The College of Judea and Samaria מכללת יהודה ושומרון

Dept. of Electrical and Electronic Engineering המחלקה להנדסת חשמל ואלקטרוניד

SPACE-FREQUENCY MODEL OF ULTRA WIDE-BAND INTERACTIONS IN FREE-ELECTRON LASERS

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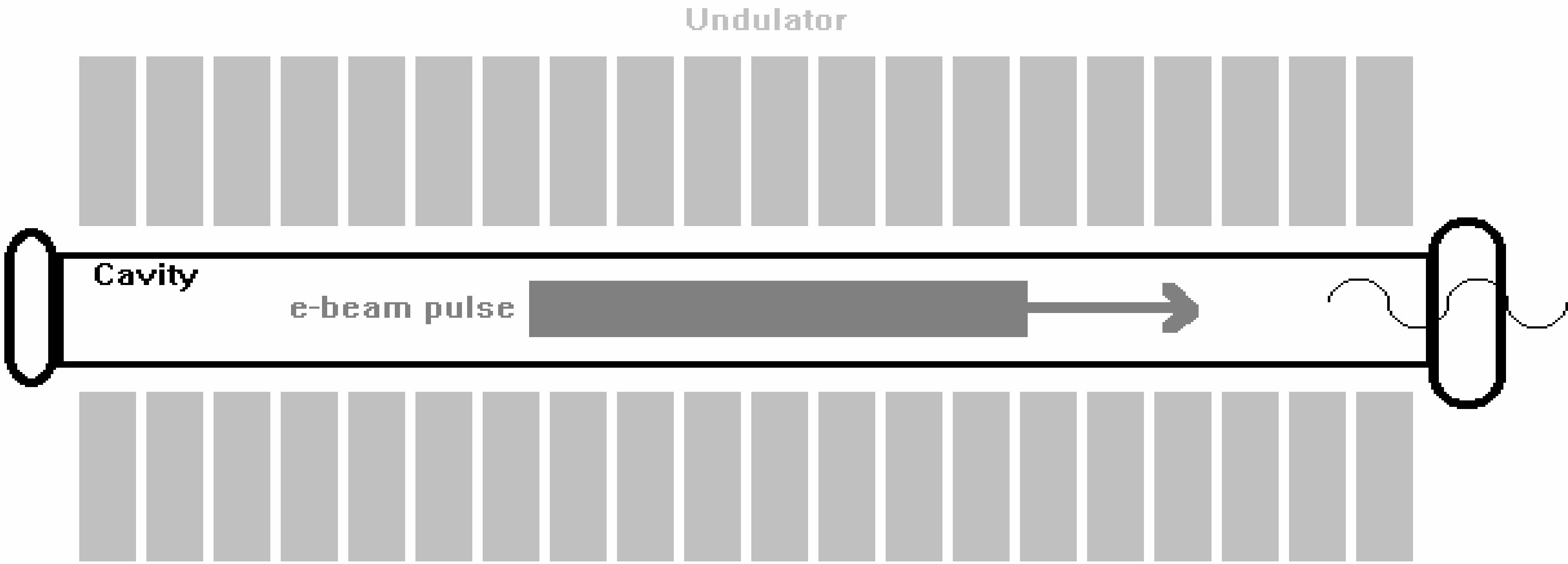
SPACE-FREQUENCY MODEL

- Excitation equations in the frequency domain
- Frequency dependent effects
(gain, absorption, dispersion)
- Consideration of statistical features
(radiation, gain medium)
- Ultra wide band interactions
- Free-space,waveguide, resonator

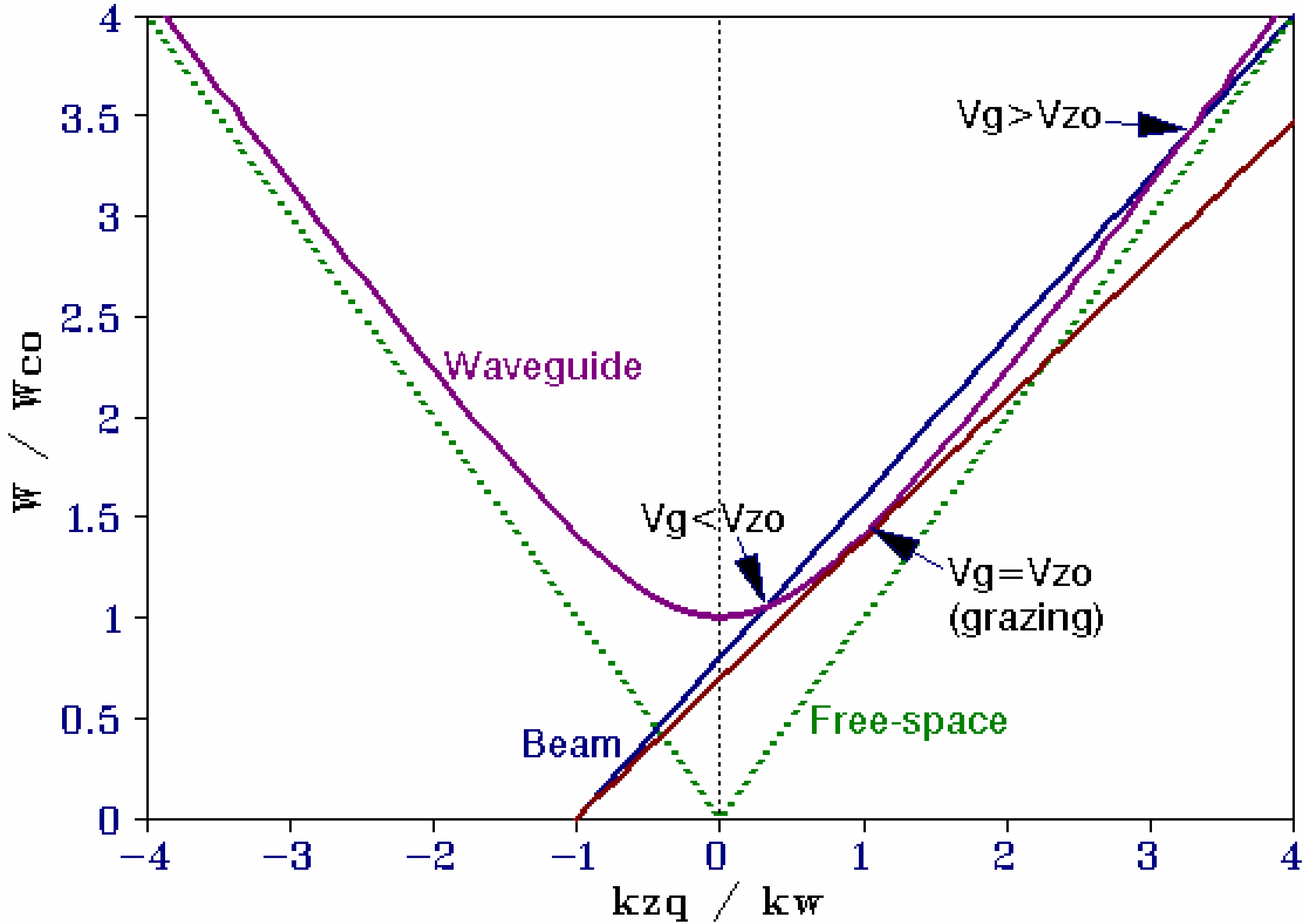
Schematic illustration of a pulsed beam free-electron laser



WIDE BAND INTERACTIONS

- Spontaneous emission and noise
- Super-radiance from ultra short bunches
- Synchrotron amplified spontaneous emission (SASE)
- Radiation excitation and buildup in an oscillator
- Single- and Multi- mode operation
  (forward and backward, ‘grazing’)

DISPERSION CURVES



MODAL REPRESENTATION OF THE ELECTROMAGNETIC FIELD

$$\tilde{E}(r,f) = \sum_q \left[C_{+q}(z) e^{+jk_{zq}z} + C_{-q}(z) e^{-jk_{zq}z} \right] \cdot \boldsymbol{\varepsilon}_q(x,y)$$

$$\frac{d}{dz} C_{\pm q}(z,f) = \mp \frac{1}{2N_q} e^{\mp jk_{zq}z} \iint \tilde{\mathbf{J}}(r,f) \cdot \boldsymbol{\varepsilon}_q^*(x,y) dxdy$$

$$N_q = \iint \left[\tilde{E}_{q\perp}(x,y) \times \tilde{H}^*_{q\perp}(x,y) \right] \cdot \hat{z} dxdy$$

ENERGY SPECTRUM

$$\frac{dW(z)}{df} = \frac{1}{2} \sum_{\substack{q \\ \text{Propagating}}} \left[\left| C_{+q}(z,f) \right|^2 - \left| C_{-q}(z,f) \right|^2 \right] \mathrm{Re}\left\{ N_q \right\} +$$

$$+ \sum_{\substack{q \\ Cut-off}} \mathrm{Im}\left\{ C_{+q}(z,f) C_{-q}{}^{*}(z,f) \right\} \mathrm{Im}\left\{ N_q \right\}$$

THE DRIVING CURRENT

$$J(r,t) = -e\sum_i \vec{v}_i \delta(x - x_i)\delta(y - y_i)\delta[z - z_i(t)]$$

$$\tilde{J}(r,f) = -2e\sum_i \frac{\vec{v}_i}{v_{zi}} \delta(x - x_i)\delta(y - y_i)e^{j2\pi f t_i(z)}u(f)$$

$$C_{\pm q}(z,f) = \pm\frac{e}{N_q}\int_0^z \sum_i \frac{1}{v_{zi}} \vec{v}_i \cdot \varepsilon_q{}^*(x_i, y_i)e^{j[2\pi f t_i(z') \mp k_{zq}(f) z']}dz'$$

PARTICLE DYNAMICS

$$\frac{d\vec{v}_i}{dz} = -\frac{1}{\gamma_i}\left\{\frac{e}{m}\frac{1}{v_{zi}}\left[E\left[r_i, t_i(z)\right] + \vec{v}_i \times B\left[r_i, t_i(z)\right] + \vec{v}_i \frac{d\gamma_i}{dz}\right]\right\}$$

$$\frac{d\gamma_i}{dz} = -\frac{e}{mc^2}\frac{1}{v_{zi}}\vec{v}_i \cdot E\left[r_i, t_i(z)\right]$$

$$t_i(z) = t_{0_i} + \int_0^z \frac{1}{v_{zi}(z')} dz'$$

Operational Parameters

Accelerator

• *Beam energy*	E_k=1÷6 MeV
• *Beam current*	I_0=1 A
• *Beam pulse duration*	T_b=0.1 pS

Wiggler

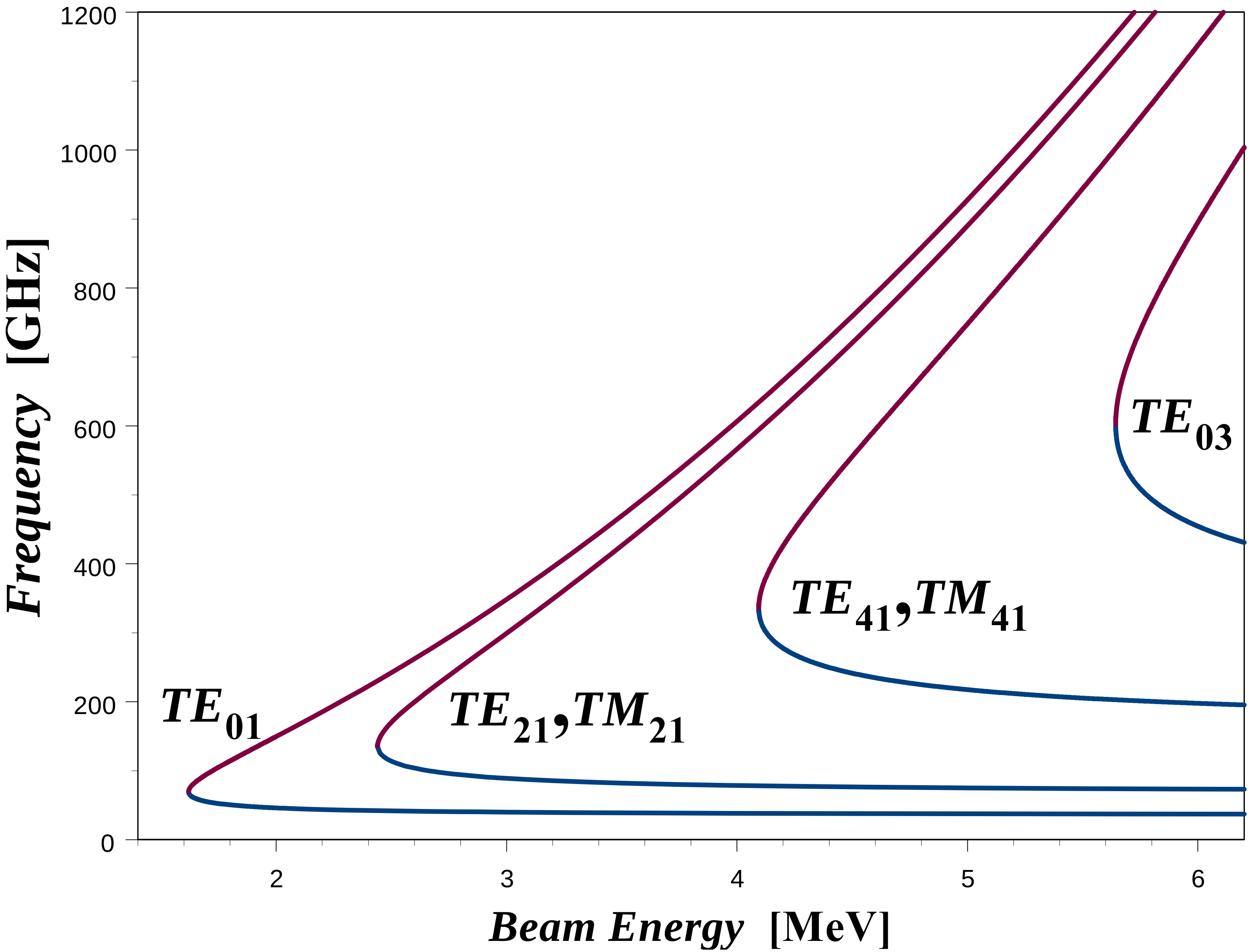
• *Magnetic induction*	B_w=2 kG
• *Period*	λ_w=5 cm
•*Number of periods*	N_w=20

Wave guide

• *Rectangular*	15mm×7.5mm

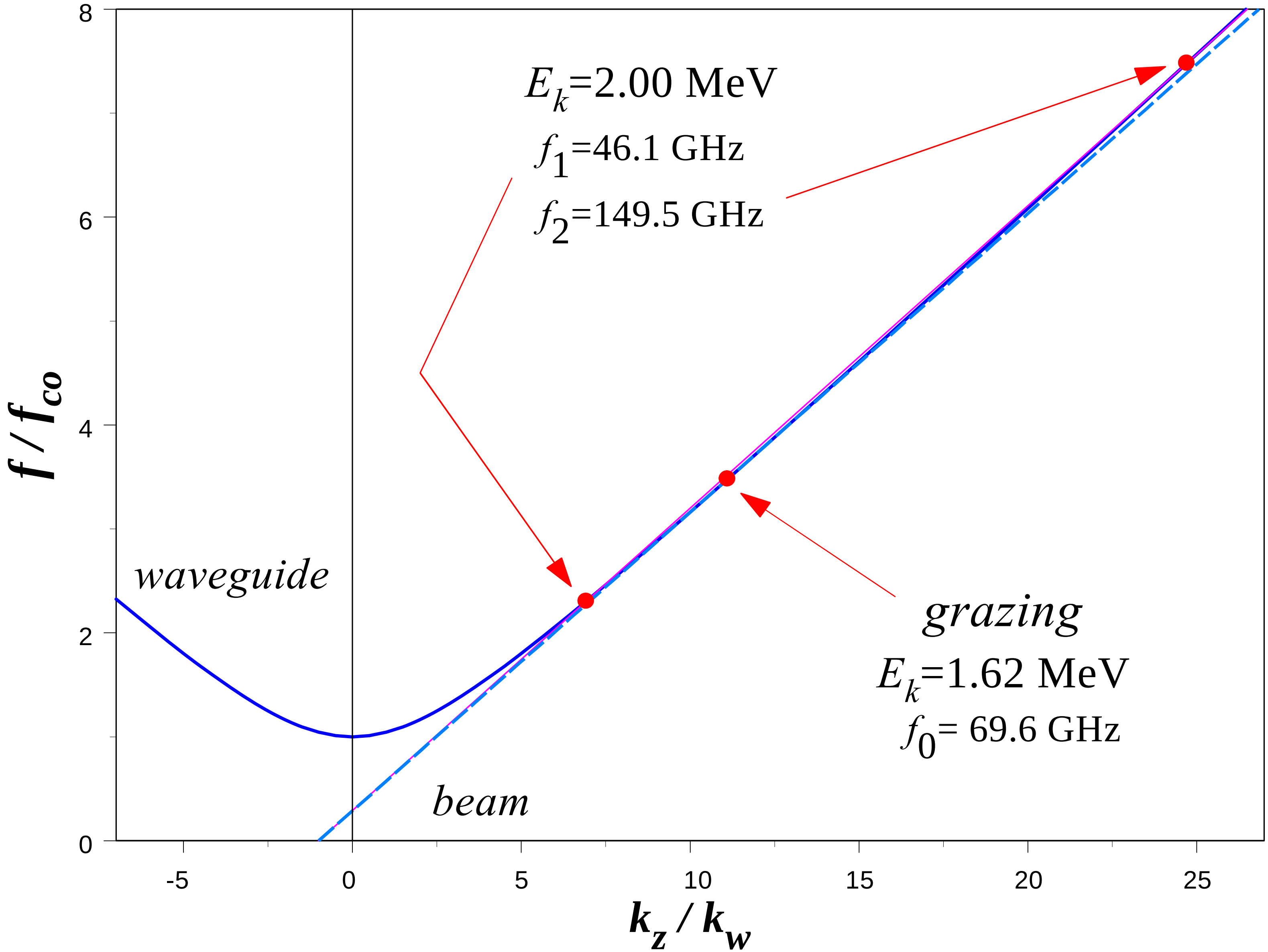
Mode	Cut-off frequency
TE_{01}	20.0 GHz
TE_{21} , TM_{21}	28.3 GHz
TE_{41} , TM_{41}	44.7 GHz
TE_{03}	60.0 GHz

Energy dependence of the dispersion solutions

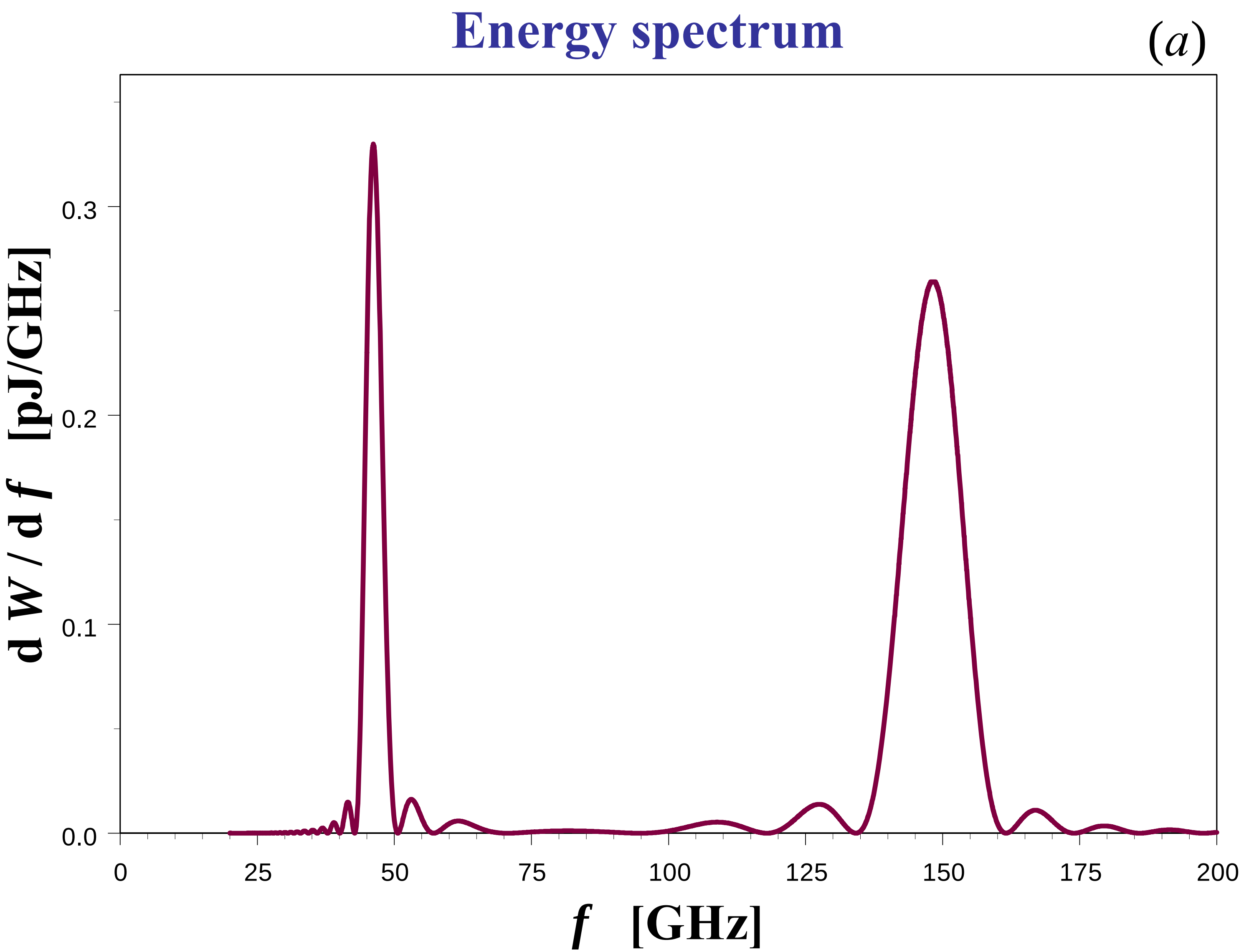


SINGLE TRANSVERSE MODE

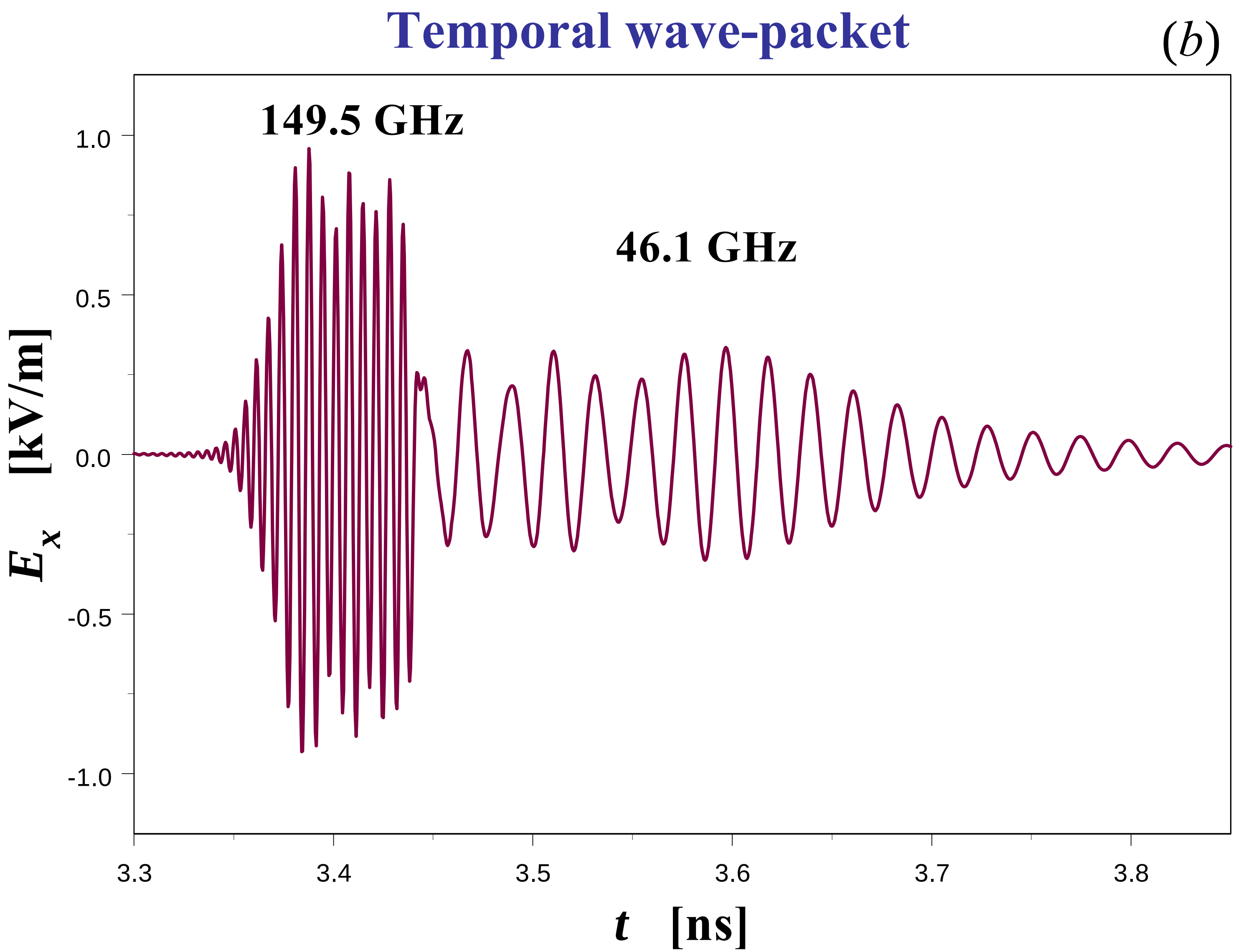
Dispersion solutions for the TE_{01} transverse mode for $E_k = 2$ MeV



Super-radiant emission from an ultra short $E_k = 2$ MeV bunch with the single TE_{01} mode



Super-radiant emission from an ultra short $E_k = 2$ MeV bunch with the single TE_{01} mode

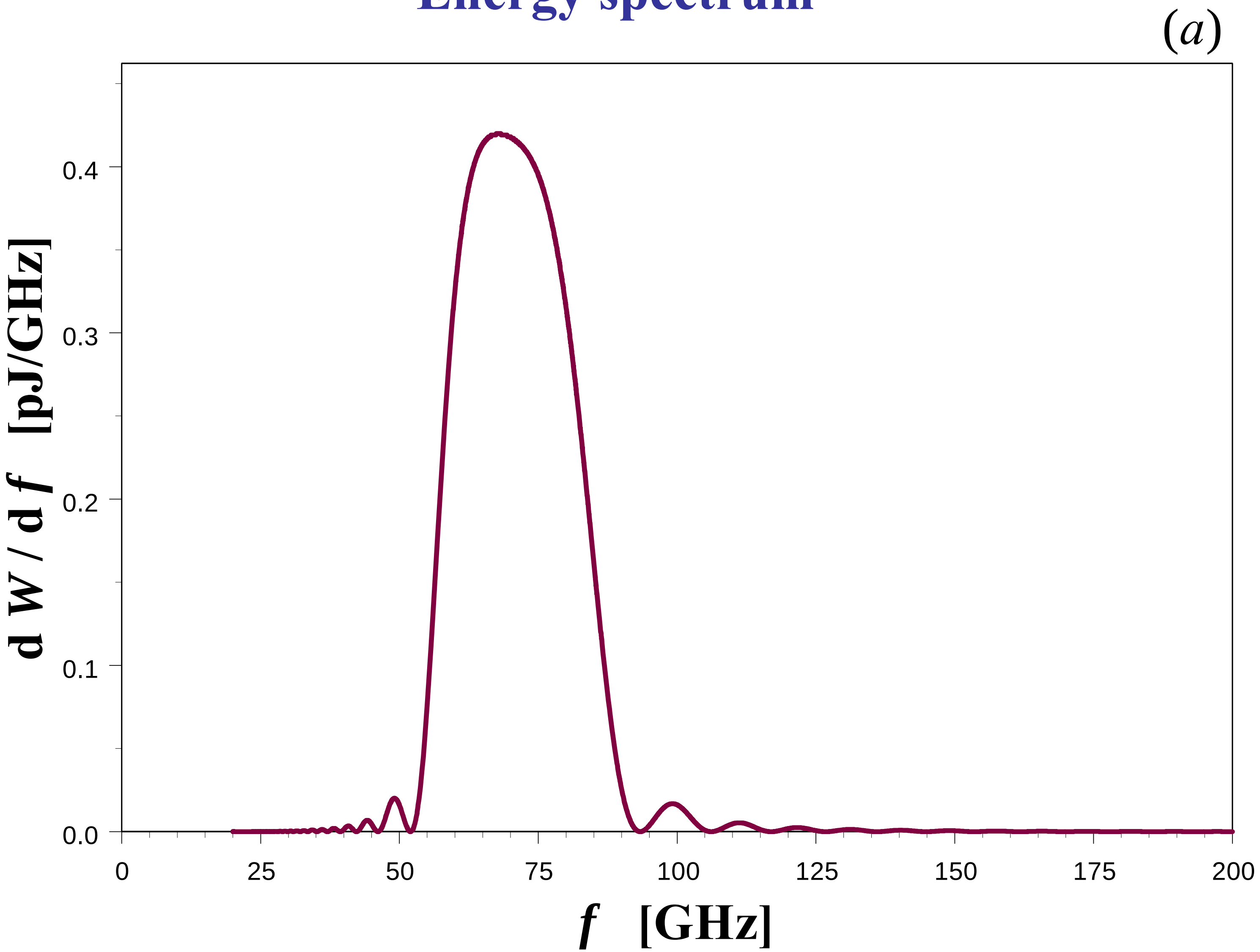


GRAZING

Super-radiant emission from an ultra short bunch at *grazing*

$$E_k \approx 1.62 \text{ MeV}$$

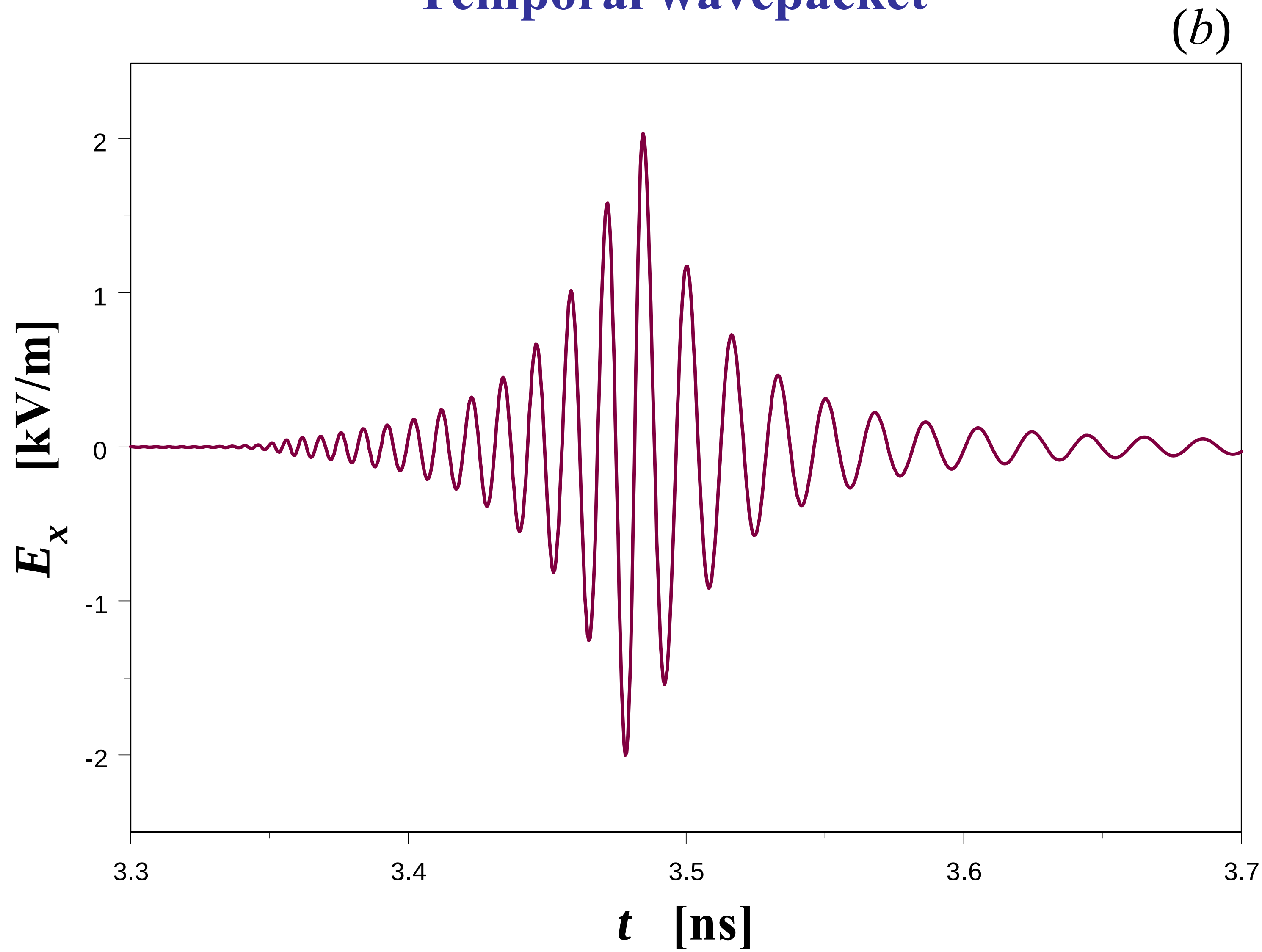
Energy spectrum



Super-radiant emission from an ultra short bunch at *grazing*

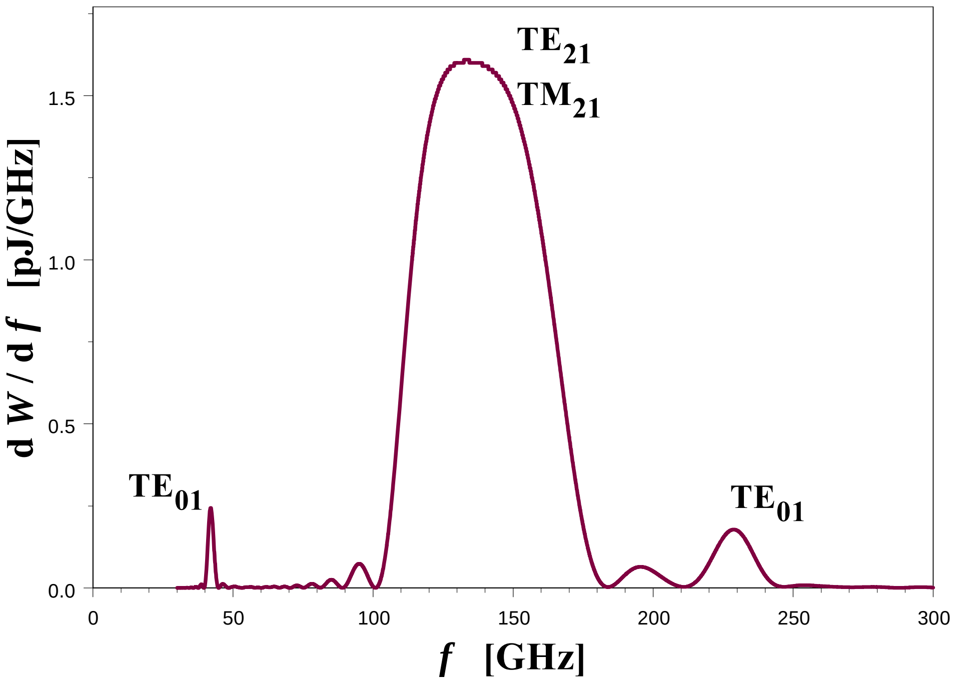
$E_k \approx 1.62$ MeV

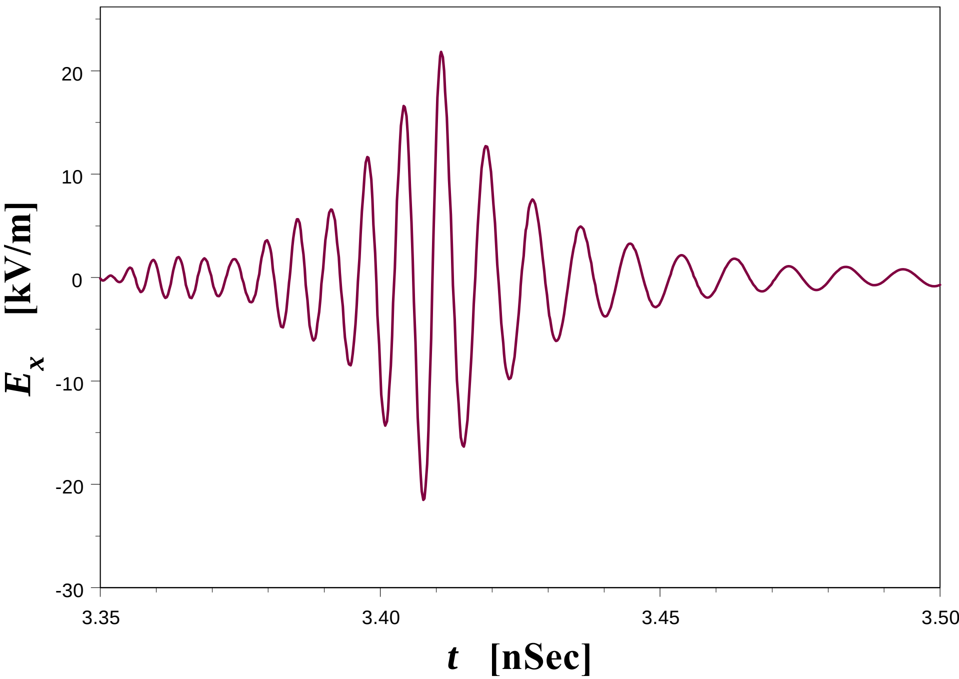
Temporal wavepacket



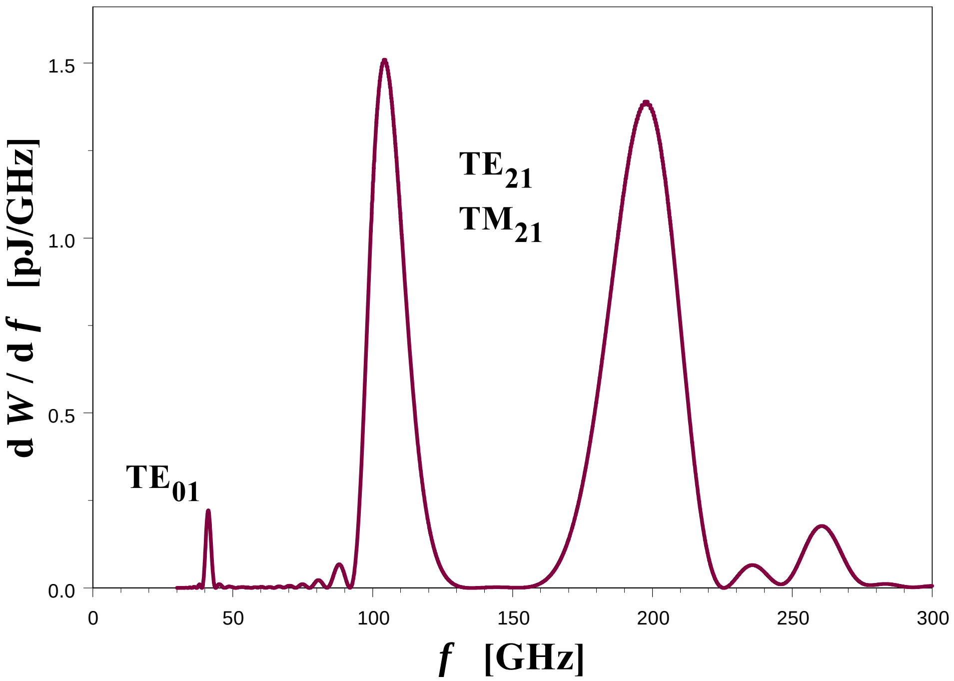
MULTI-TRANSVERSE MODES

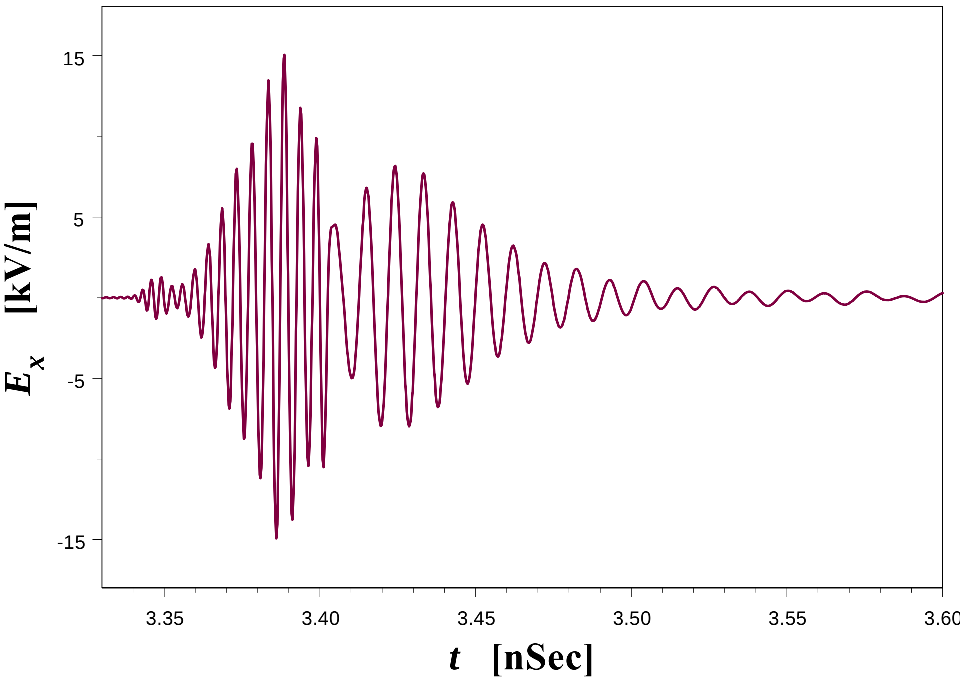
$E_k \approx 2.44$ MeV (grazing in the TE_{21}, TM_{21} modes)





$E_k \approx 2.6$ MeV

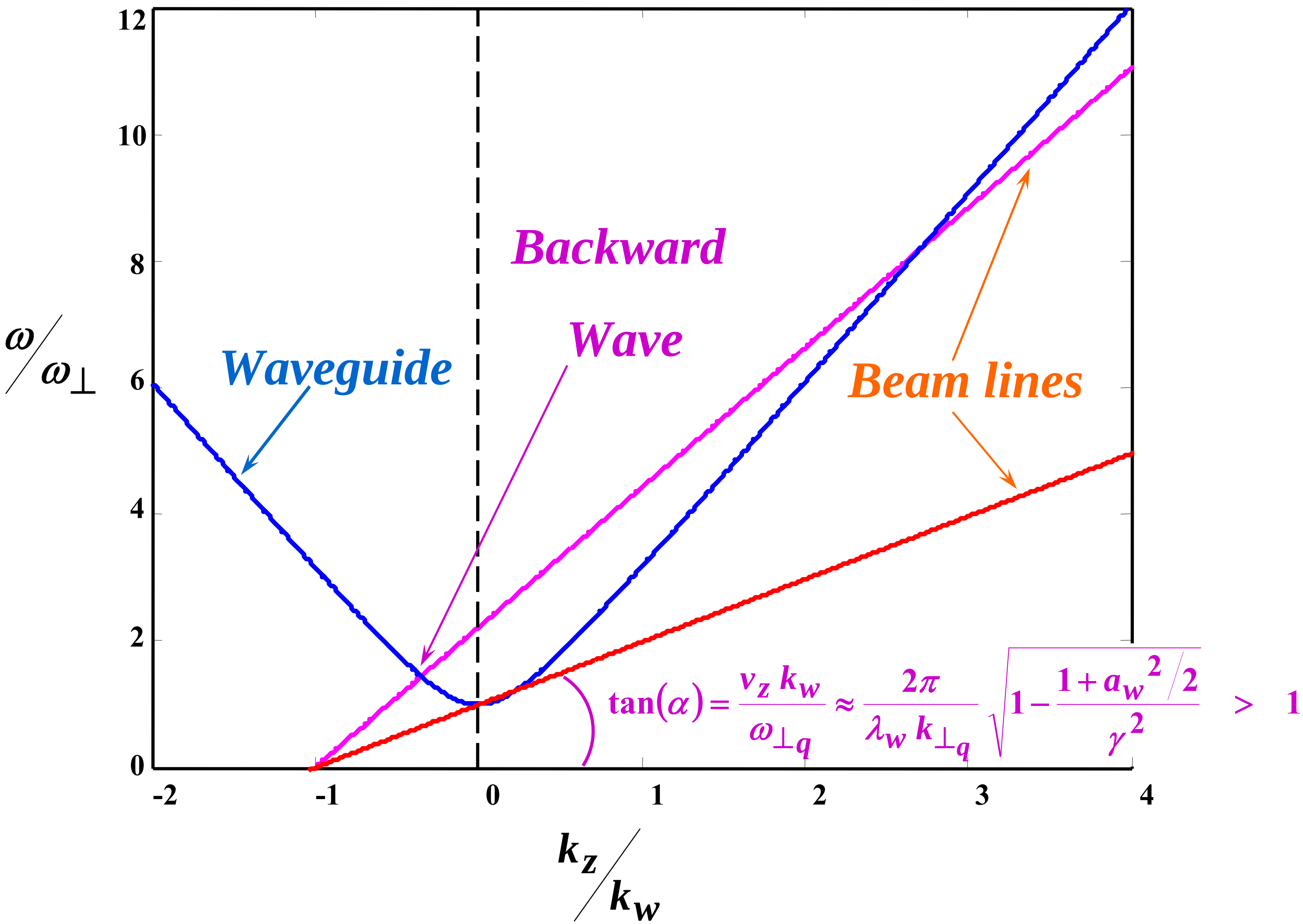




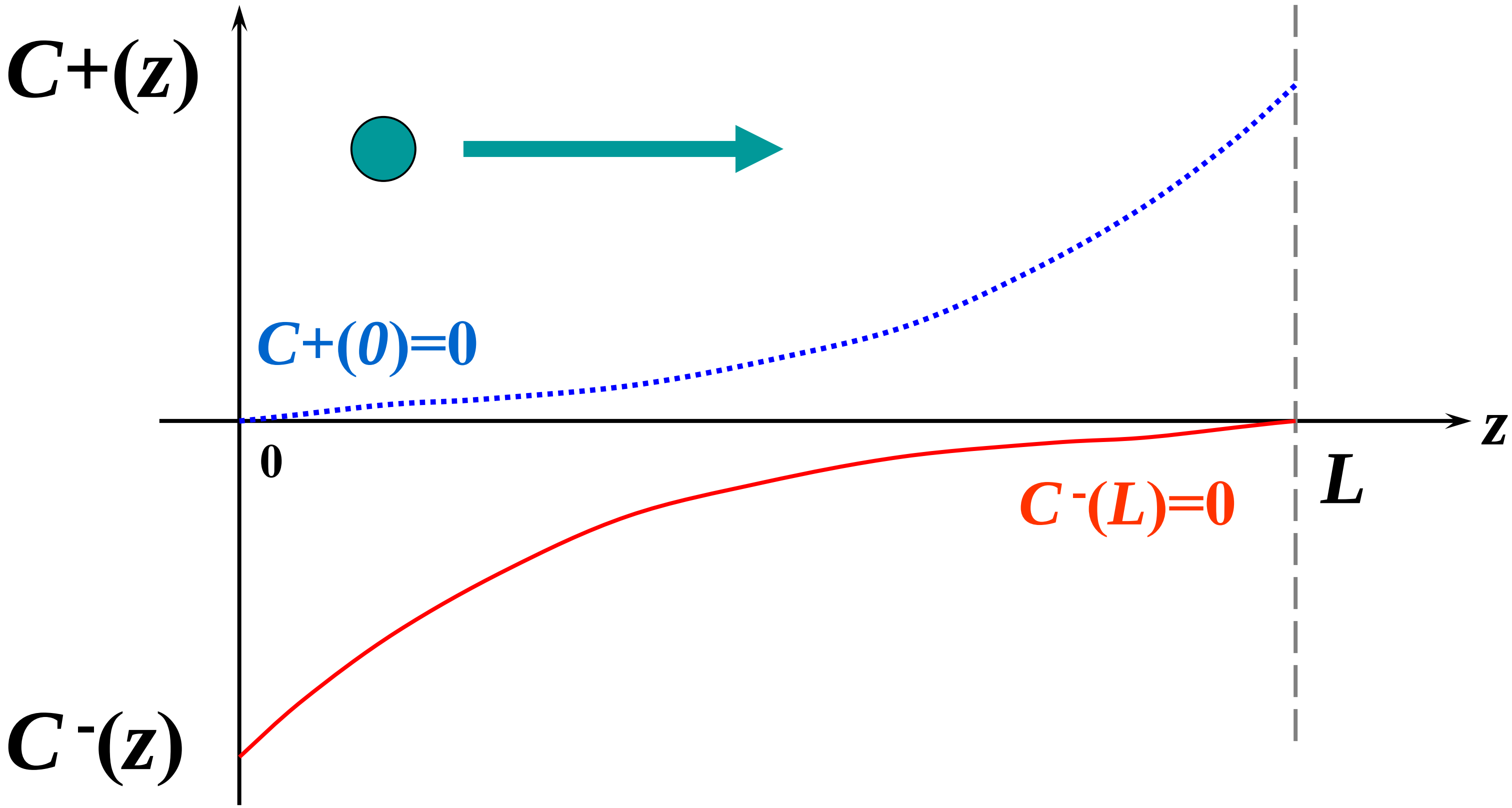
BACKWARD WAVES

Two-point boundary value problem

Dispersion Solutions



Boundary conditions



Operational parameters

Accelerator

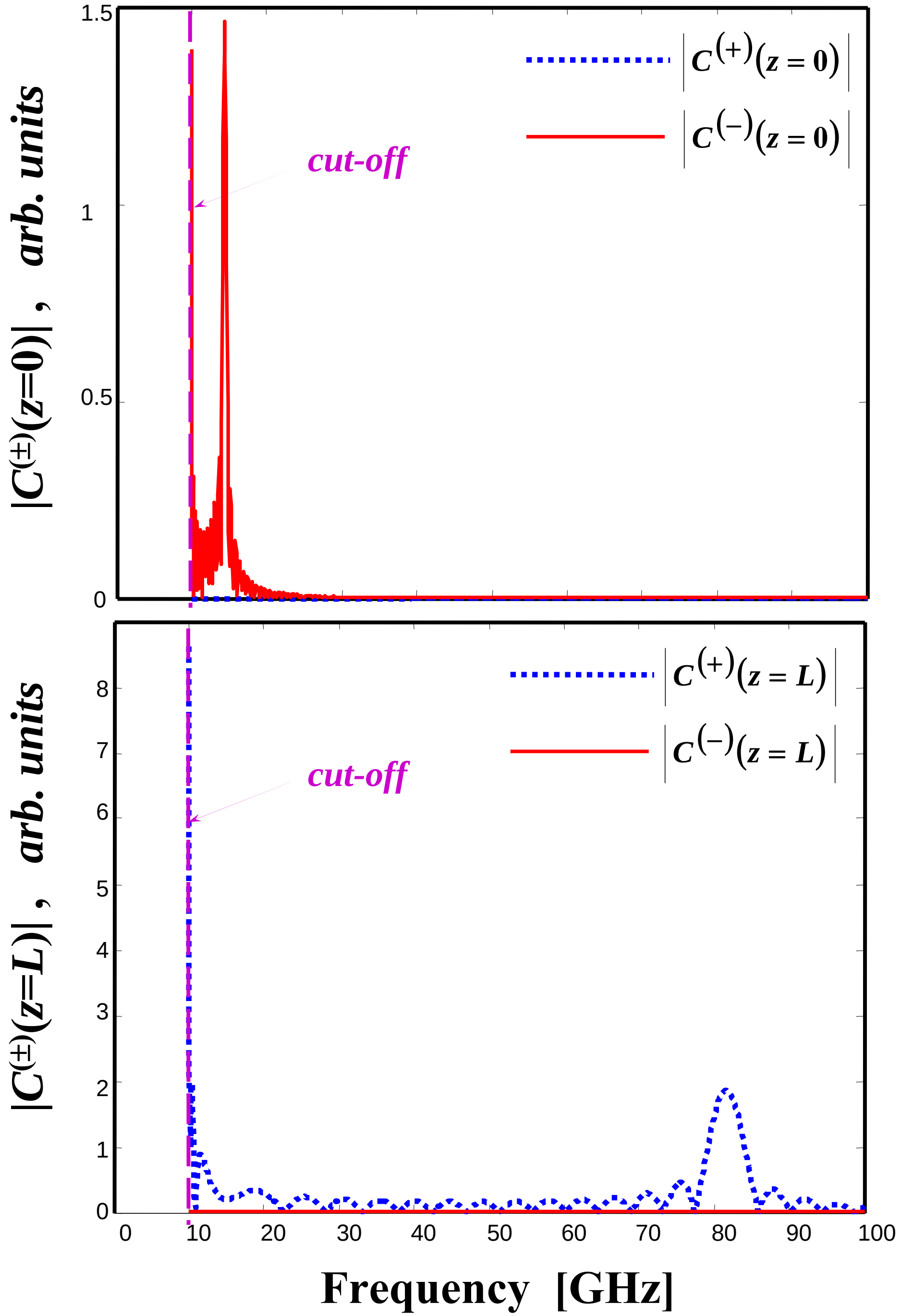
- *Beam energy* — E_k=250 keV

Wiggler

- *Magnetic induction* — B_w=2 kG
- *Period* — λ_w=10 mm
- *Number of periods* — N_w=20

Wave guide

- *Rectangular* — 15×15 mm^2
- *Fundamental mode* — TE_{01}



1.5
1
0.5
0
$|C^{(\pm)}(z=0)|$, arb. units
$\left|C^{(+)}(z=0)\right|$
$\left|C^{(-)}(z=0)\right|$
cut-off
8
7
6
5
4
3
2
1
0
$|C^{(\pm)}(z=L)|$, arb. units
$\left|C^{(+)}(z=L)\right|$
$\left|C^{(-)}(z=L)\right|$
cut-off
0
10
20
30
40
50
60
70
80
90
100
Frequency [GHz]

Summary and conclusions

- Coupled-mode theory, formulated in the frequency domain, enables development of a three-dimensional model, which accurately describes wide-band interactions between radiation and electron beam.

- The approach is applied in a numerical simulation WB3D !