difficult.

Wavelet transform is a commonly used time-varying frequency analysis algorithm. There already have some related researches on beam analysing using Wavelet. The signal is analysed by selecting an appropriate base function with limited energy on the time axis. The continuous wavelet transform is defined as following quotation.

$$WT(a,\tau) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} f(t) * \psi\left(\frac{t-\tau}{a}\right) dt$$

where ψ is the mother wavelet, τ is the delay coefficient, and α is the scale coefficient which can be converted to corresponding frequency. The final result is a two-dimensional array of coefficients (a, τ) [5, 6].

This article uses Morlet wavelet as the mother wavelet to analyse the turn-by-turn data from storage ring and booster of SSRF during injection. Morlet wavelet is a complex wavelet whose envelope is the Gauss function. Its analytical formula is:

$$m(t) = e^{j\omega_0 t} e^{-t^2/2}$$

The real part image of the function is shown in the Figure 1.

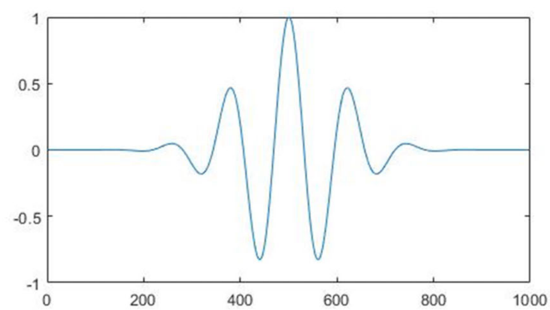

Figure 1: Morlet Wavelet.

ALGORITHM EVALUATION

Before use this new algorithm, we must test this wavelet method with Monte Carlo method. We established some test signals to simulate the real sampling.

It can be seen that FFT is restricted by the number of sampling points, and the frequency resolution is always in a relatively poor range in the Fig. 2. It cannot capture the subtle changes of tune. However, the morlet wavelet method has a relatively high value when the number of points is relatively small. The resolution is concentrated in the vicinity of the true frequency, and there is a lot of jitter when the SNR is low. As the number of sampling points increases, the stability of the morlet wavelet method is greatly improved, but the resolution of the FFT does not change significantly.

There is a special parameter in morlet wavelet analysis called analysis length, which affects the degree of time-frequency emphasis of wavelet analysis. The larger the analysis length, the higher the frequency resolution and the lower the time resolution, and vice versa. Use the simulation signal to analyse, get Fig. 3.

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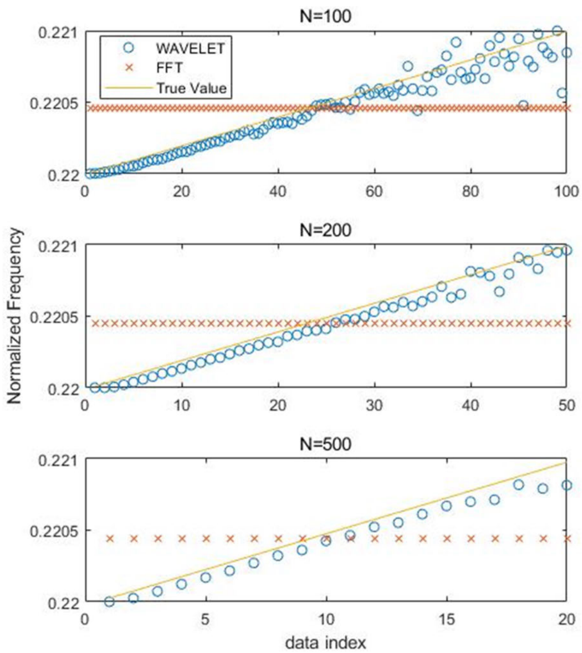

Figure 2: The relationship between test results and the number of sampling points.

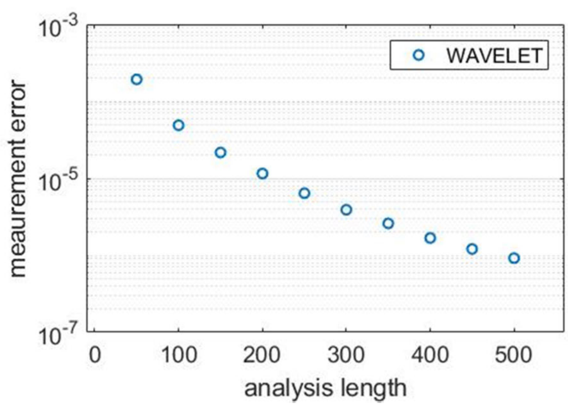

Figure 3: The relationship between test results and analysis length.

In Fig. 3, it can be seen that as the analysis length increases, the frequency resolution increases significantly. Considering the time resolution requirement, a minimum analysis length should be selected after a certain frequency resolution requirement is met.

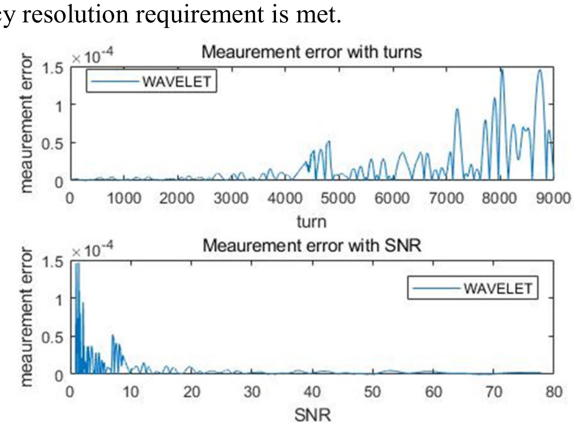

Figure 4: The relationship between test results and SNR.

Figure 4 shows the variation of measurement uncertainty with the number of turns. As the number of turns increases, the SNR decreases, and the frequency uncertainty increases significantly. The horizontal axis is replaced by the SNR in the figure below, which is easy to get. When it is greater than 10, the algorithm is available and has a higher frequency resolution. Combining the measurement error of the instrument and the collected value, a meaningful number of analysis points can be obtained.

Fortunately, such data will be deliberately avoided when designing the tune, so the systematic error can be small.

Figure 5 simply shows the variation of frequency uncertainty with test frequency.

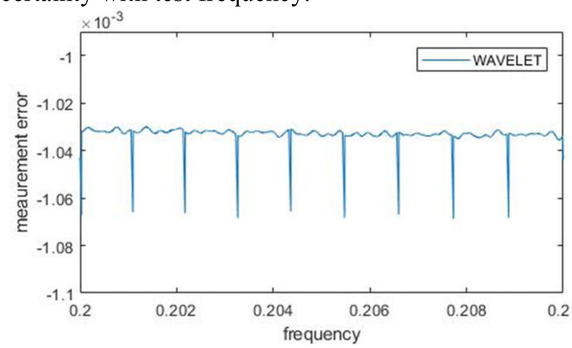

Figure 5: Test results and frequency dependence.

APPLICATION IN STORAGE RING

The test results show that the application of morlet wavelet algorithm in the extraction of storage ring tune is feasible.

The sampling result of the beam position during the injection of SSRF storage ring is shown in Fig. 6.

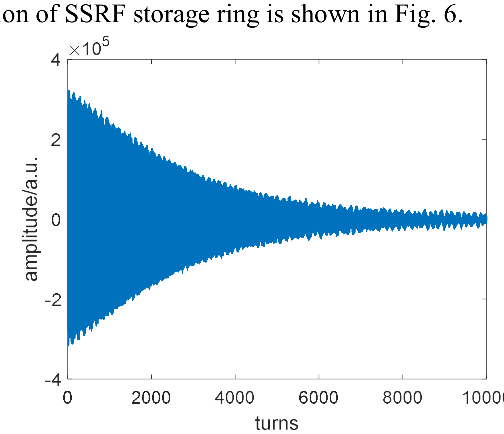

Figure 6: Injection horizontal turn-by-turn beam position data from SSRF storage ring.

The measurement resolution of turn-by-turn position is 2.1 μm, and the corresponding random noise peak-to-peak value is 7.6 μm. Before the SNR dropped to 10. We get about 9000 sampling points. Calculations within this range can be considered effective.

Here, the wavelet analysis algorithm is used to analyse the circle-by-lap horizontal position data of SSRF storage ring injection.

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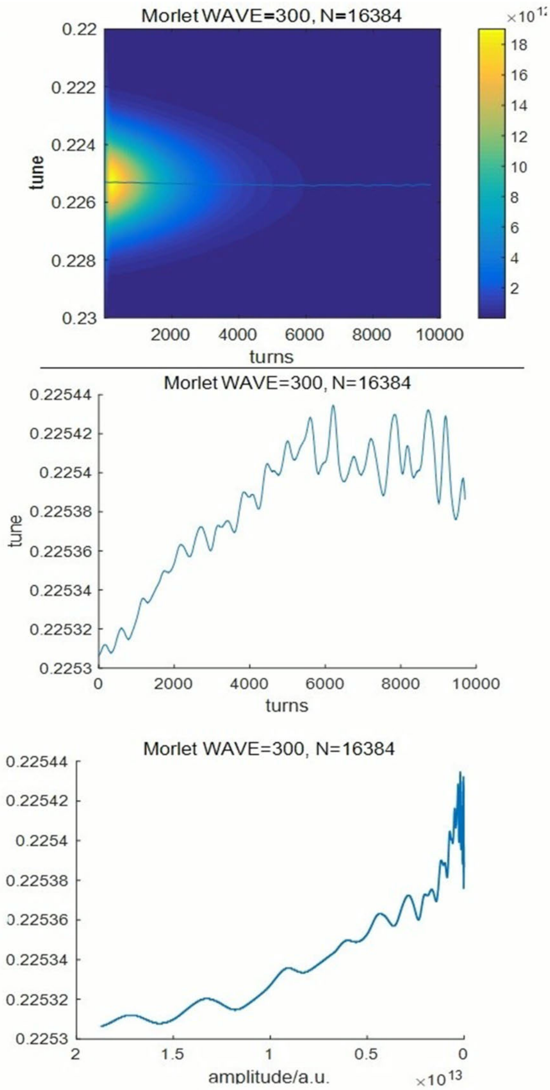

Figure 7: Wavelet analysis of horizontal tune drift during SSRF storage ring injection.

Figure 7 shows tune during the injection increased from 0.22530 to 0.22543. Before applying this algorithm to tune measurement, we think that the average value of the whole process is the actual tune seen by the bunch, but after considering the nonlinearity, this is not accurate. After replaced the horizontal axis with the bunch amplitude, it can be seen that as the amplitude decreases, tune increases approximately linearly. When the amplitude is 0, tune is corrected to 0.22542. Compared with the original correction of 0.0001, this is very meaningful.

APPLICATION IN SSRF BOOSTER

Transverse oscillations are also introduced during the injection process of the booster. Figure 8 shows the horizontal and vertical turn-by-turn position data of the SSRF booster during the injection and boosting period. Compared with the storage ring, the transverse oscillation caused by the injection on the booster is not so obvious.

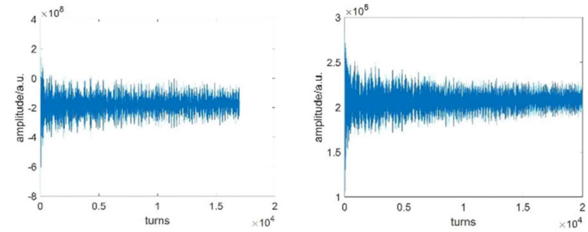

Figure 8: Injection horizontal (left) and vertical (right) turn-by-turn beam position data from SSRF booster.

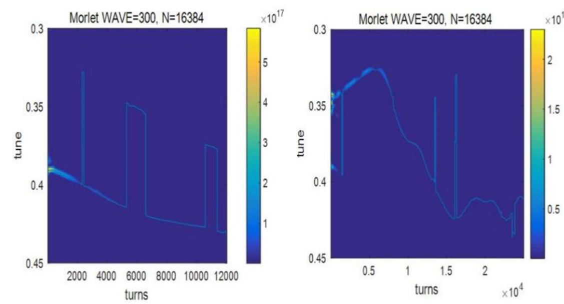

Figure 9: Morlet Wavelet analysis of horizontal (up) and vertical (bottom) tune drift measurement during SSRF storage ring injection.

Figure 9 is the use of wavelet analysis algorithm to extract the horizontal and vertical tune changes during the energy-up period of the booster. It can be seen that except for some of the weak oscillation amplitudes Part of the calculation results are unreliable, the changing trajectories of tune in the horizontal and vertical directions are gradually clear. Among them, the transverse tune increases unidirectionally, and the vertical tune decreases first and then increases. Due to the weak amplitude of the transverse oscillation, it is difficult to calculate the tune with some data, and obvious noise can be seen.

CONCLUSION

The simulation proves that the morlet wavelet algorithm can measure the time-varying tune. When the data analysis window is selected as 100 cycles, tune change as small as 0.00001 can be detected.

During the period of storage ring injection and booster's energy upgrade, tune has obvious change. By applying the morlet wavelet algorithm, the complete trajectory of the tune drift during the booster's energy upgrade process can be tracked.

Therefore, this new algorithm can be used to perform undisturbed tune measurement during the top-up injection period for SSRF, which provides an effective tool for the beam instability research and performance optimization of the accelerator.

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