Coherence of E-Beam Radiation Sources and FEL – A Theoretical/Tutorial Overview

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The FEL Knowledge Center for Radiation Sources and Applications

FEL Prize talk –Berlin 2006

Dedication

Zofia Piotrowska (1909-1997)
Josef Pietrowski (1906-1989)

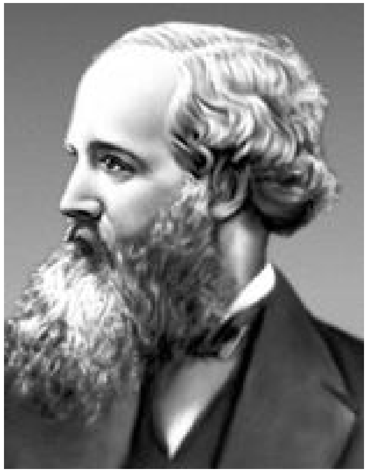
Righteous among nations

Sharing the FEL prize
with the Piotrowski family
Gdansk, Poland, Sept. 2005 –

Maxwell Equation

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} - \mu_0 \frac{\partial \mathbf{M}}{\partial t}$$

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} + \frac{\partial \mathbf{P}}{\partial t}$$

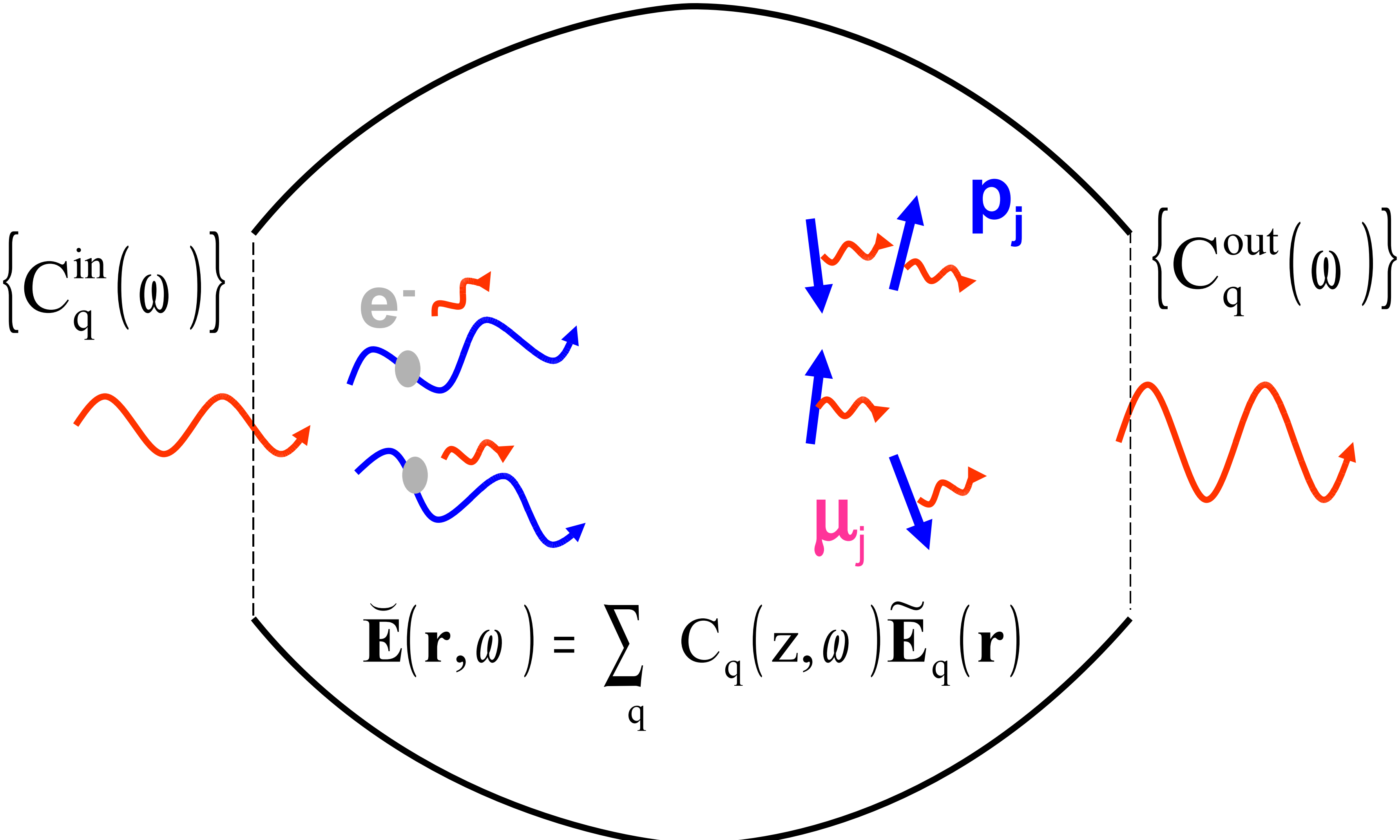


Particulate Radiation Sources

$$\mathbf{J} = \sum_{j=1}^{N} -e\mathbf{v}_j \delta\left(\mathbf{r} - \mathbf{r}_j(t)\right)$$

$$\mathbf{P} = \sum_{j=1}^{N} \mathbf{p}_j \delta\left(\mathbf{r} - \mathbf{r}_j(t)\right) \qquad \mathbf{M} = \sum_{j=1}^{N} \boldsymbol{\mu}_j \delta\left(\mathbf{r} - \mathbf{r}_j(t)\right)$$

Modal Expansion of the Radiation Field (Frequency Domain)



Excitation of Modes by Particulate Charges

$$C_q^{out}(\omega) - C_q^{in}(\omega) = \sum_{j=1}^{N} \Delta C_{q_j} =$$

$$= -\frac{1}{4}\sum_{j=1}^{N}\left\{-e\int_{-\infty}^{\infty} v_j(t)\cdot \tilde{E}_q^{*}\left(\mathbf{r}_j(t)\right)e^{i\omega t}dt\right\}$$

$$C_q^{out}(\omega) = C_q^{in}(\omega) + \Delta C_{qe}^{(0)}(\omega)\sum_{j=1}^{N} e^{i\omega t_{oj}} + \sum_{j=1}^{N} \Delta C_{q_j}^{st}$$

$$P_q = \left|C_q^{out}(\omega)\right|^2$$

OR

$$\frac{dW_q}{d\omega} = \frac{2}{\pi}\left|C_q^{out}(\omega)\right|^2$$

$$C_q^{out}(\omega) =$$

$$\frac{dW_q^{out}}{d\omega} = \frac{2}{\pi}\left\langle \left| C_q^{out}(\omega) \right|^2 \right\rangle_j$$

DEFINITIONS

- Spontaneous emission = Shot noise emission:

$$C_q^{in}(\omega) = 0 \quad , \quad \left\langle \left| \sum_{j=1}^{N} e^{i\omega t_{oj}} \right|^2 \right\rangle_j = N$$

- Superradiant* emission = Coherent emission:

$$C_q^{in}(\omega) = 0 \quad , \quad \left\langle \left| \sum_{j=1}^{N} e^{i\omega t_{oj}} \right|^2 \right\rangle_j = N^2$$

- Stimulated emission:

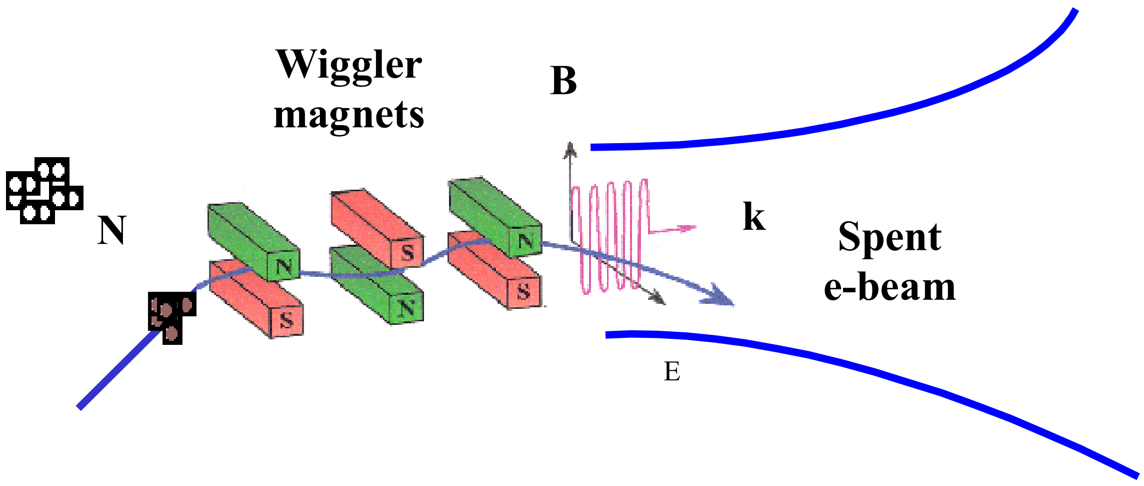
$$C_q^{in}(\omega) \neq 0 \quad , \quad \sum_{j=1}^{N} \Delta C_{qj}^{st} \propto C_q$$

(!In the sense of Dicke (Classical*

Radiation Emission Schemes

- Undulator Synchrotron
- Smith-Purcell
- Cerenkov Radiation
- Transition Radiation
- Cyclotron Resonant Emission (CRE)

UNDULATOR /SYNCHROTRON RADIATION SCHEMES



Excitation of Modes by a Single Electron

$$C_q^{out}(\omega) = -\frac{1}{4}\left\{-e\int_{-\infty}^{\infty}\mathbf{v}_j(t)\cdot\tilde{\mathbf{E}}_q^{\,*}\left(\mathbf{r}_j(t)\right)e^{i\omega t}dt\right\}$$

$$\mathbf{v}_j(t) = \mathrm{Re}\left\{\tilde{\mathbf{v}}_w e^{-ik_w z_j(t)}\right\} + v_z\hat{\mathbf{e}}_z$$

Single Electron Emission Frequency Domain

$$C_{qj}^{out}(\omega) = e\frac{\mathbf{v}_w \cdot \mathbf{E}_q^*(\mathbf{r}_{\perp 0})}{8v_z} L\,\underbrace{\mathrm{sinc}(\theta L/2)\, e^{i\theta L/2}}\, \underbrace{e^{i\omega t_{0j}}}$$

$$\theta(\omega) \equiv \left(\frac{\omega_0}{v_z} - k_z(\omega_0) - k_w\right) = (\omega - \omega_0)t_{sl} \equiv 2\pi\frac{\omega - \omega_0}{\Delta\omega}$$

Where:

$$\theta(\omega_0) = \frac{\omega_0}{v_z} - k_z(\omega_0) - k_w = 0$$

- Synchronism

$$t_{sl} = \frac{2\pi}{\Delta\omega} = \frac{L}{v_z} - \frac{L}{v_g}$$

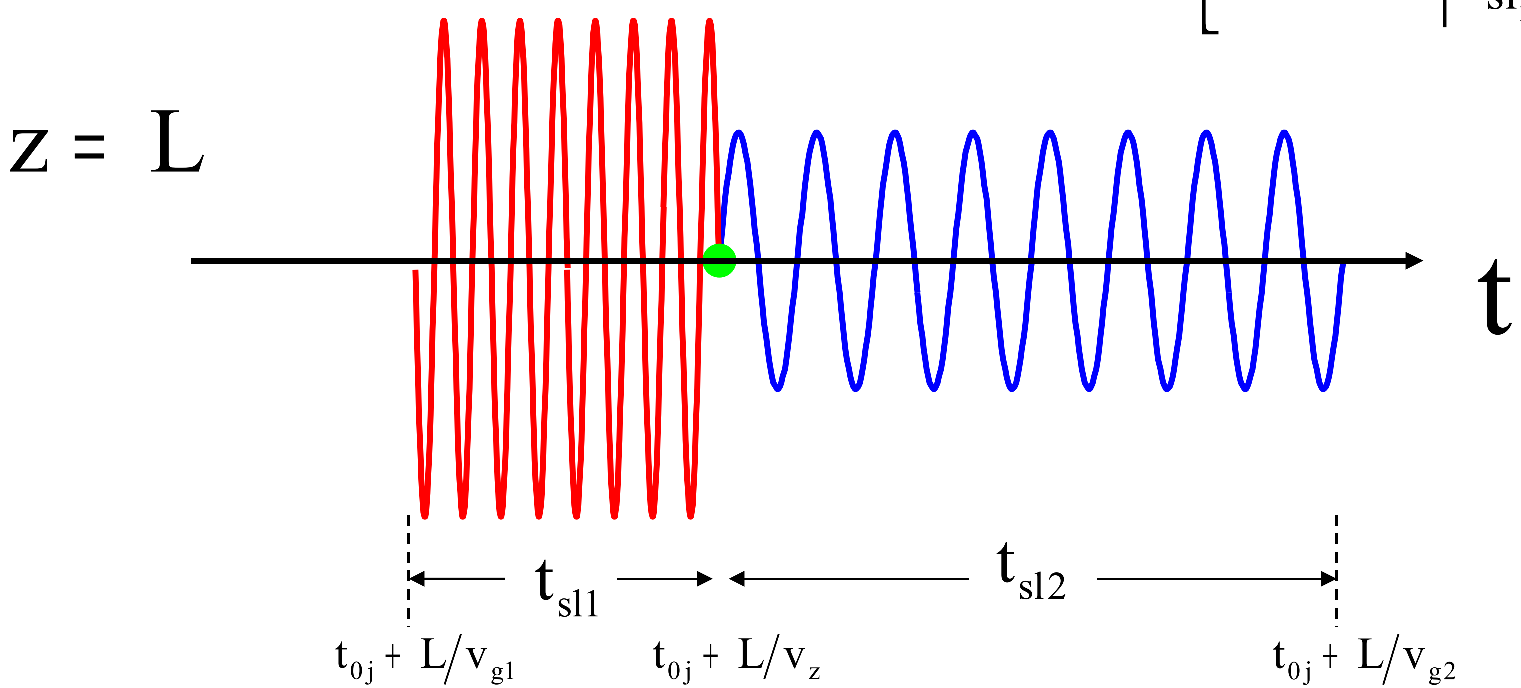
- Slippage Time

$_g(=cv)$

Single Electron Emission
Time Domain

$$\mathbf{E}(\mathbf{r},t) = \mathcal{F}^{-1}\left\{C_q(z,\omega)\,\mathbf{E}_q(\mathbf{r}_\perp)\,e^{ik_z(\omega)z}\right\} =$$

$$= z\sum_{i=1}^{2} A_i\,\mathbf{E}_q(\mathbf{r}_\perp)\cos\left(k_z(\omega_i)z - \omega_i t\right)\,\mathrm{rect}\left[\frac{t - z/v_z + t_{sl_i}/2}{\left|t_{sl_i}\right|}\right]$$



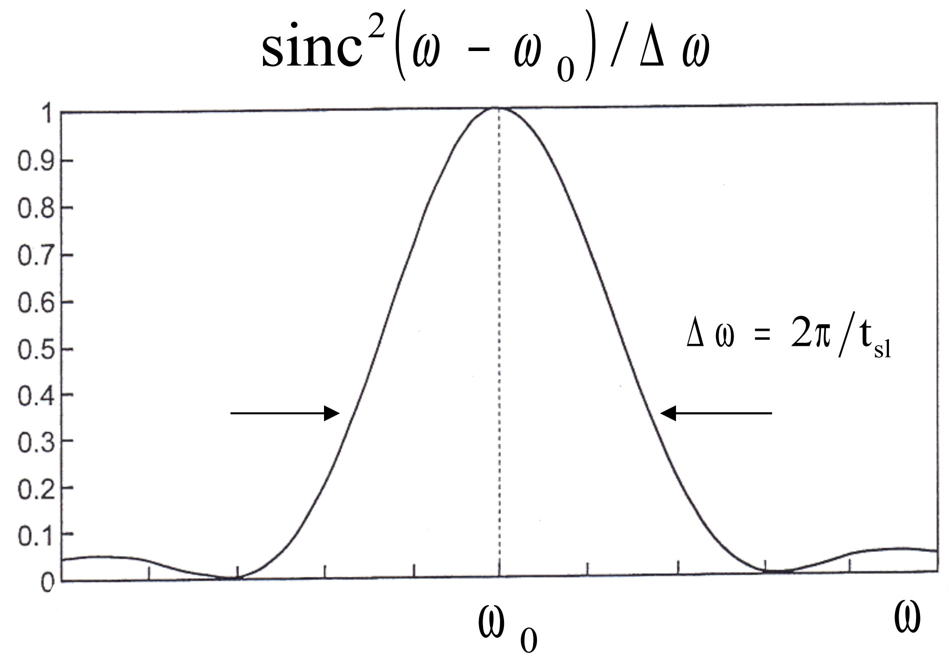
Superradiance and Spontaneous Emission from a Single Bunch of N Electrons

Spontaneous (t_{oj} random):

$$\left\langle \frac{dW_q(\omega)}{d\omega} \right\rangle_{SP} = \frac{2}{\pi} \left| C_{qe}^0(\omega) \right|^2 \cdot N$$

Superradiant ($| t_{0j} - t_0 | < 2\pi/\omega$):

$$\left\langle \frac{dW_q(\omega)}{d\omega} \right\rangle_{SR} = \frac{2}{\pi} \left| C_{qe}^0(\omega) \right|^2 \cdot N^2$$

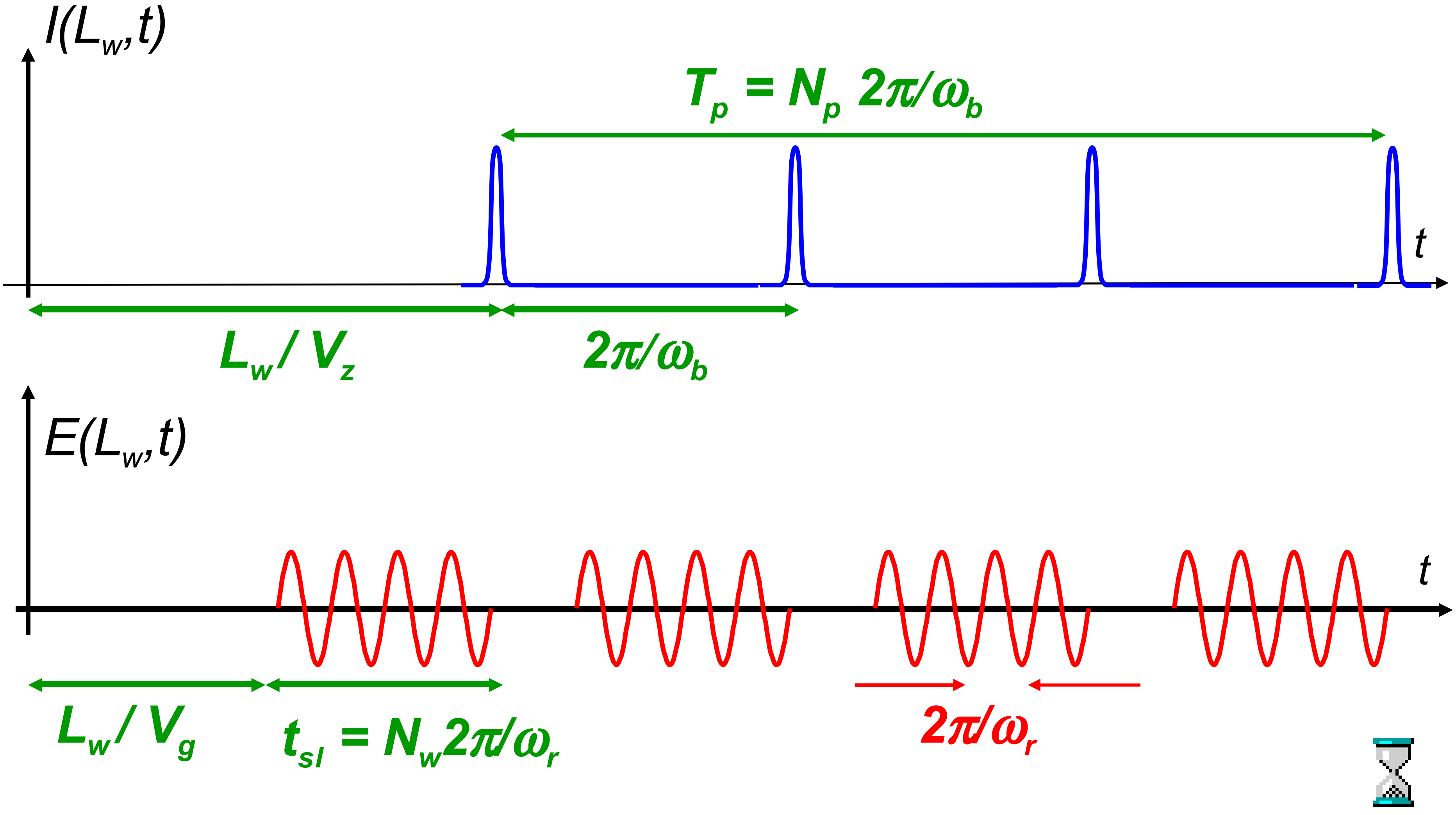


Where:

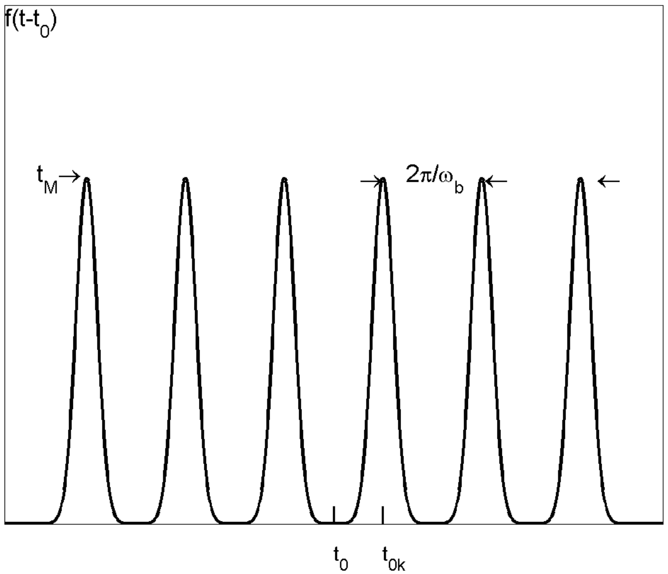
$$\left| C_{qe}^0(\omega) \right|^2 = \left| e \frac{\tilde{\mathbf{v}}_w \cdot \mathbf{E}_q^*(\mathbf{r}_{\perp 0})}{8 v_z} L \right|^2 \operatorname{sinc}^2(\theta L/2)$$

Superradiant Emission from a Pulse Composed of a Train of Bunches ($t_{sl} < 2\pi/\omega_b$)

Time Domain



A Train of Bunches (Macro-Pulse)



$$\sum_{j=1}^{N} e^{i\omega t_{0j}} = \sum_{k=1}^{N_P} \sum_{j=1}^{N_{bk}} e^{i\omega t_{0j}} =$$

$$\boxed{N \cdot M_b(\omega) \cdot M_M(\omega) \cdot e^{i\omega t_0}}$$

:Bunch form factor

$$M_b(\omega) = \frac{1}{N_b} \sum_{J=1}^{N_b} e^{i\omega t_{0j}} = \int_{-\pi/\omega_b}^{\pi/\omega_b} f(t_0')e^{i\omega t_0'} dt_0' = F\{f(t_0')\}$$

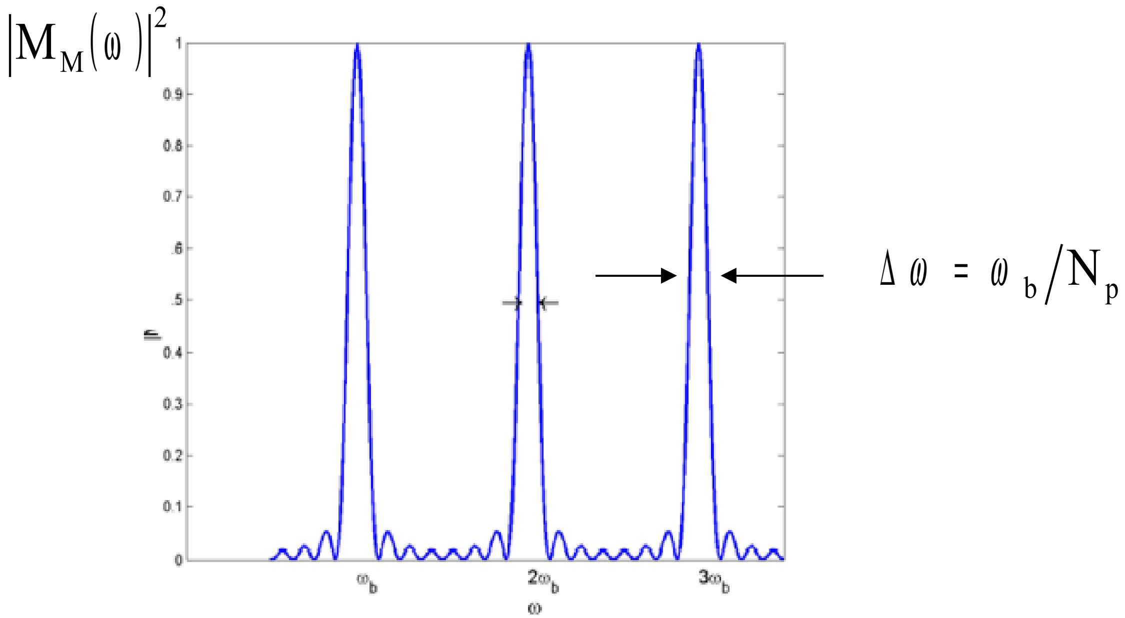
:Macro-pulse form factor

$$M_M(\omega) = \frac{1}{N_p} \sum_{k=1}^{N_p} e^{i\omega t_{0_k}} = \frac{\sin(N_p \pi\omega/\omega_b)}{N_p \sin(\pi\omega/\omega_b)}$$

Macropulse Form Factor

(for: $N_p = 8$)

$$M_M(\omega) = \frac{\sin(N_p \pi \omega / \omega_b)}{N_p \sin(\pi \omega / \omega_b)}$$

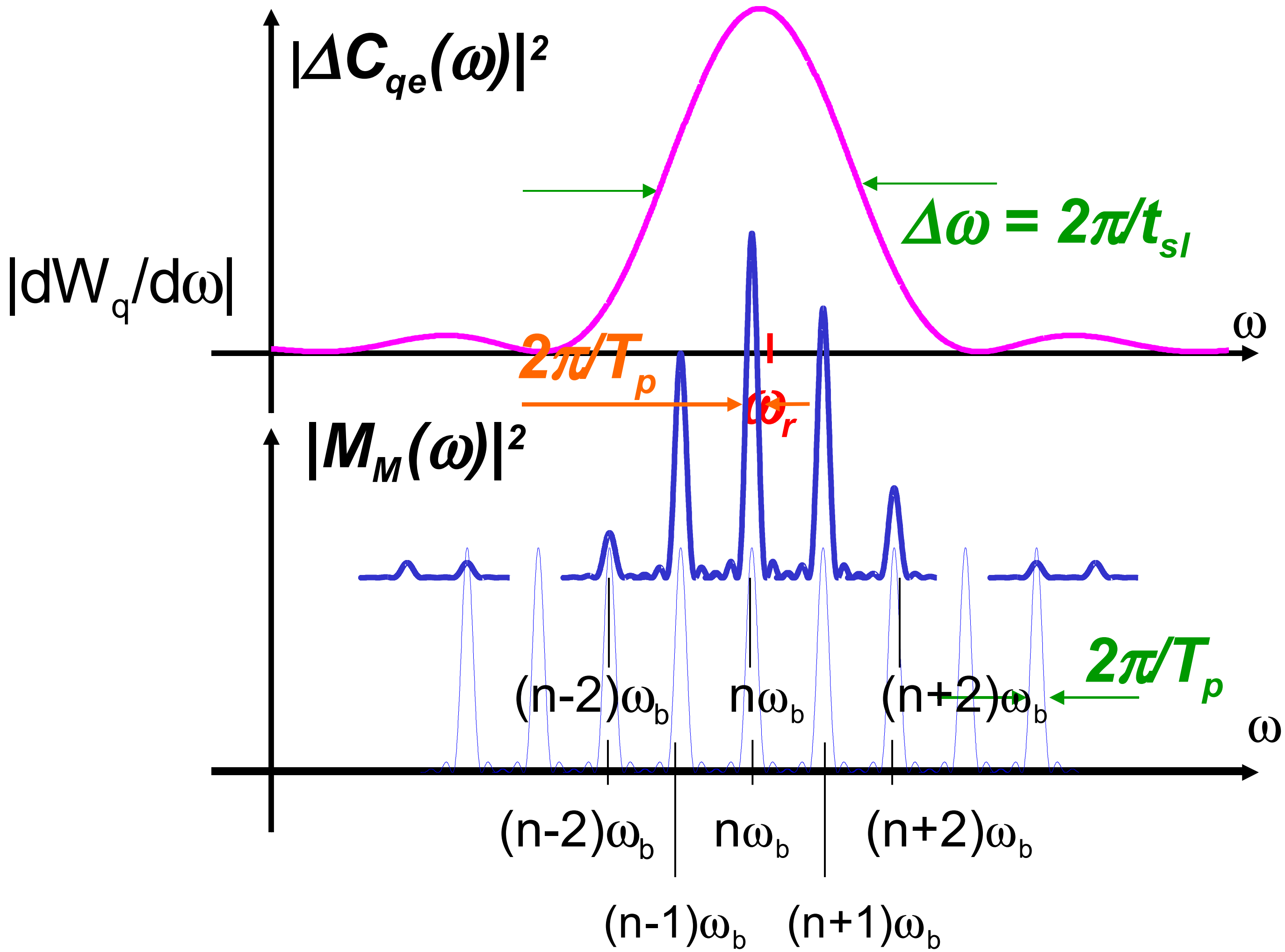


$t_b < 1/\omega \Rightarrow 1$

$$\left\langle \frac{dW_q(\omega)}{d\omega} \right\rangle_{SR} = \left\langle \frac{dW_q(\omega)}{d\omega} \right\rangle_{Single} \cdot |M_b(\omega)|^2 \cdot |M_M(\omega)|^2 \cdot N^2$$

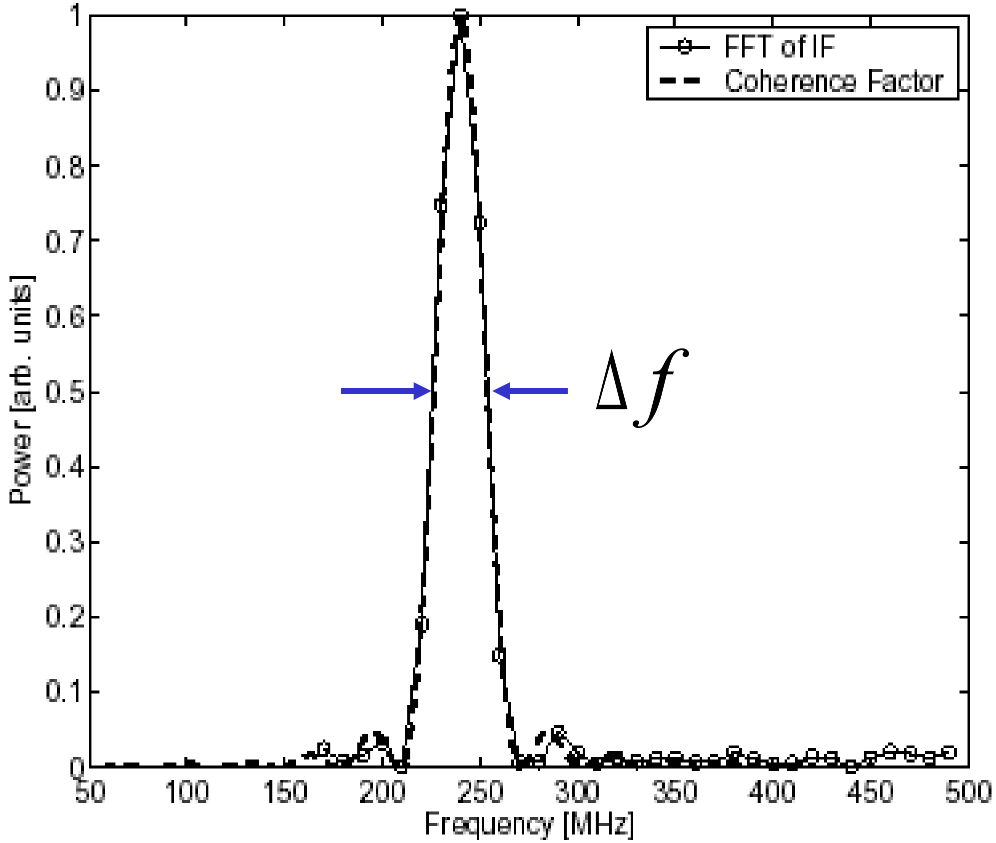
Superradiant Emission from a Pulse Composed of a Train of Bunches ($t_{sl} < 2\pi/\omega_b$)

Frequency Domain



Mesured Multi-bunch Coherent Smith-Purcell Linewidth

(MIT - S.E. Korbly et al PRL 2005)



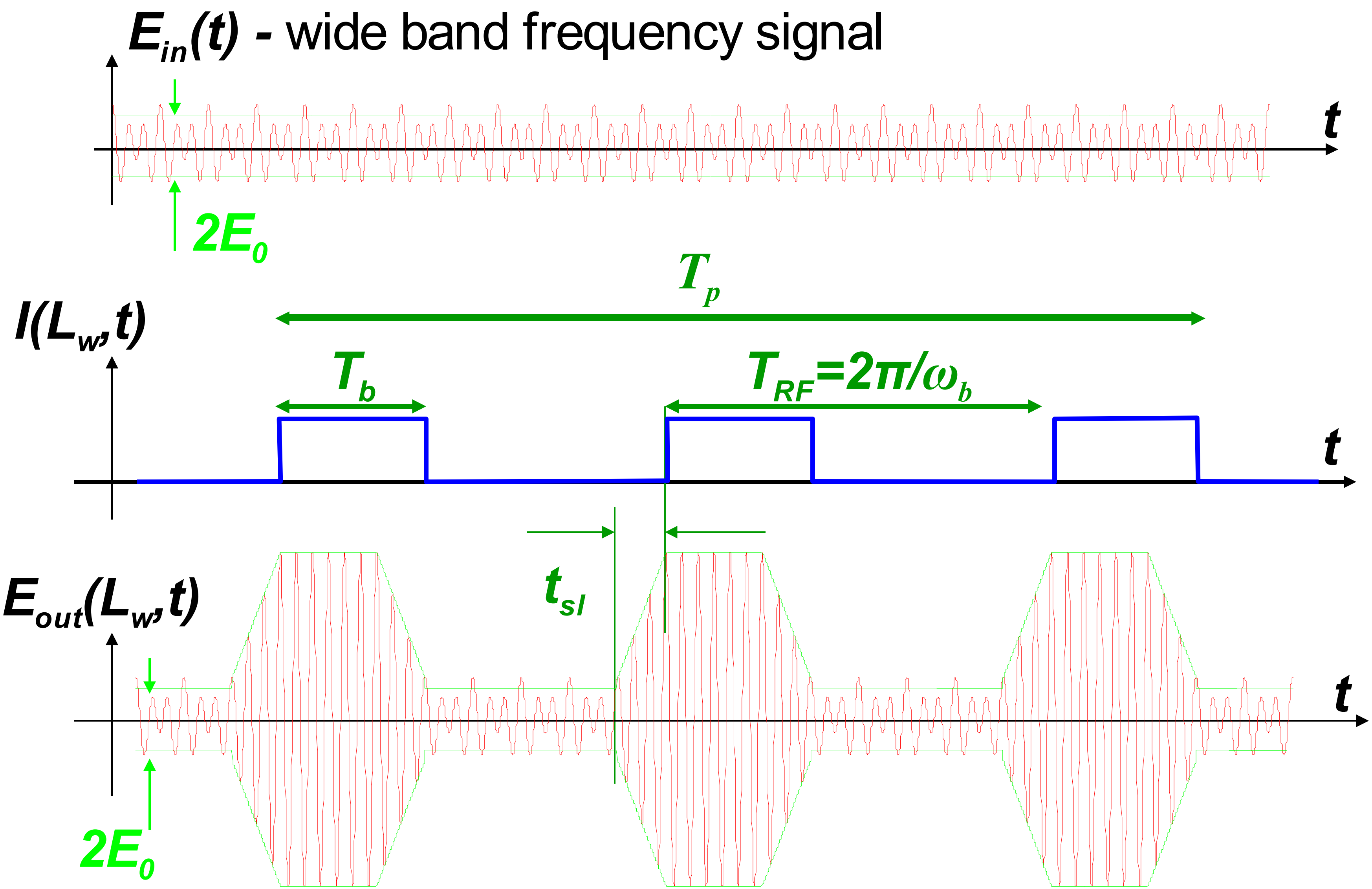
$f_{or} = mf_b = 240GHz$ $\Delta f = f_b / N_M = 1/T_p = 28MHz$ $\Delta f / f_{or} = 1/mN_M = 1.3 \cdot 10^{-4}$

GHz **1.7 GHz** **550** **14** **550 1.7** **14**

Stimulated Emission

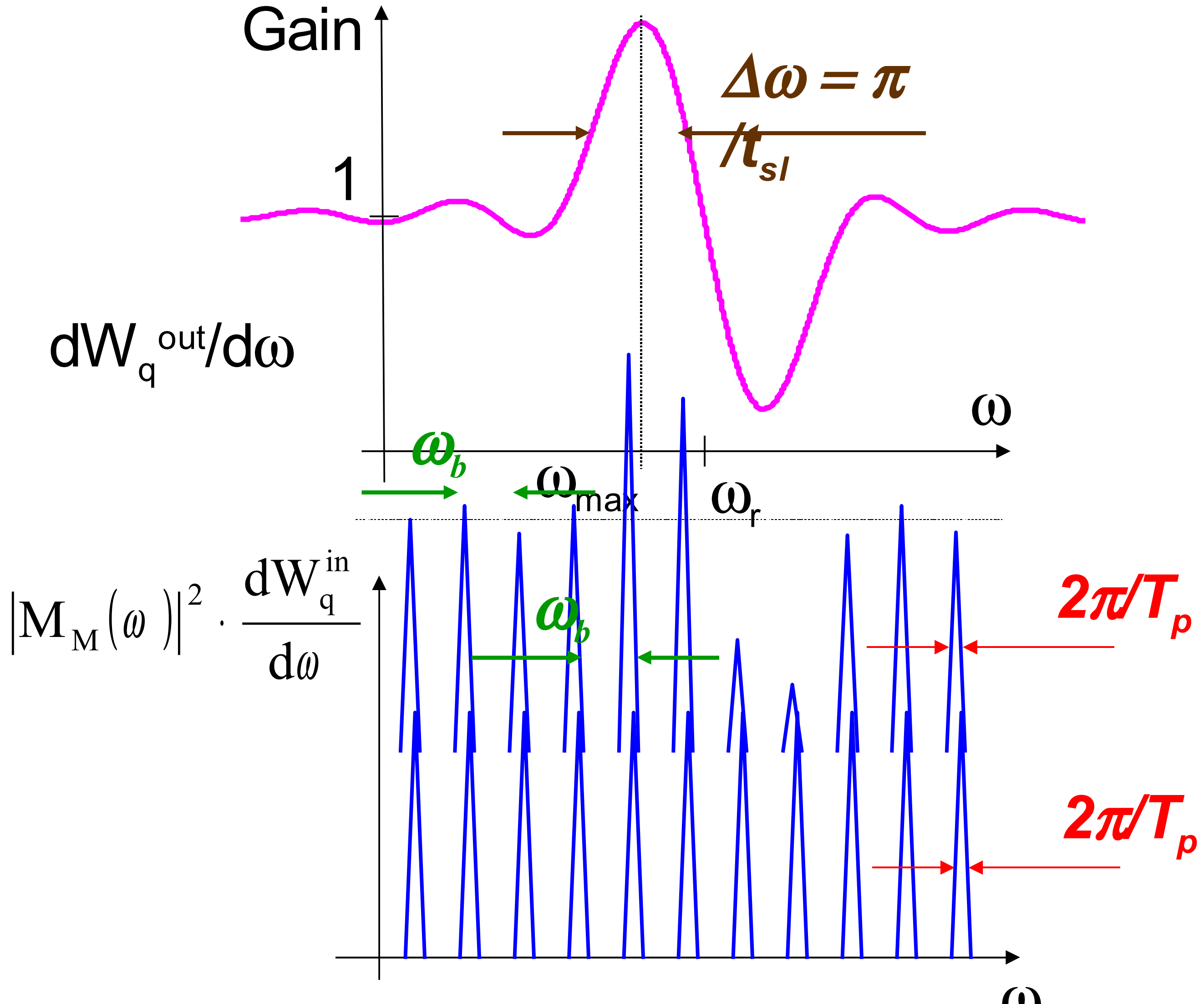
RF-Linac FEL Amplifier

Time Domain ($T_p >> T_{RF} >> T_b >> t_{sl}$)

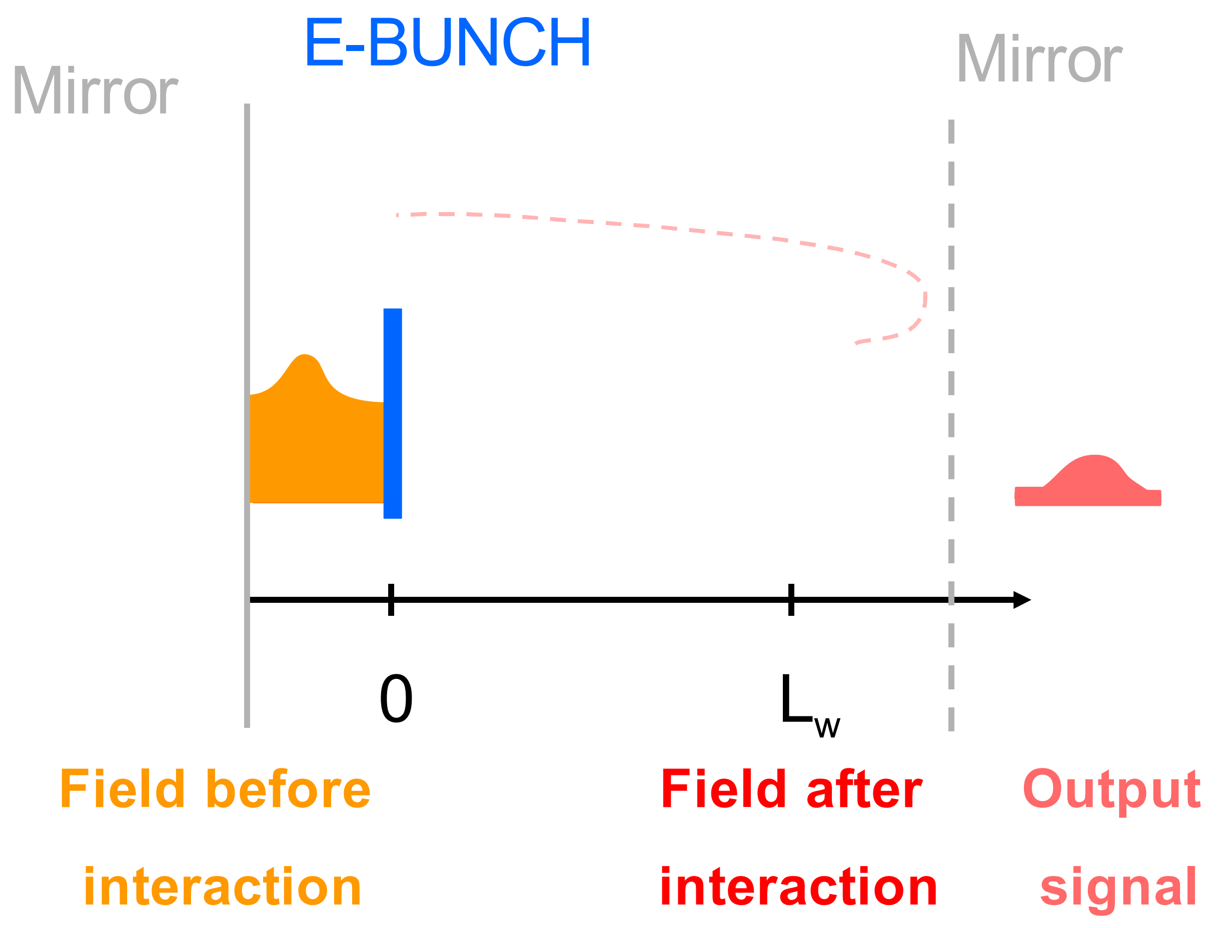


RF-Linac FEL Amplifier

Frequency Domain $(T_p \gg T_{RF} \gg T_b \gg t_{sl})$



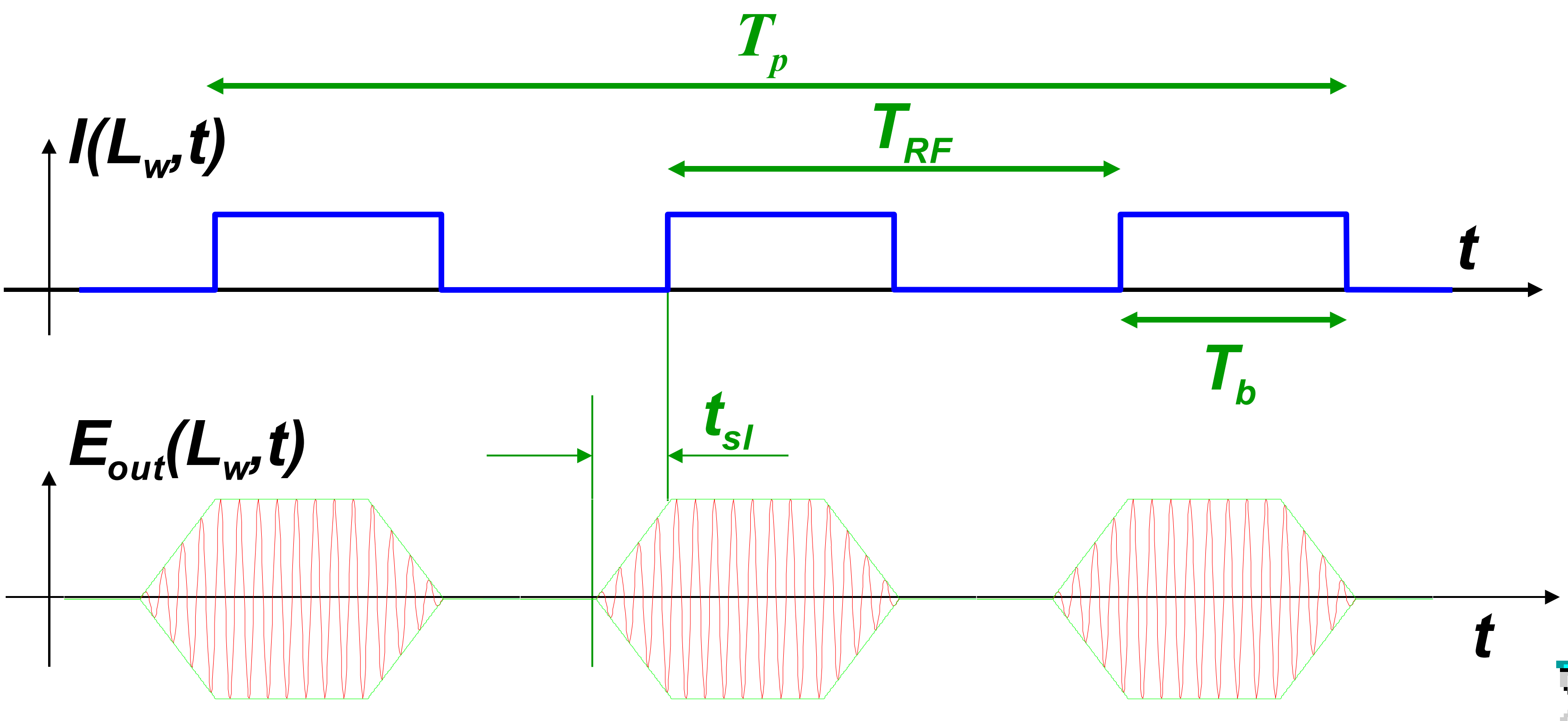
Oscillation Build-Up in a RF-LINAC FEL Oscillator



RF-Linac FEL Oscillator at Steady-state

Time Domain

In a resonator : $\boldsymbol{\omega_b} = \omega_{RF} = 2\pi/t_{rt}$



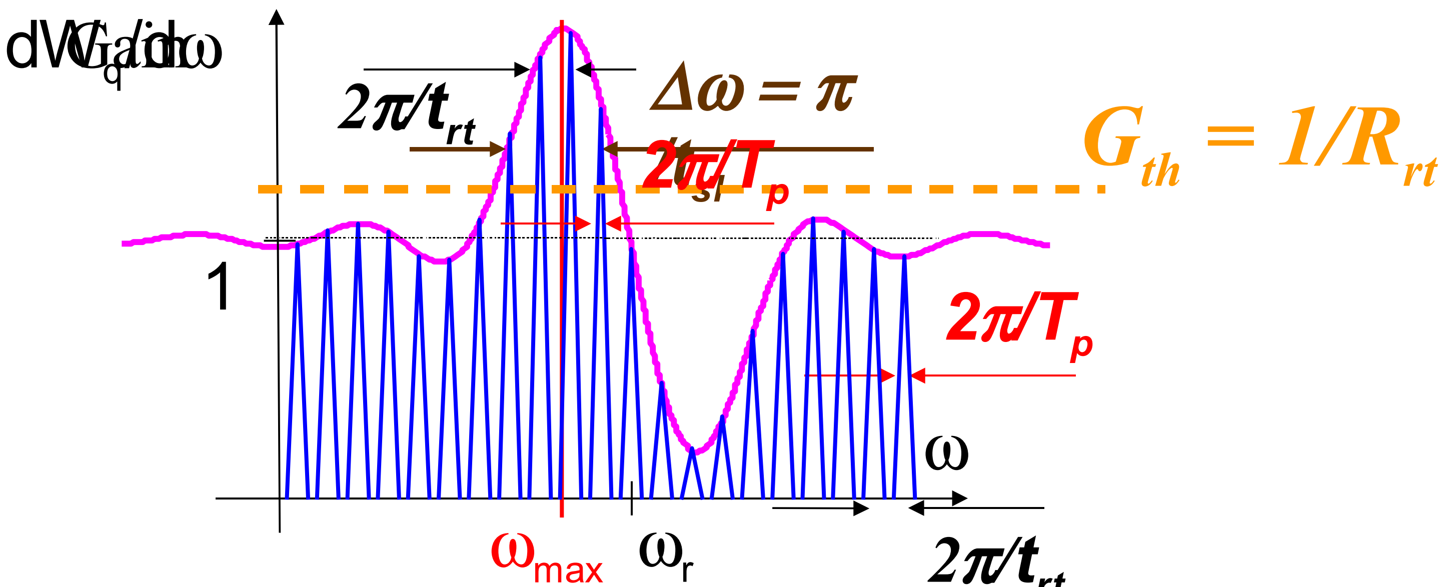
RF-Linac FEL Oscillator

Frequency Domain ($\omega_b < \Delta\omega$)

In a resonator : $\omega_b = \omega_{RF} = 2\pi/t_{rt}$

Oscillator build-up process

Steady state (saturation)



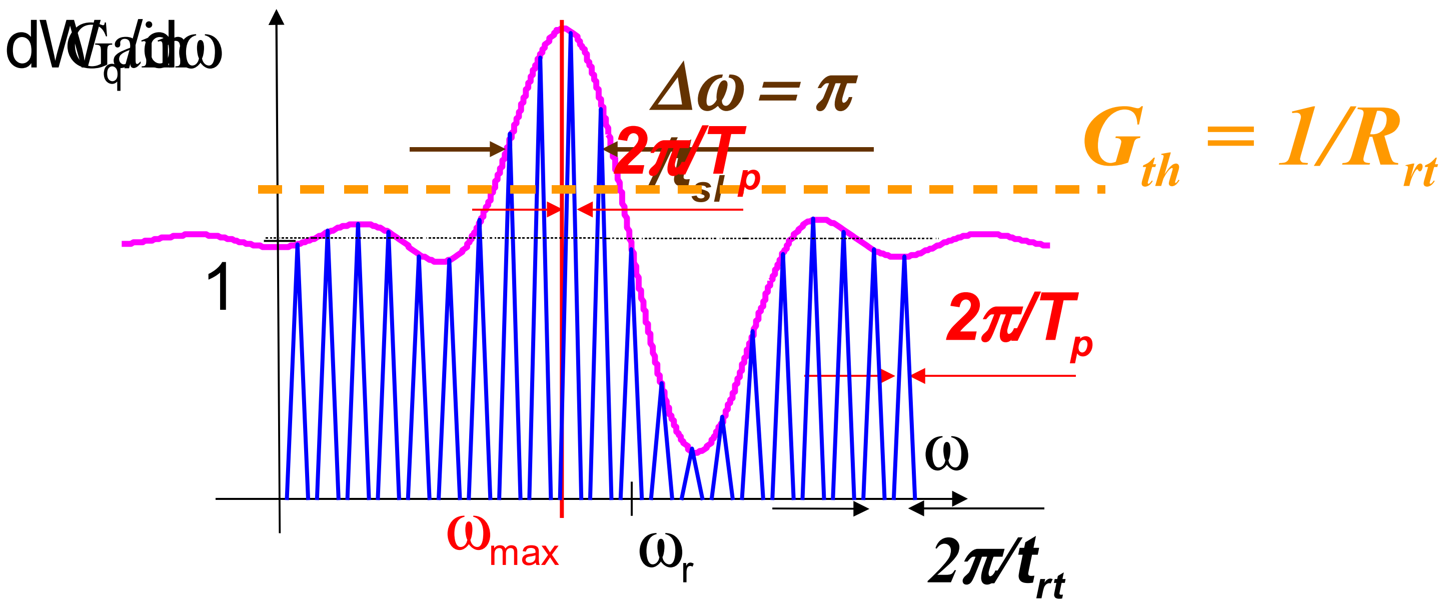
RF-Linac FEL Oscillator → Mode Locked Laser!

Electrostatic FEL (CW) Oscillator

Frequency Domain ($T_p > t_{sat} >> t_{rt}$)

In a resonator : $\omega_b \longrightarrow 2\pi / t_{rt}$ (no RF Bunches!)

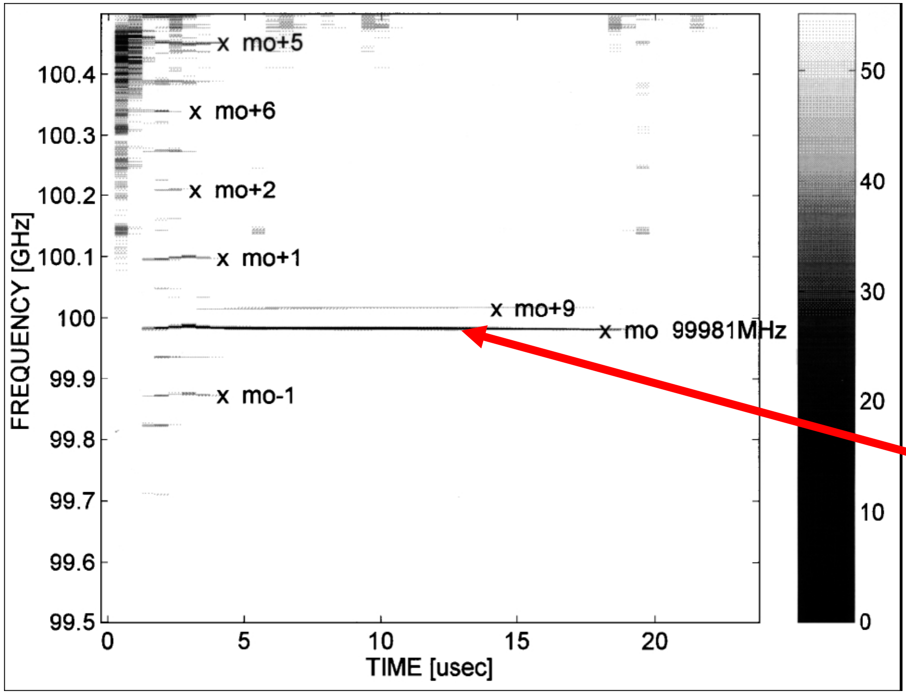
Oscillator build-up process / Steady state (saturation)

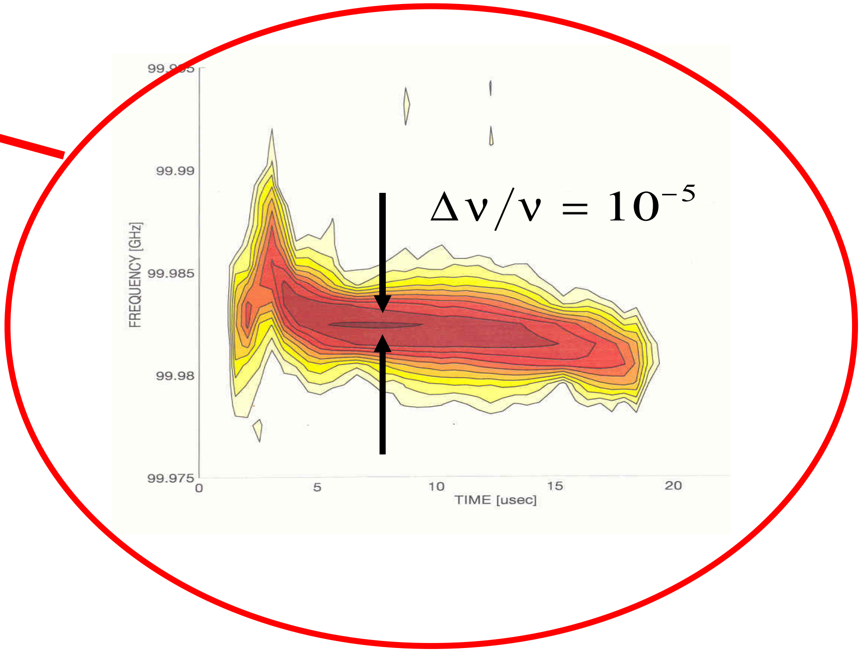


Electrostatic FEL Oscillator →

Homogeneously Broadened Single Mode Laser!

Spectrogram of Quasi-CW FEL: Mode Competition → Single Mode Operation

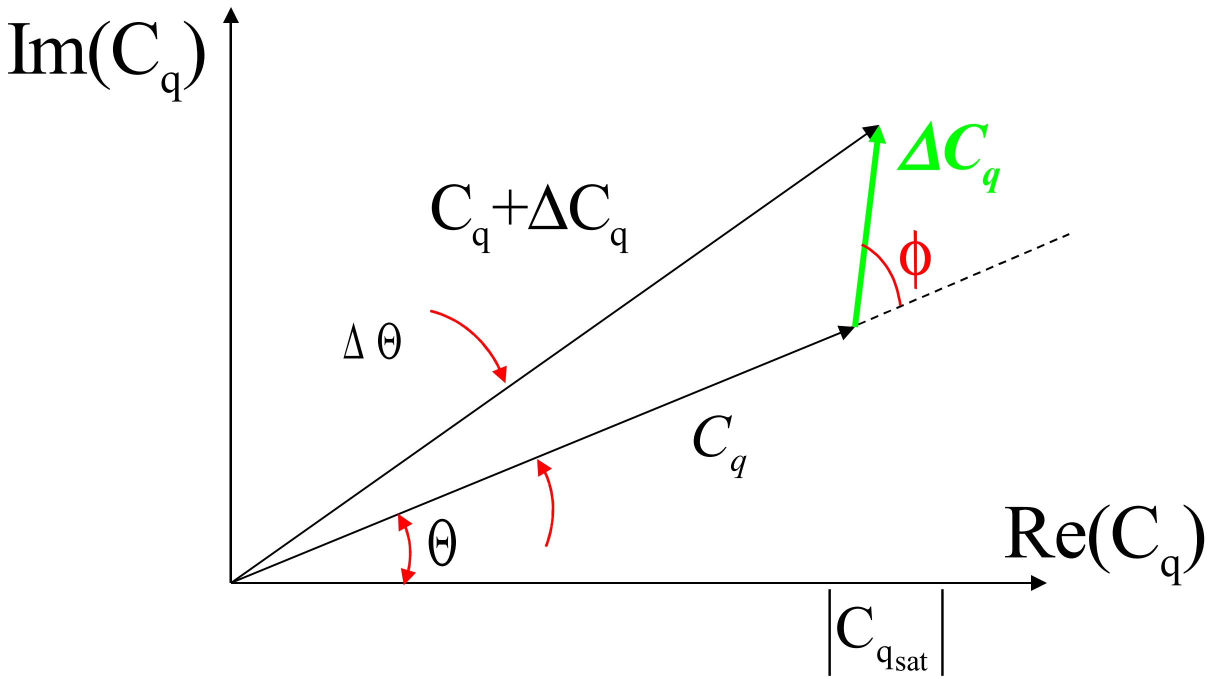




(A. Abramovich et al, P.R.L. 82, p.5257 (1999

Schawlow - Towns Relation

What happens in CW FEL ($T_p \rightarrow \infty$) ? $\Delta\omega = 0$?



Fluctuating phasor phase" model for Saturated Laser linewidth"

(A. Gover A. Amir, L. Elias, Phys. Rev. A35 (1987

Intrinsic Linewidth Conditions

Maser $$\Delta\nu_{maser} = 2\pi kT \frac{\left(\Delta\nu_{sp}\right)^2}{P_{gen}}$$

Gordon, Zeiger, Townes

PR, 99, 1264 (1955)

Laser $$\Delta\nu_{laser} = 2\pi h\nu \frac{\left(\Delta\nu_{1/2}\right)^2}{P_{gen}}$$

Schawlow and Townes

PR, 112, 1490 (1958)

FEL $$\Delta\nu_{FEL} = \frac{\left(\Delta\nu_{1/2}\right)^2}{I_0/e}$$

A. Gover A. Amir, L. Elias,
PR-A, 35, 164 (1987)

FEL IN THE HIGH GAIN REGIME

Pierce Linear Response Model

(Cubic Equation)

$$C_q(L,\omega) = H^E(\omega)\, C_q(\omega,0) + H^v(\omega)\, v(\omega,0) + H^I(\omega)\, I(\omega,0)$$

$$H^E(\omega) = \sum_{j=1}^{3} \mathrm{Res}\left(\frac{(\delta k - i\theta)^2 + \theta_{pr}^{\ 2}}{\Delta} \right)$$

$$H^v(\omega) = \sum_{j=1}^{3} \mathrm{Res}\left(\frac{ik_z L / v_{0z}}{\Delta} \cdot P_b^{1/2} \right)$$

$$H^I(\omega) = \sum_{j=1}^{3} \mathrm{Res}\left(\frac{(\delta k - i\theta)}{\Delta} \cdot P_b^{1/2} / I_b \right)$$

$$\left(P_b = \frac{I_b^{\ 2} \sqrt{\frac{\mu_0}{\varepsilon_0}}}{32} \cdot \left(\frac{a_w}{\gamma \beta_z} \right)^2 \cdot \frac{L^2}{A_{em}} \right)$$

$$\Delta = \delta k \left(\delta k - \theta - \theta_{pr} \right)\left(\delta k - \theta + \theta_{pr} \right) + \Gamma^3$$

High Gain FEL / PB-FEL

:Power $P_q(L,\omega) = \left|\widetilde{C}_q(L,\omega)\right|^2$

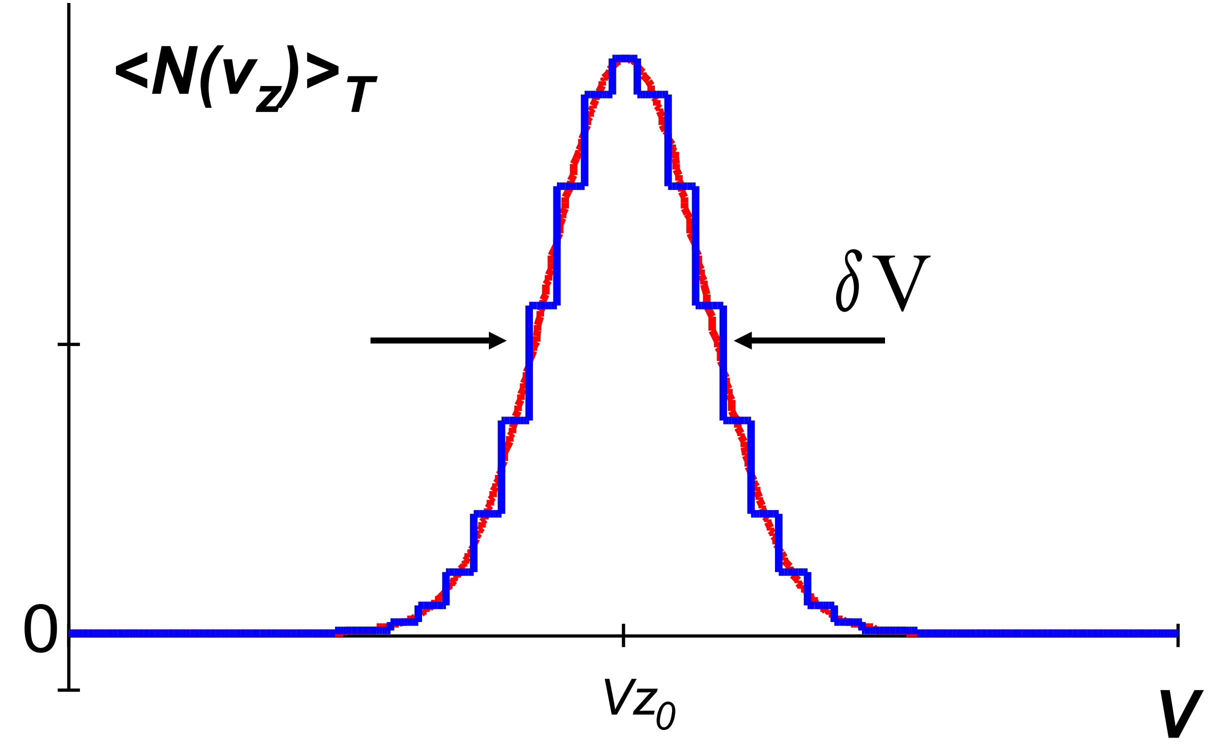
$$(\Gamma = 2\rho k_w)$$

$$P_q(L,\omega_0) = \frac{1}{9} e^{\sqrt{3}\Gamma L} \cdot \left|\widetilde{C}_q(0,\omega_0)\right|^2$$ - FEL Amplifier

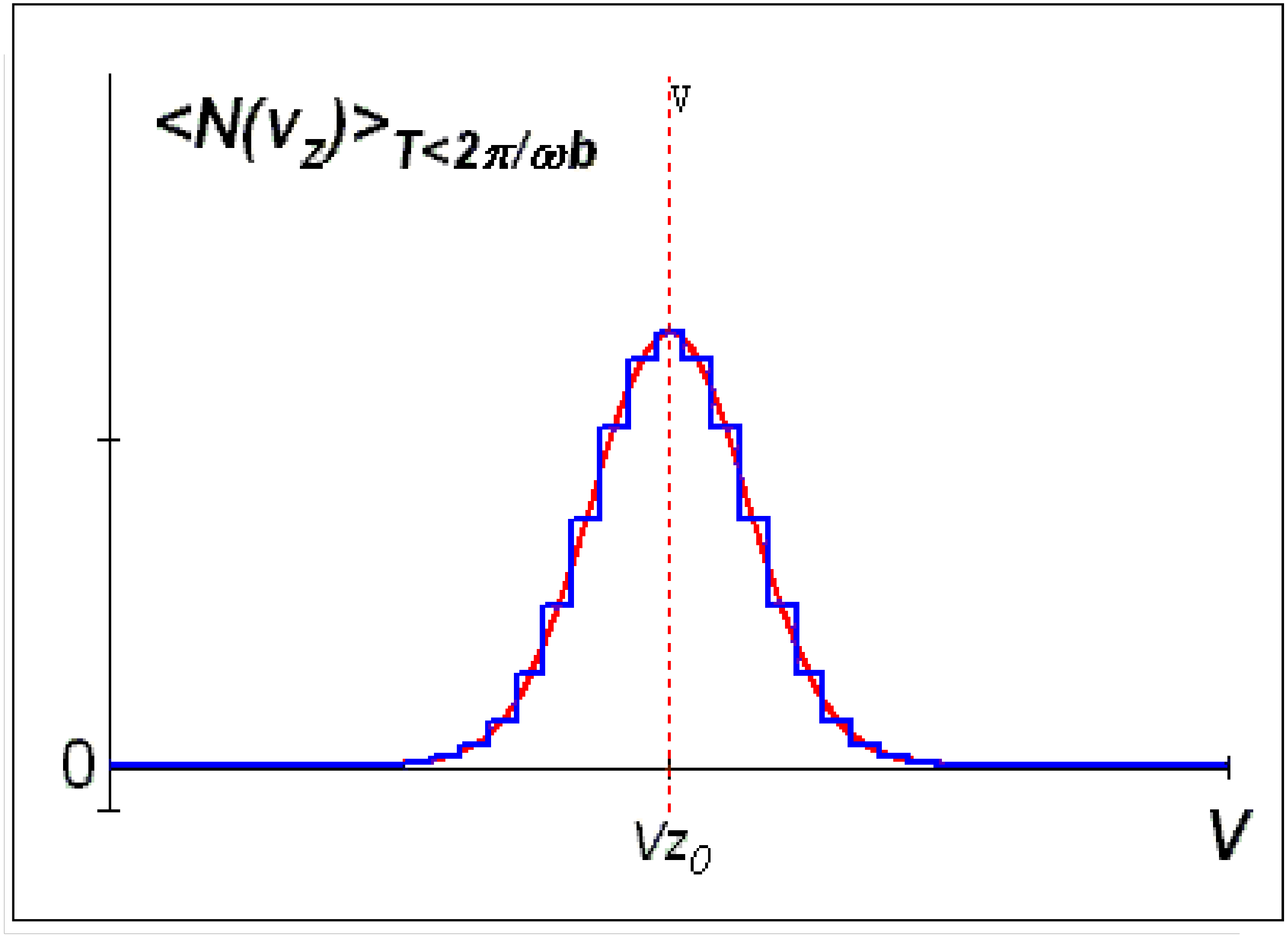
$$P^{pb-I}(L,\omega_0) = \frac{1}{9} e^{\sqrt{3}\Gamma L} \cdot P_b \left(\frac{1}{I_b \Gamma L}\right)^2 \left|\widetilde{I}(0,\omega_0)\right|^2$$ - I – PB-FEL

$$P^{pb-v}(L,\omega_0) = \frac{1}{9} e^{\sqrt{3}\Gamma L} \cdot P_b \left(\frac{k_z L}{V_{0z}(\Gamma L)^2}\right)^2 \left|\widetilde{v}_z(0,\omega_0)\right|^2$$ - v – PB-FEL

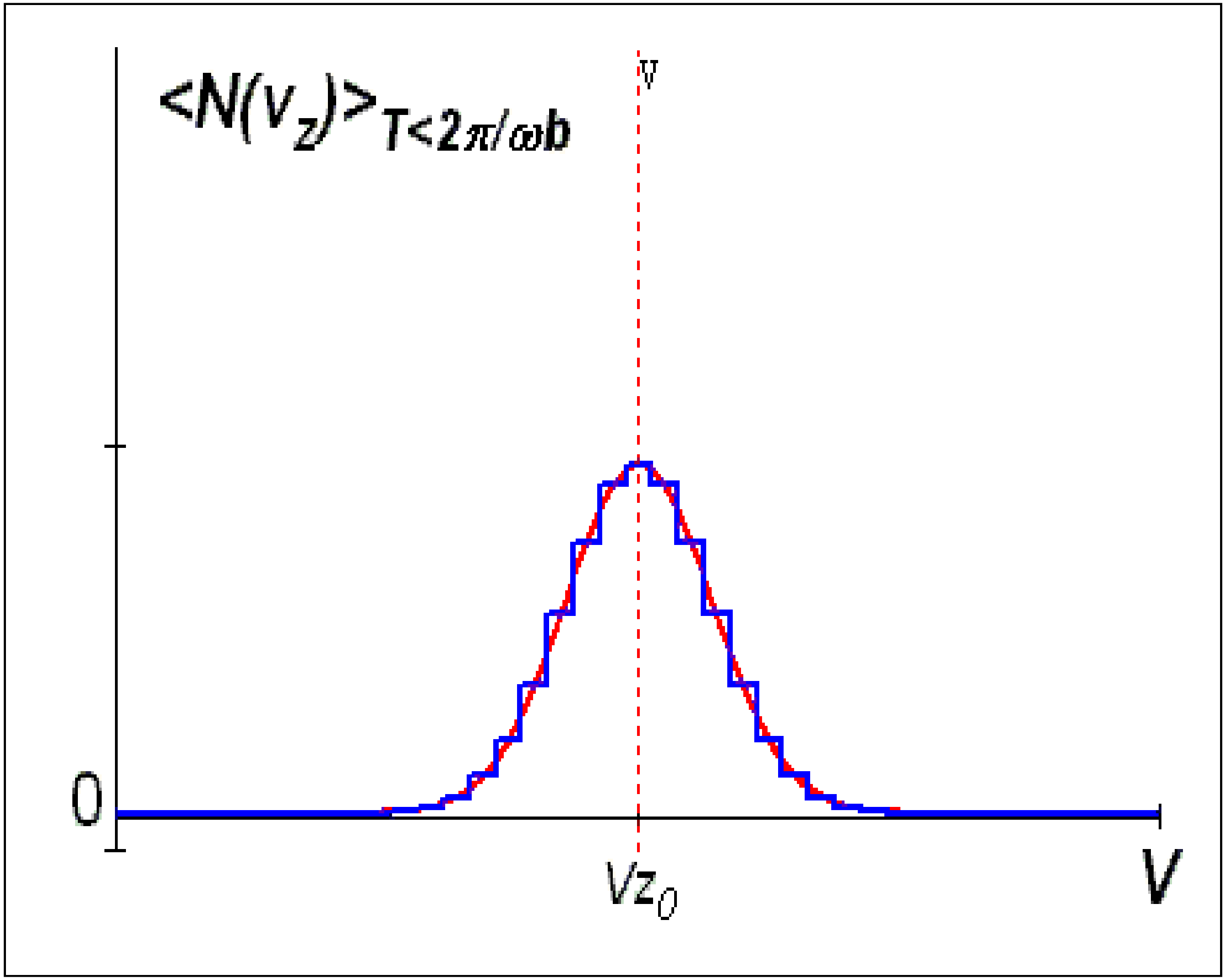
Axial Velocity Distribution of the E-beam Density Averaged over Time



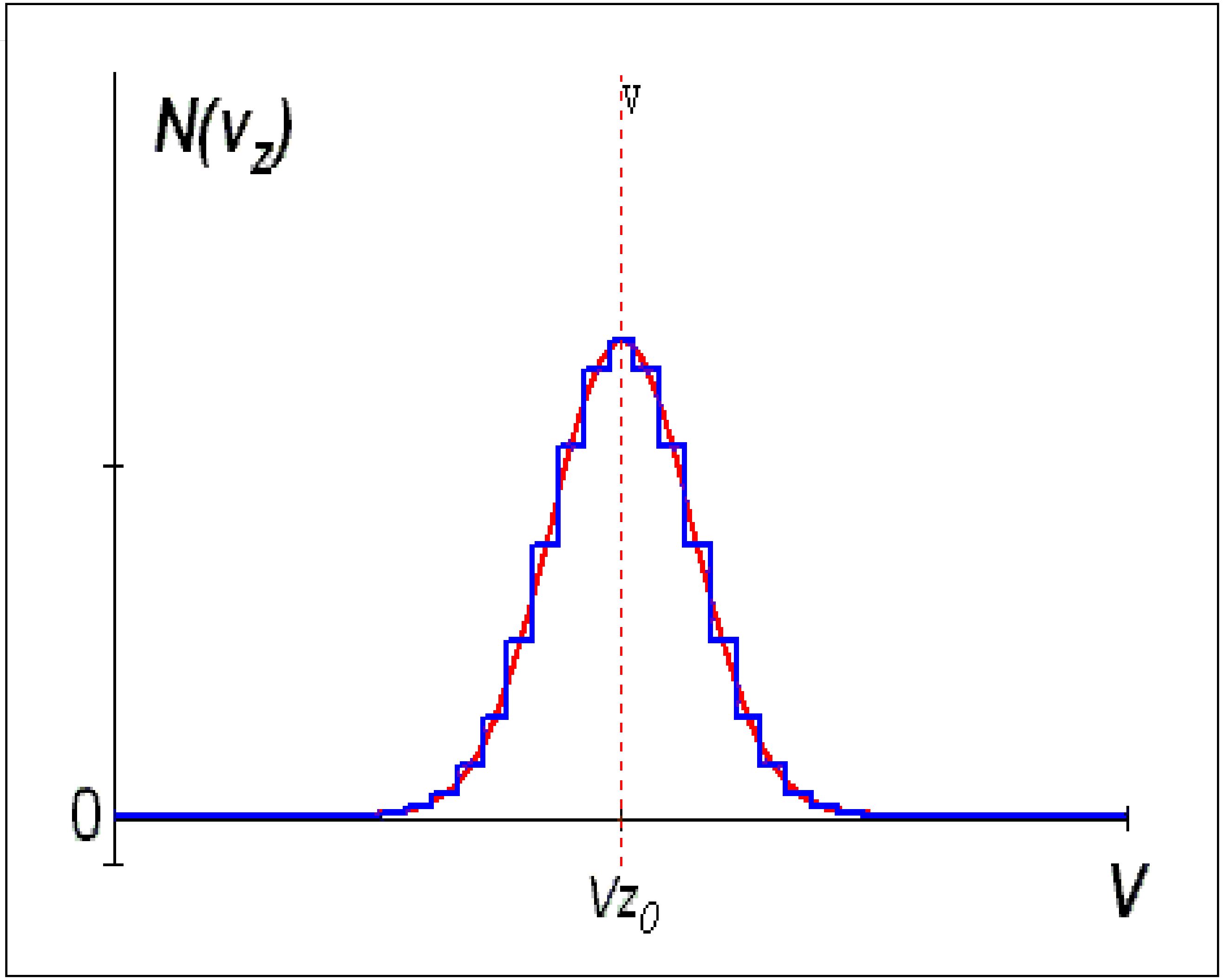
Current modulation



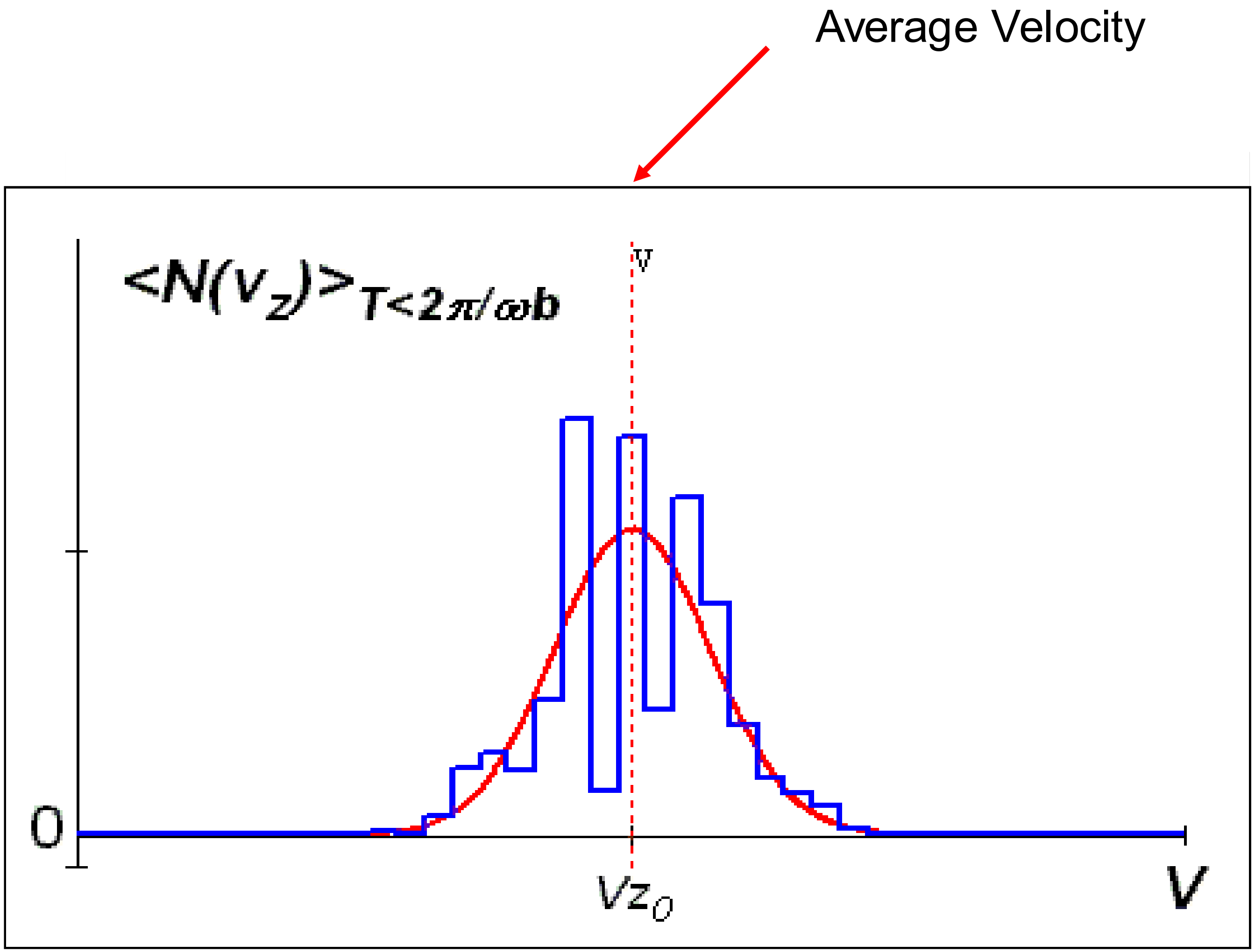
Velocity modulation



Current Fluctuations (Shot Noise)



Velocity Fluctuations



SASE-FEL

:Spectral Energy $$\frac{dW_q}{d\omega} = \frac{2}{\pi}\left\langle \left| \breve{C}_q(\omega) \right|^2 \right\rangle$$

:Spectral Power $$\frac{dP_q}{d\omega} = \frac{1}{T}\left\langle \frac{dW_q}{d\omega} \right\rangle$$

SASE (shot noise amplification):

:Current Fluctuations

$$\frac{dP_q^{\,I}}{d\omega} = \frac{2}{\pi}\frac{1}{9}e^{\sqrt{3}\Gamma L}\cdot P_b\left(\frac{1}{I_b \Gamma L}\right)^2 \frac{\left\langle \left| \breve{I}(0,\omega) \right|^2 \right\rangle}{T}$$

$$\left(\frac{\left\langle \left| \tilde{I}(0,\omega) \right|^2 \right\rangle}{T} = eI_b \right)$$

Velocity Fluctuations:

$$\frac{dP_q^{\,V}}{d\omega} = \frac{2}{\pi}\frac{1}{9}e^{\sqrt{3}\Gamma L}\cdot P_b\left(\frac{k_z L}{V_{0z}(\Gamma L)^2}\right)^2 \frac{\left\langle \left| \breve{v}_z(0,\omega) \right|^2 \right\rangle}{T}$$

$$\left(\frac{\left\langle \left| \tilde{V}_z(0,\omega) \right|^2 \right\rangle}{T} = \frac{e}{I_b}\left|\delta V\right|^2 \right)$$

SASE-FEL Velocity Spread Noise

vs. Current Shot-noise

:I – SHOT

$$\frac{dP^{I}(L,\omega)}{d\omega} = \frac{2}{\pi}\frac{1}{9}e^{\sqrt{3}\Gamma L}\cdot P_b \frac{e}{I_b(\Gamma L)^2}$$

:V – SHOT

$$\frac{dP^{v}(L,\omega)}{d\omega} = \frac{2}{\pi}\frac{1}{9}e^{\sqrt{3}\Gamma L}\cdot P_b \left(\frac{k_z L}{v_{0z}(\Gamma L)^2}\right)^2 \frac{e|\delta v|^2}{I_b}$$

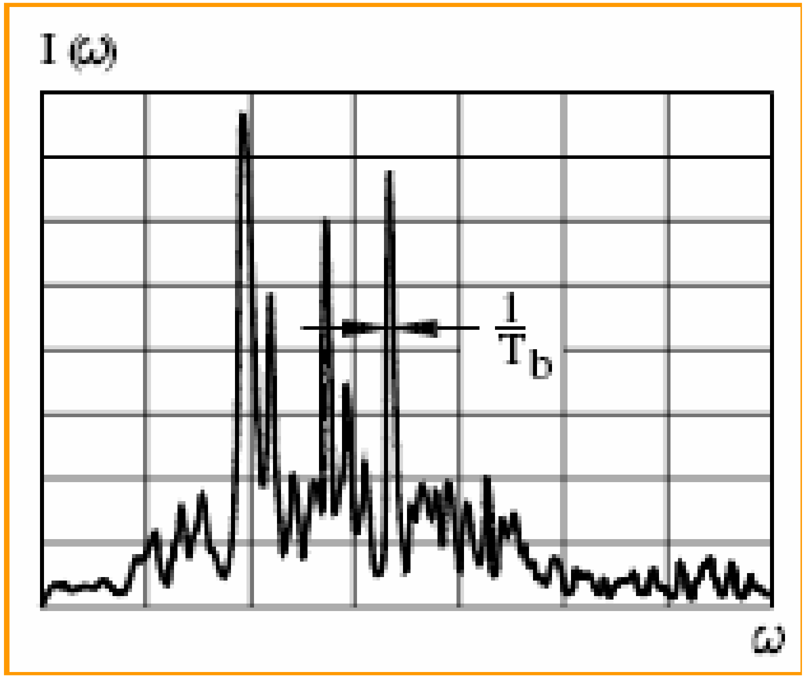
Dominance of current shot noise:

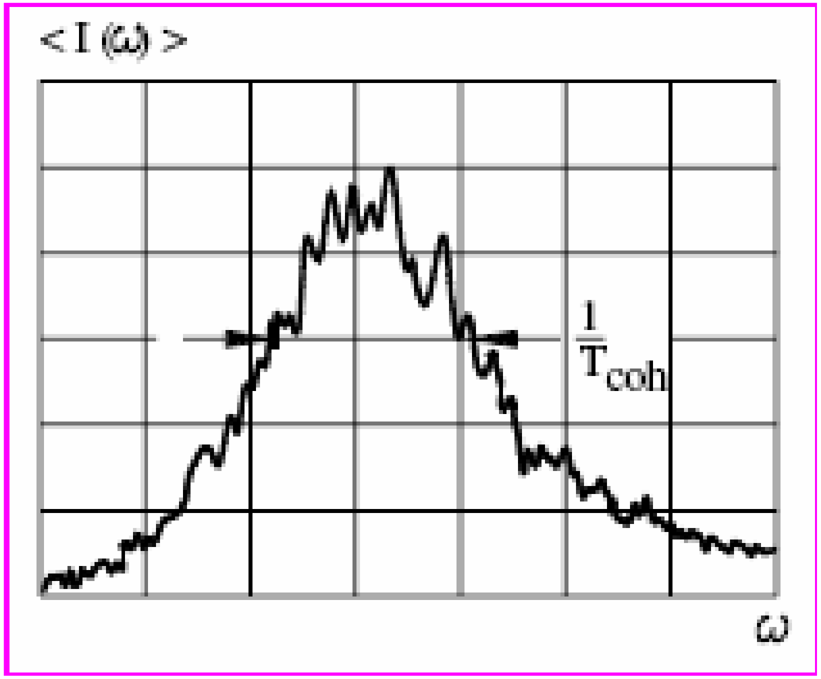
$$\frac{dP_q^{I}}{d\omega} > \frac{dP_q^{v}}{d\omega} \quad \Rightarrow \quad \boxed{\frac{\delta v_z}{v_{oz}} < \frac{\Gamma}{k_z}}$$

SASE –
COHERENCE and SPIKING

SASE FEL

Frequency Domain





Single-shot spectrum. The width of the peaks is inversely proportional to the electron bunch duration T_b.

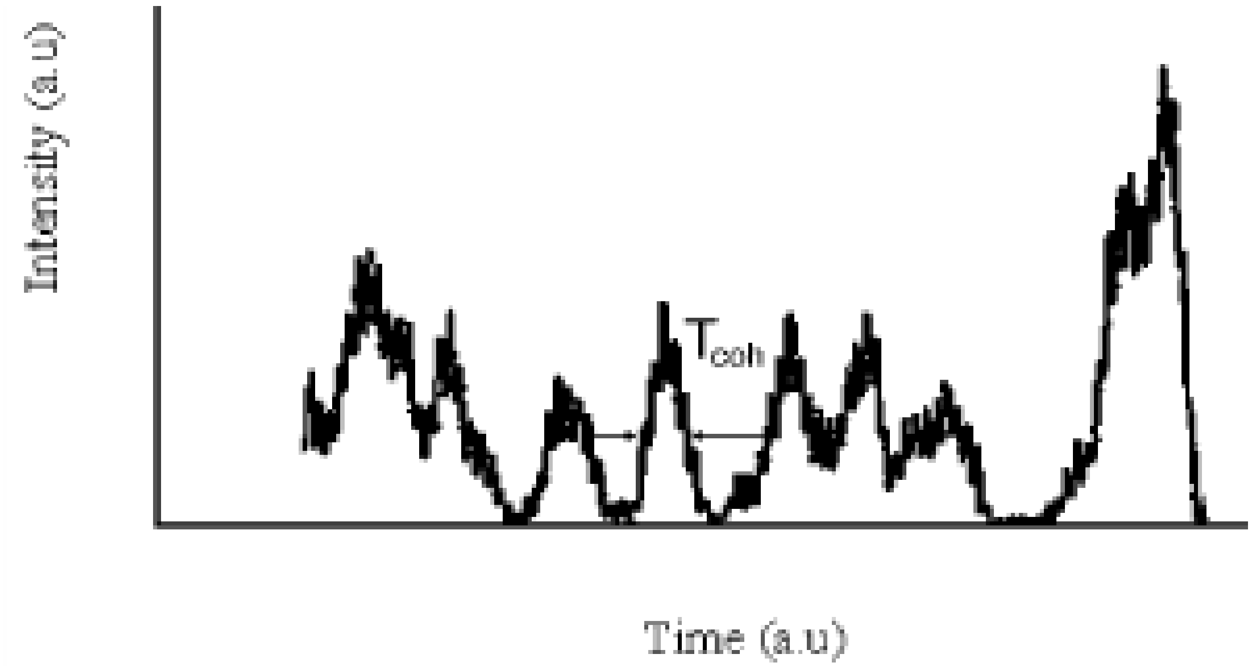
The average of many SASE pulses. The width is proportional to the gain bandwidth.

$$\sigma_{\omega} = \sqrt{\pi} / T_{coh}$$

S. Krinski and R.L.Gluckstern, PRST-AB, 6, 050701, 2003

Spiking” in SASE FEL“

Time Domain



Intensity spiking in the time domain (arbitrary units).

The width of the peaks is characterized by the SASE coherence time

$$T_{coh} = \sqrt{\pi}\,\sigma_{\omega}$$

S. Krinski and R.L.Gluckstern, PRST-AB, 6, 050701, 2003

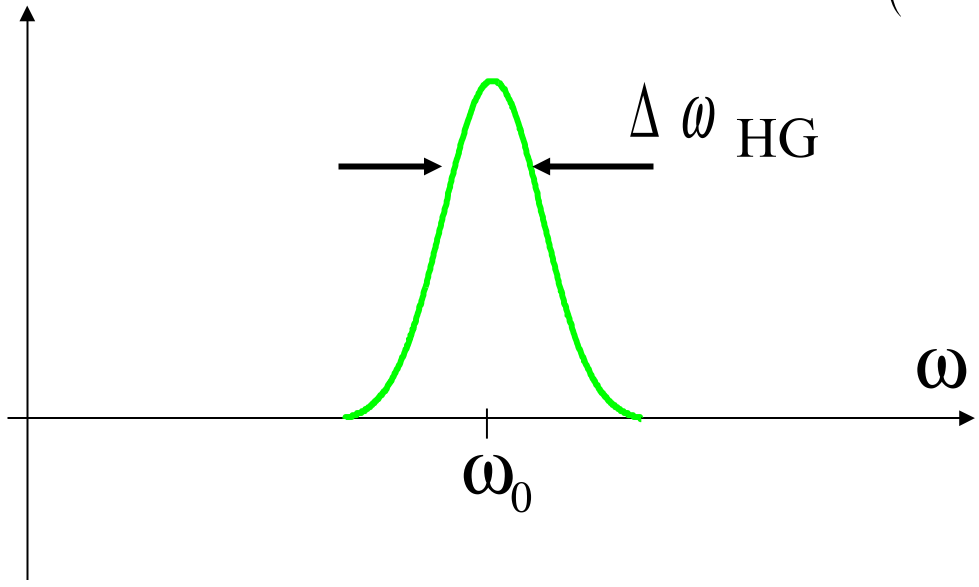
SASE FEL – Frequency Domain

:Quadratic expansion of the exponent

$$\tilde{H}^{I}(z,\omega) \cong \frac{P_b}{3I_b\Gamma z} e^{-i\pi/12} e^{\left(\sqrt{3}+i\right)\Gamma z/2} e^{-\left(1+i/\sqrt{3}\right)(\omega-\omega_0)^2/2(\Delta\omega_{HG})^2} e^{ikz}$$

$$\left(\Delta\omega_{HG} = \frac{3^{3/4}}{2\pi}\lambda_w\sqrt{\frac{\Gamma}{L}}\omega_0\right)$$

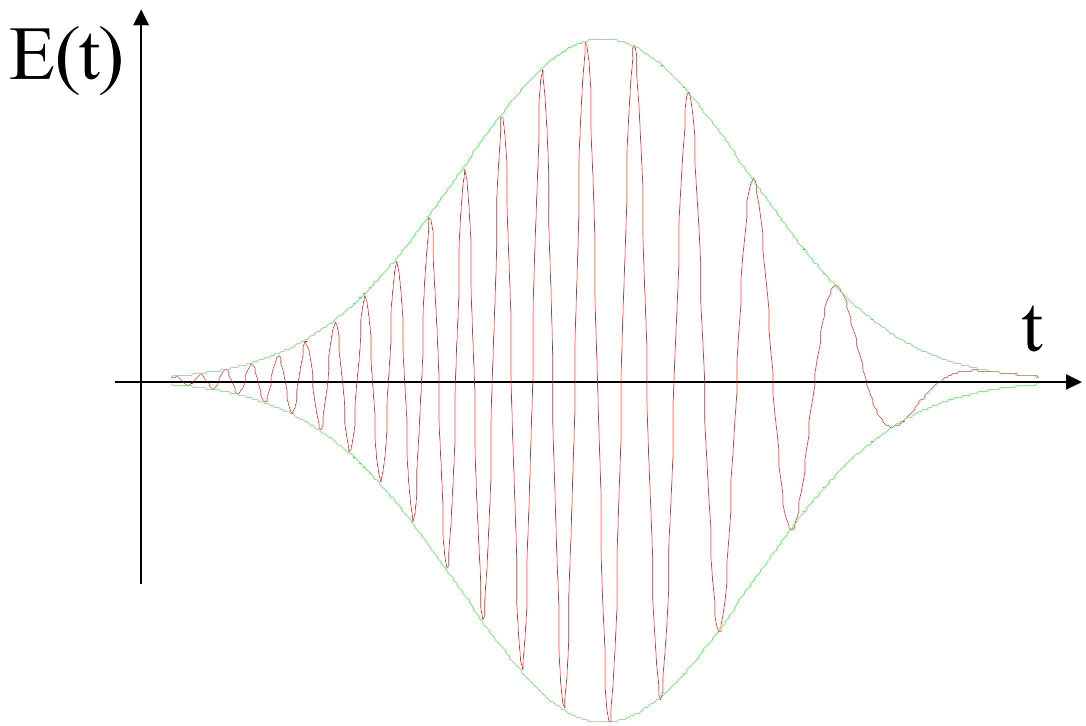
$$\left|\tilde{H}^{I}(z,\omega)\right|^2$$



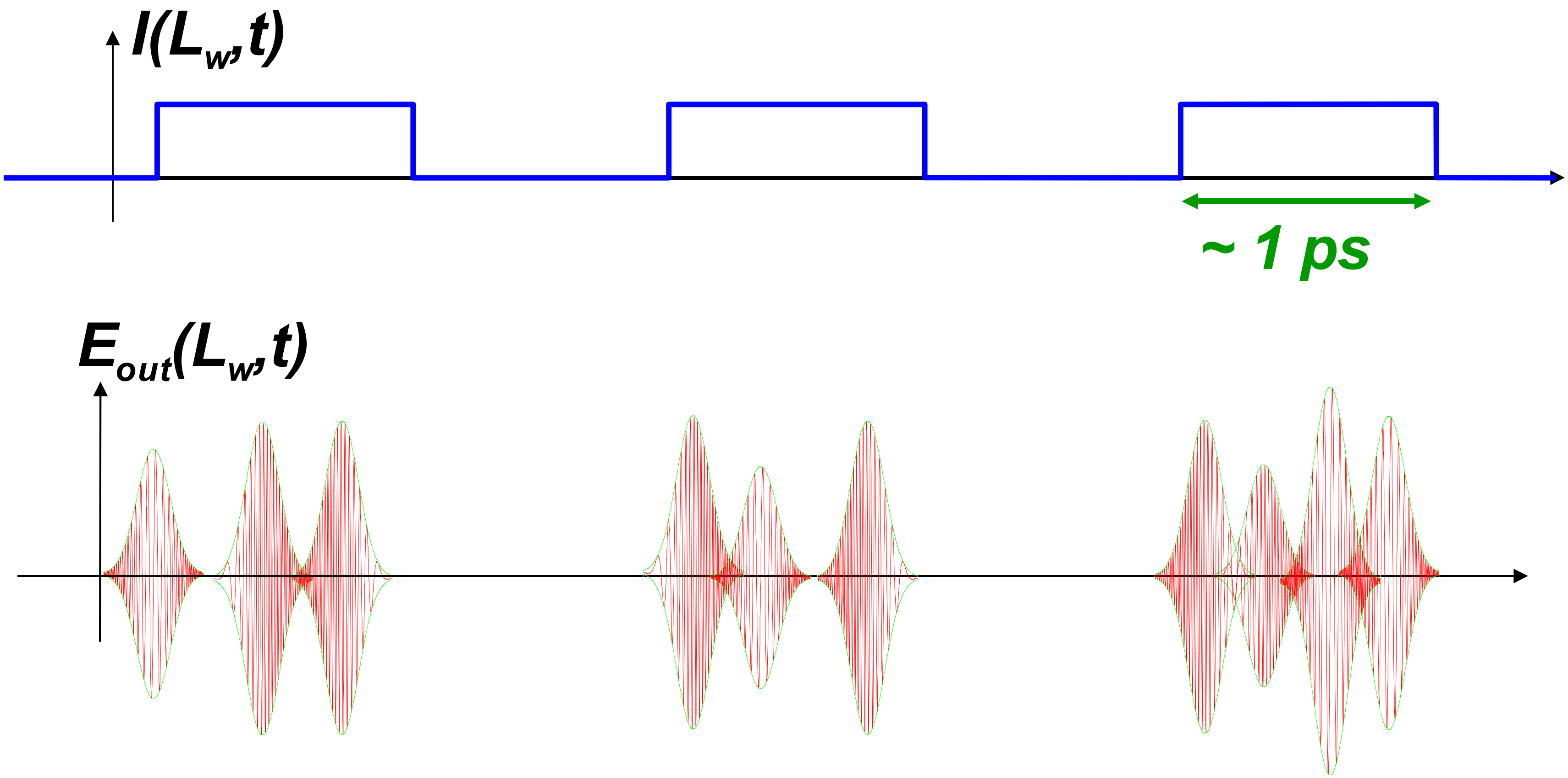
High Gain Impulse Response Function

$$E(z,t) = \mathrm{Re}\left\{ \left(P_b \Delta\omega_{HG} / 3^{3/4} \sqrt{2\pi\left(i + \sqrt{3}\right) I_b \Gamma z} \right) \cdot \exp\left[-i\pi/12 + \left(\sqrt{3} + i\right)\Gamma z/2 \right] \cdot \right.$$

$$\left. \exp\left[i\left(\omega_0 \left(t - \frac{z}{c} - t_0\right) - \left(\Delta\omega_{HG}\right)^2 \left(t - \frac{z}{c} - t_0\right)^2 / 8 \right) - \left(\Delta\omega_{HG}\right)^2 \left(t - \frac{z}{c} - t_0\right)^2 / 8 \right] \right\}$$



SASE FEL – Time Domain



How to Make SASE FEL Coherent?

- Seed Radiation Injection (Amplification).
- Filtering and Re-injection.
- Sub-Harmonic micro-Bunching.
- High Gain High Harmonic Generation.

?What is Required

Seed radiation injection:

Find coherent or filtered sources in the SASE emission frequency range.

Seed power: larger than the current and velocity shot noises.

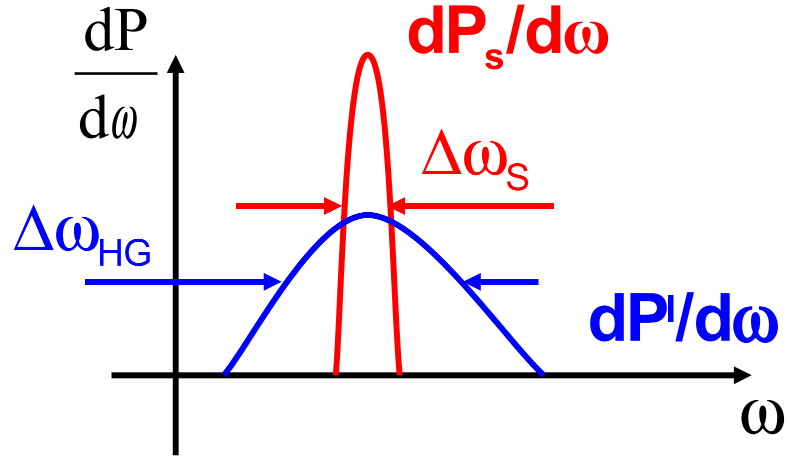
Sub-harmonic μ-bunching:

Find a sub-harmonic μ-bunching scheme that produces periodic short micro-perturbation over a uniform current during the LINAC micro-bunch.

Harmonic bunching power: larger than the current and velocity shot noise.

SASE-FEL

SEED-Radiation Power *vs.* Current Shot-noise

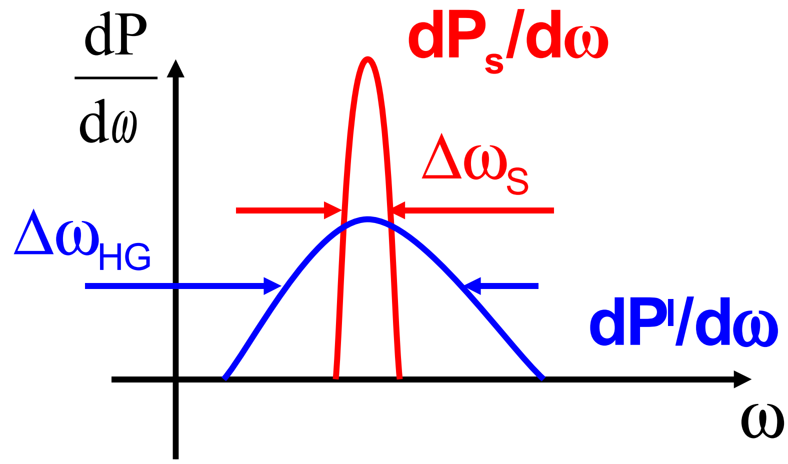


Seed *SPECTRAL*: power dominates

$$P^{seed} = \frac{dP^I}{d\omega}\Delta\omega_s \quad \Rightarrow \quad P_s(0) > P_b \frac{e}{I_b(\Gamma L)^2}\Delta\omega_s$$

SASE-FEL

SEED-Radiation Power *vs.* Current Shot-noise



Seed *TOTAL*: power dominates

$$P^{seed} = \frac{dP^I}{d\omega}\Delta\omega_{HG} \Rightarrow$$

$$P_s(0) > P_b \frac{e}{I_b(\Gamma\ L)^2}\Delta\omega_{HG}$$

High Harmonic μ-Pre-bunching

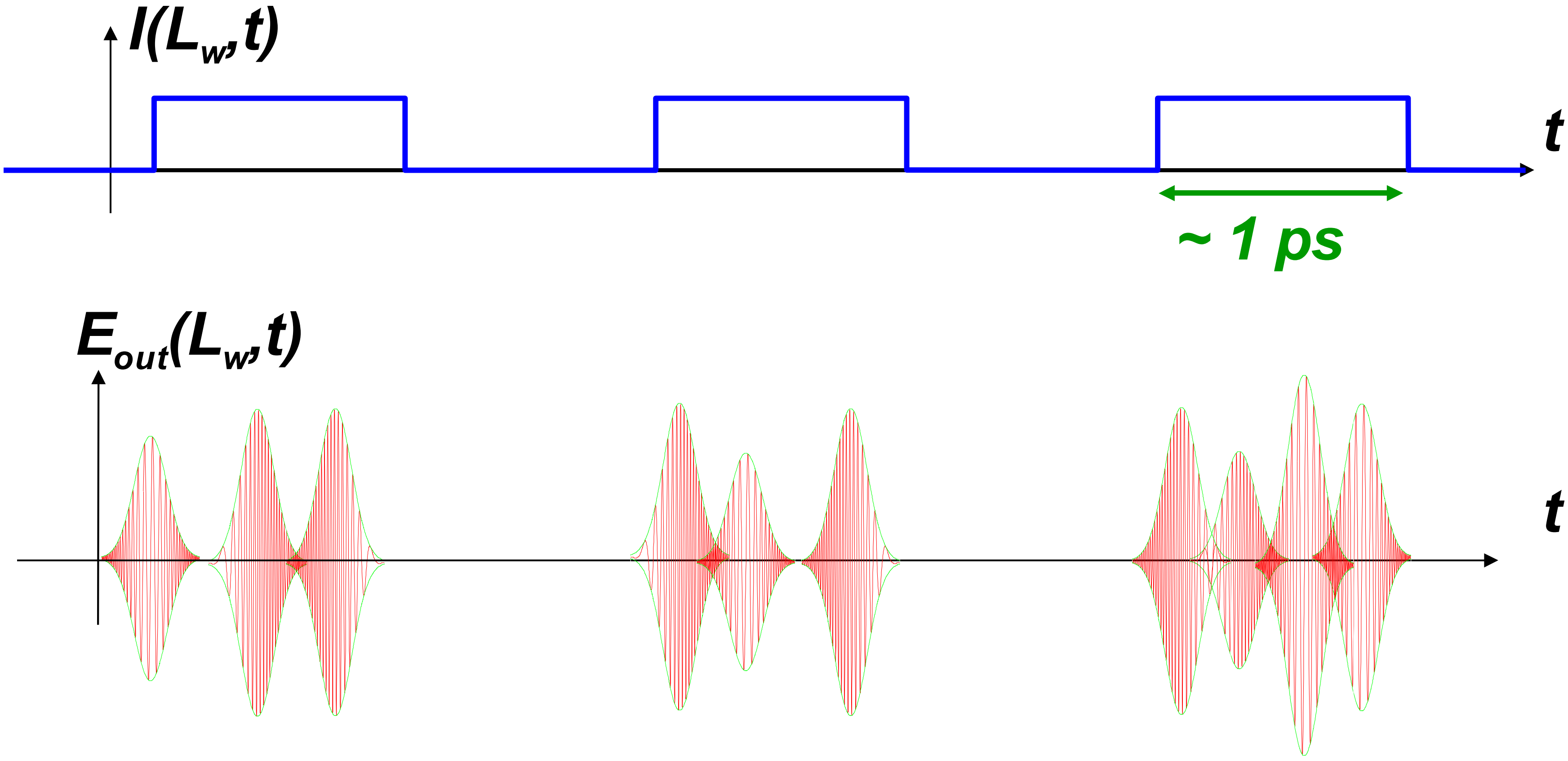
<u>*vs.*</u> Current Shot-noise SASE-FEL

μ-Pre-bunching: $$P(L,\omega) = \left|\tilde{H}^{I}(\omega)\right|^2 \cdot \left|\tilde{I}_{in}(\omega)\right|^2$$

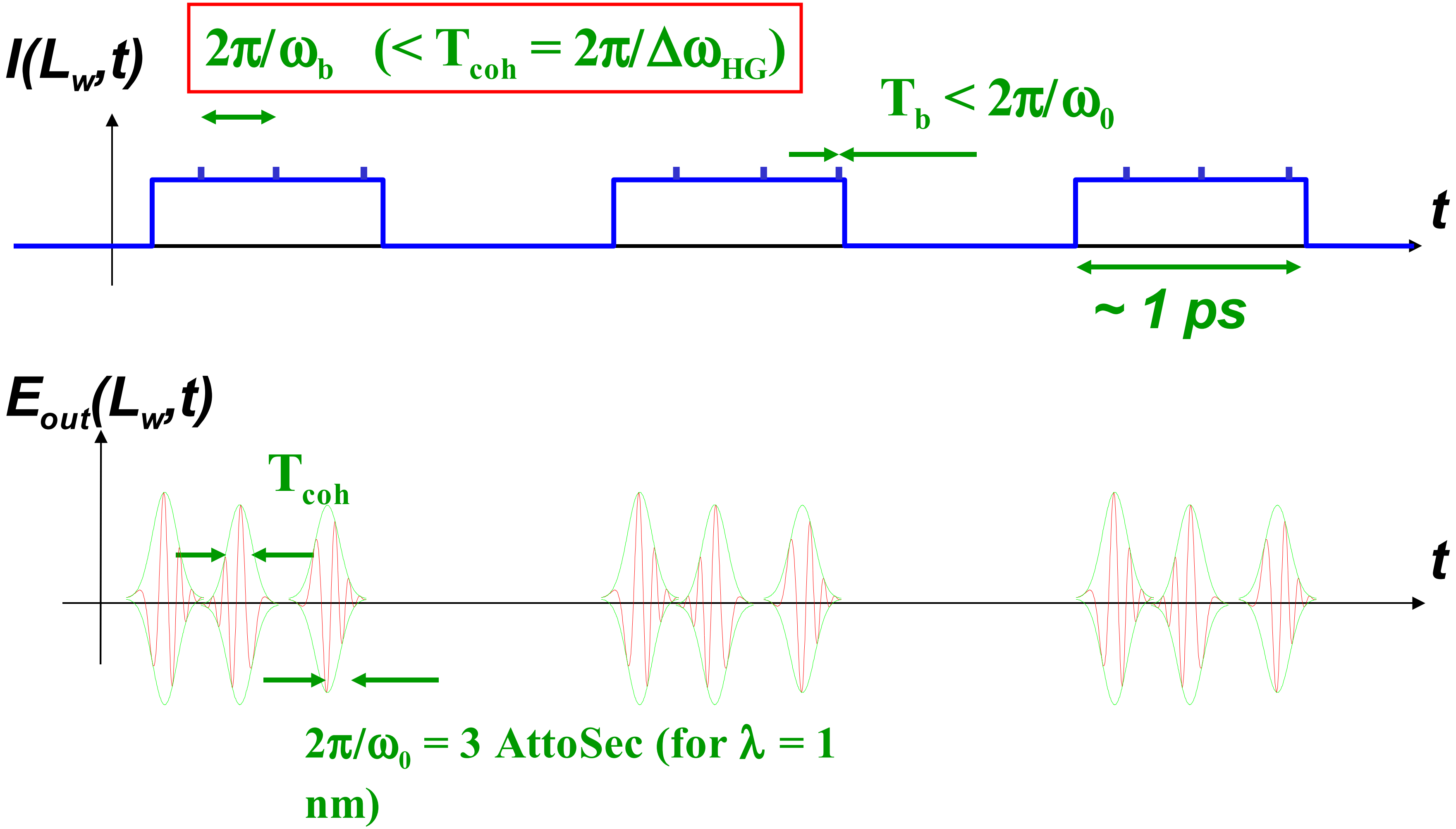
Shot noise: $$\frac{dP^{I}(L,\omega)}{d\omega} = \frac{2}{\pi}\frac{1}{9}e^{\sqrt{3}\Gamma L} \cdot P_b \frac{e}{I_b(\Gamma L)^2}$$

$$P_b > \frac{dP^{I}}{d\omega}\Delta\omega_{HG} \quad \Rightarrow \quad \left|\tilde{I}'_{in}(\omega_0)\right| > \sqrt{\frac{3^{3/4} e\lambda_w}{2\pi}\sqrt{\frac{\Gamma}{L}}\omega_0 I_b}$$

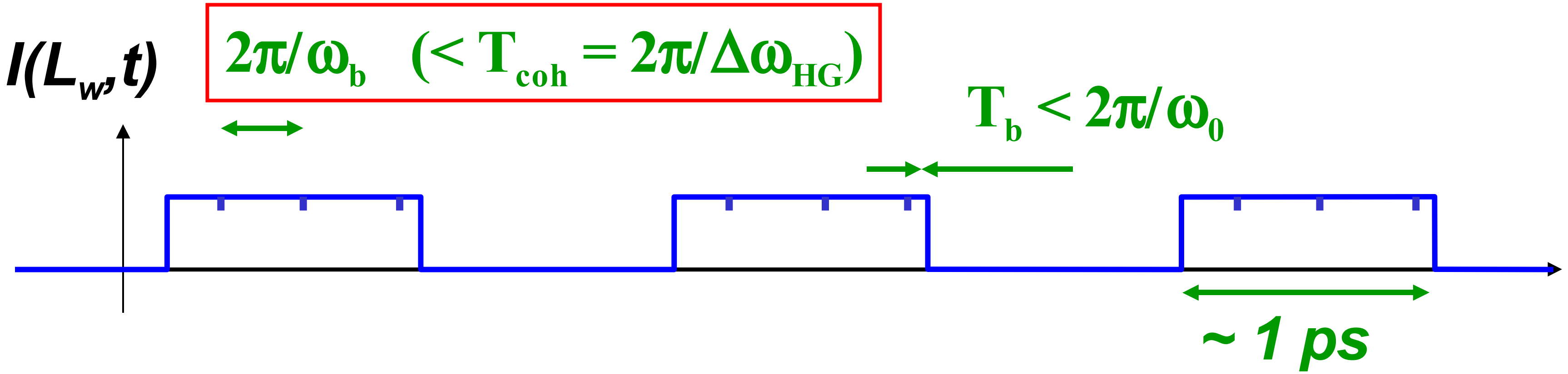
SASE FEL – Time Domain

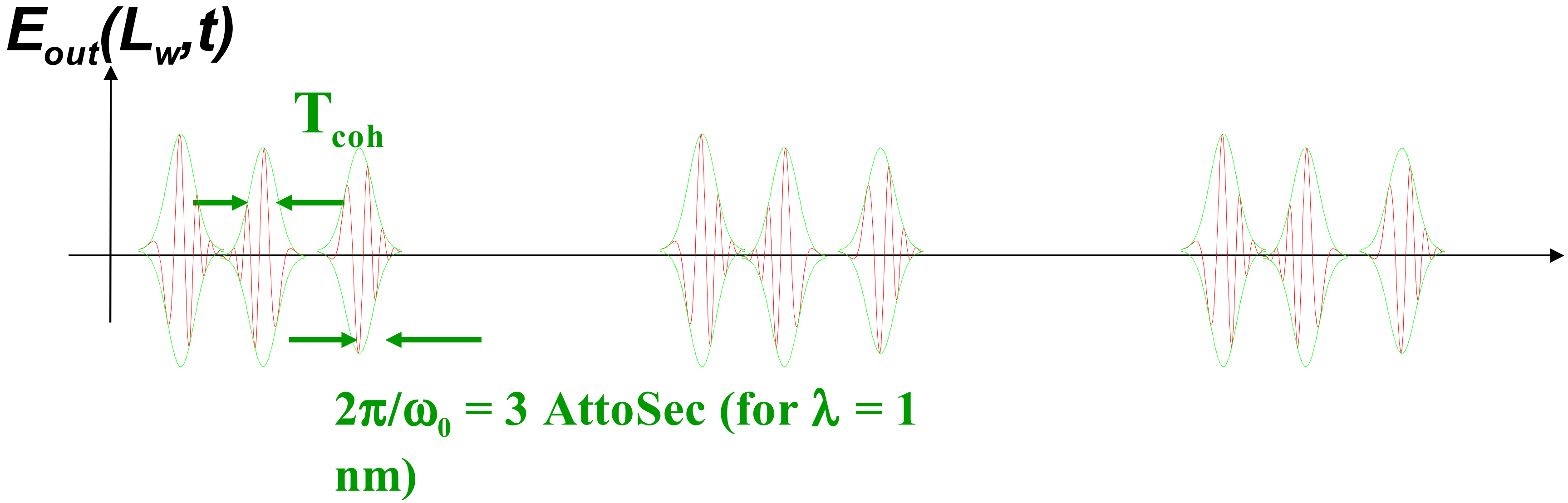


Sub-Harmonic μ-Pre-Bunching

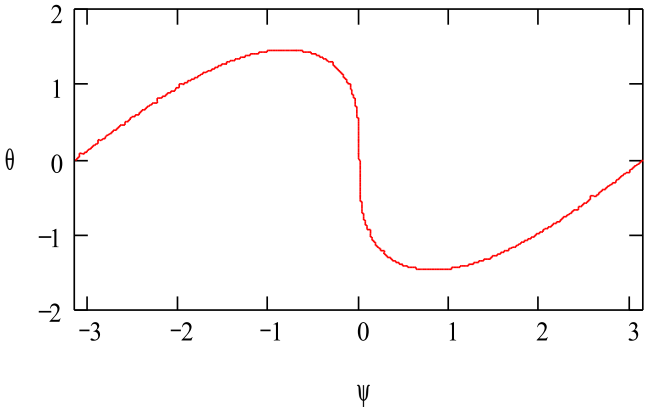


Sub-Harmonic μ-Pre-Bunching

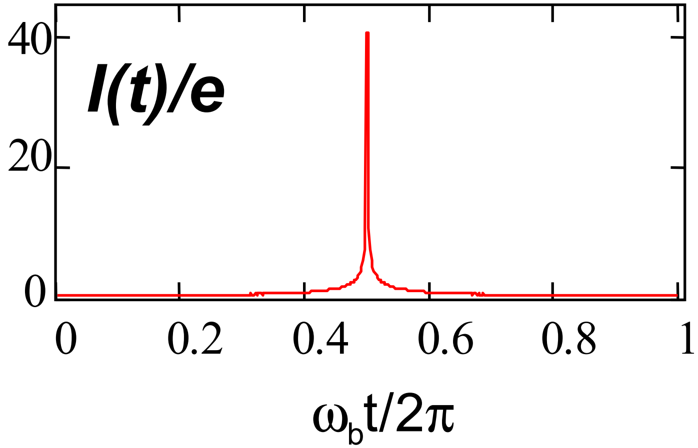




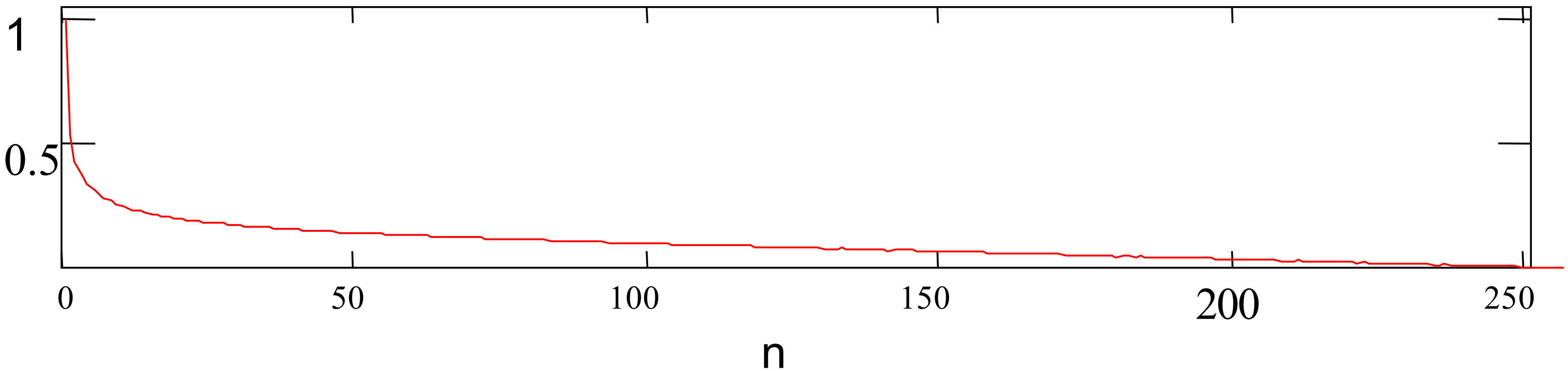
Subharmonic prebunching



Maximum bunching at L = $\lambda_{sinch}/4$
(detuning–phase space)



Bunch current in one prebunch period



Current harmonic amplitude

Conclusion

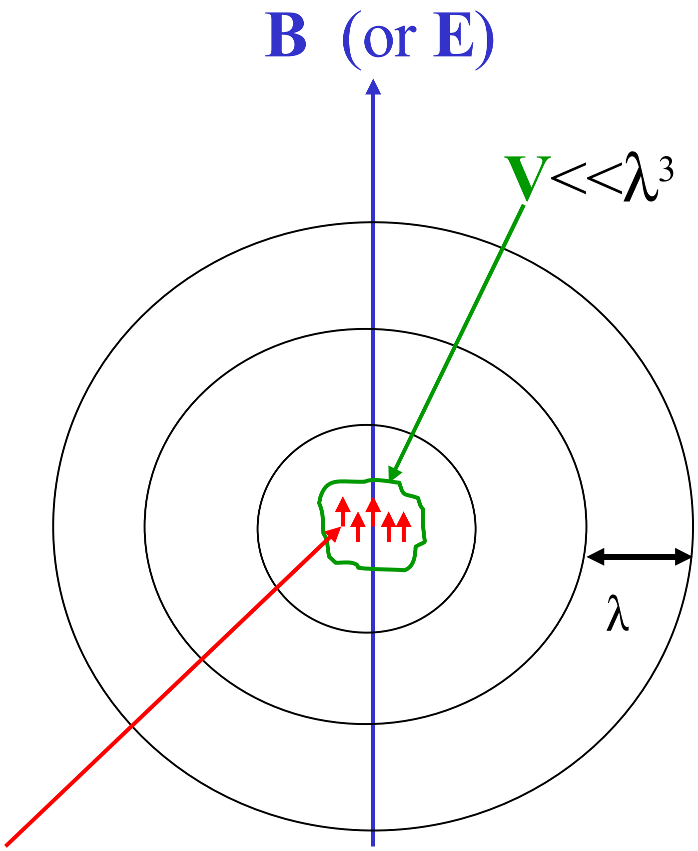
- A General modal excitation model for radiation from particulate charges was applied to analyze the coherence properties of e-beam radiation sources.

- The formulation is applied to characterize superradiant sources, FEL amplifier, oscillator and SASE devices.

- conditions for turning X-UV SASE FELs into coherent radiation sources were derived. It is likely that only partial coherence will be attainable, and statistical averaging over bunches will be needed in spectroscopic applications.

THANKS TO EGOR DYUNIN FOR HELPING TO PREPARE THE PRESENTATION

Dicke's Superradiance



N Electric or magnetic dipoles prepared by means of a "π/2 pulse"

$$\mathbf{I = (r+m)(r-m+1)I_0}$$

r = 1/2, 1, 3/2.....N/2

m = -r.....0.....r

Spontaneous emission (quantum)

N=1, r = m = 1/2 => $I = I_0$

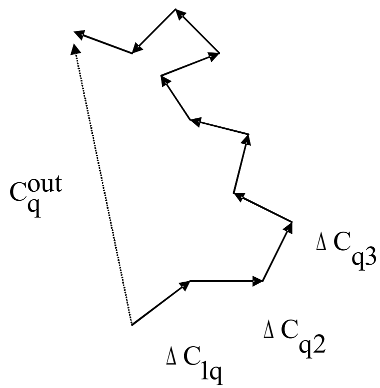
Superradiant emission (classical)

r = N/2 >>1, m = 0 => $I = \frac{1}{4}N^2 I_0$

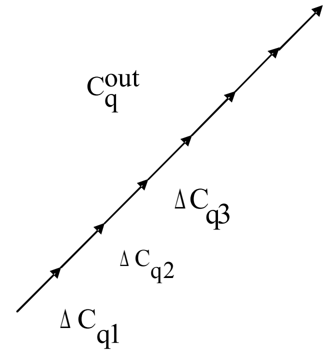
R. H. Dicke, PR, **93**, 99 (1954); A. Gover, "Spin-Flip" PR-ST 9, 060703 (2006)

Superposition of Radiation Wavepackets in the Complex C_q Plane

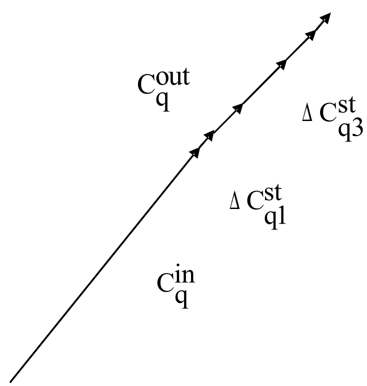
"Spontaneous Emission"



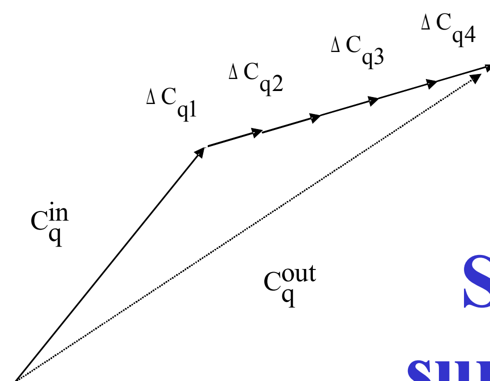
Superradiance



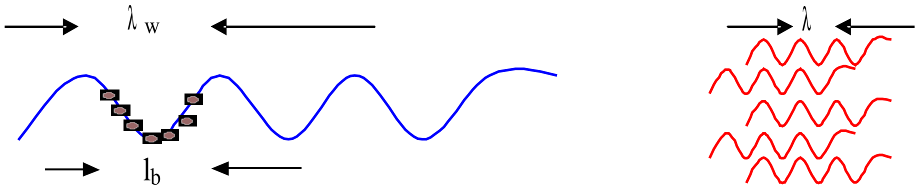
Stimulated emission



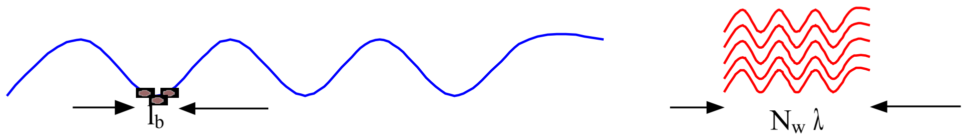
Stimulated superradiance



“Spontaneous Emission”



P α N



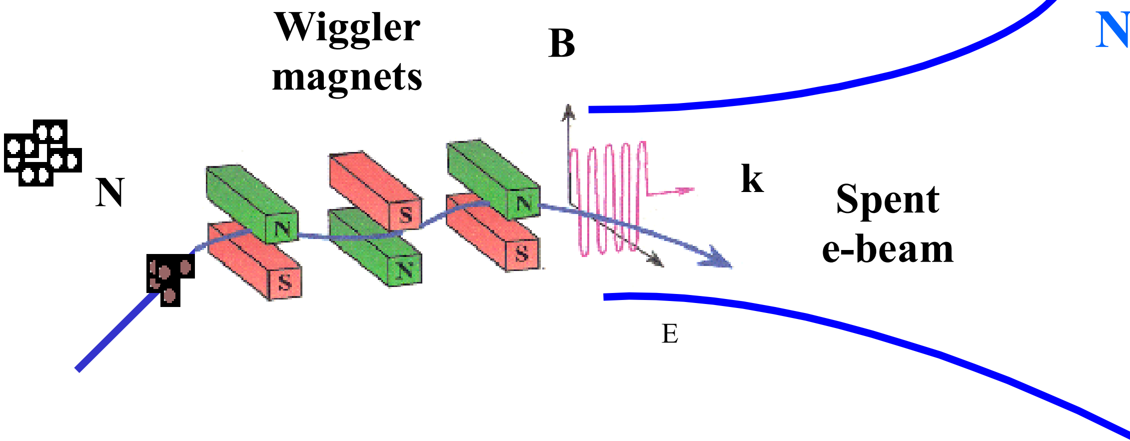
P α N²

Superradiant Emission*

*R. H. Dicke, PR, **93**, 99 (1954)

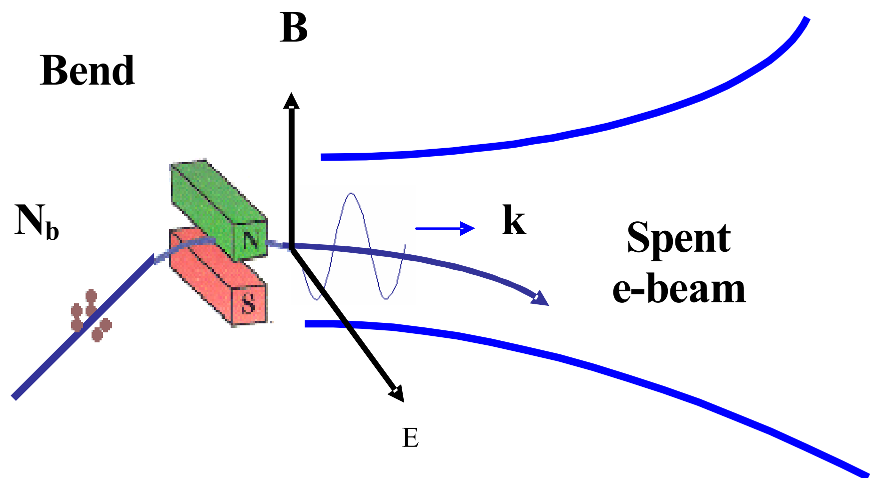
PB-FEL

I. Schnitzer, A. Gover,
“The Prebunched FEL…”,
NIMPR A237(, 124 (1985

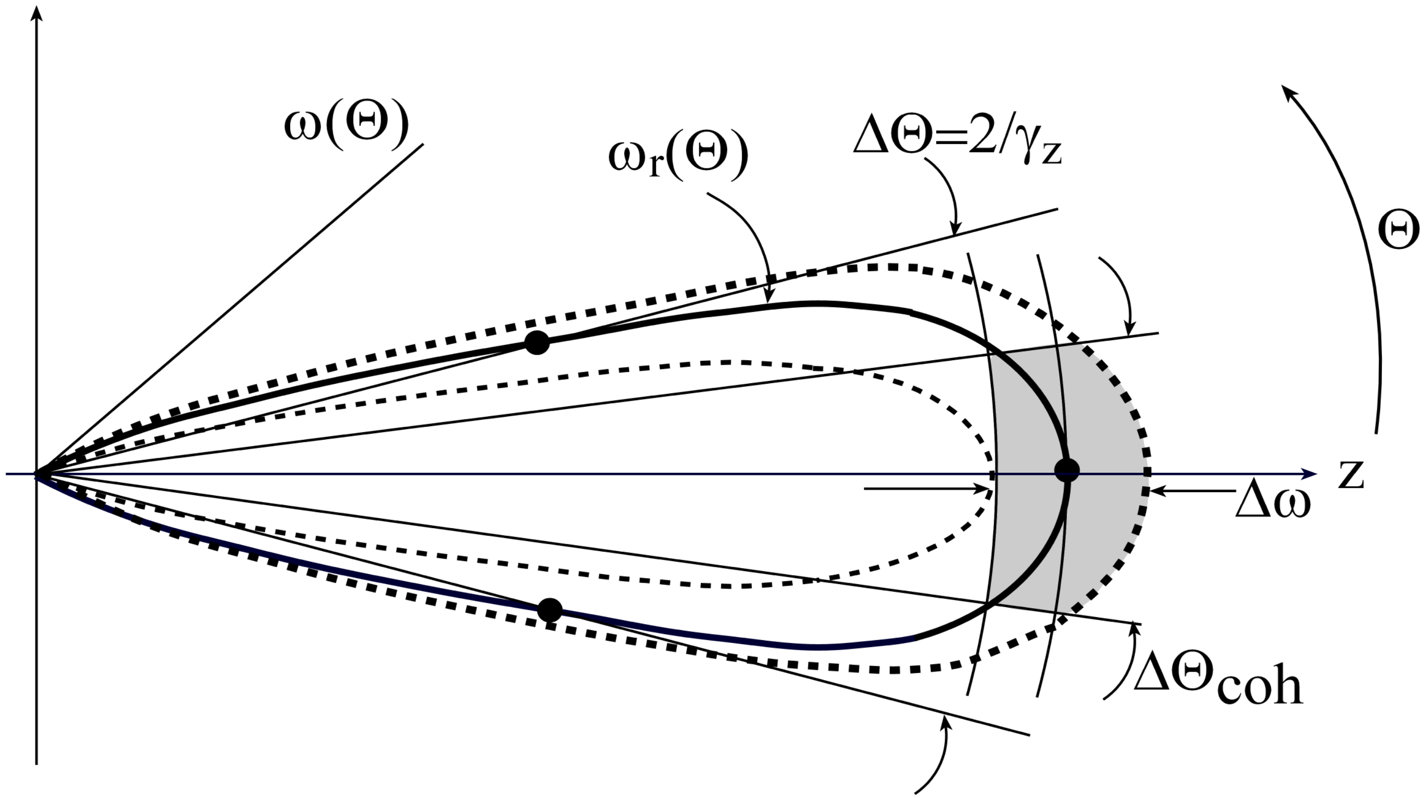


CSR

G.L. Carr et al, “High power
THz radiation…”,
Nature 420, 153 (2002)



Radiation Patern in Free Space (Plane Waves)

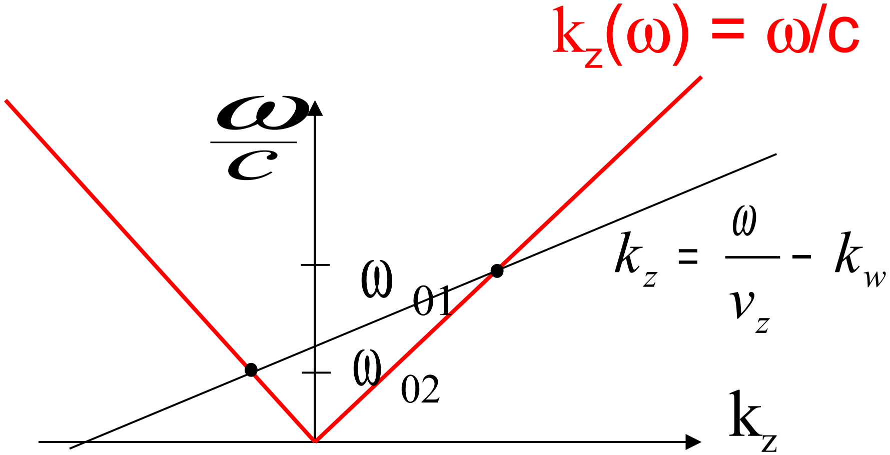


$$q = \left(\mathbf{k}_{\perp}, \sigma\right) = \left(k \sin\Theta\ \cos\Psi\ ,\ k \sin\theta\ \sin\Psi\ ,\ \sigma\right)$$

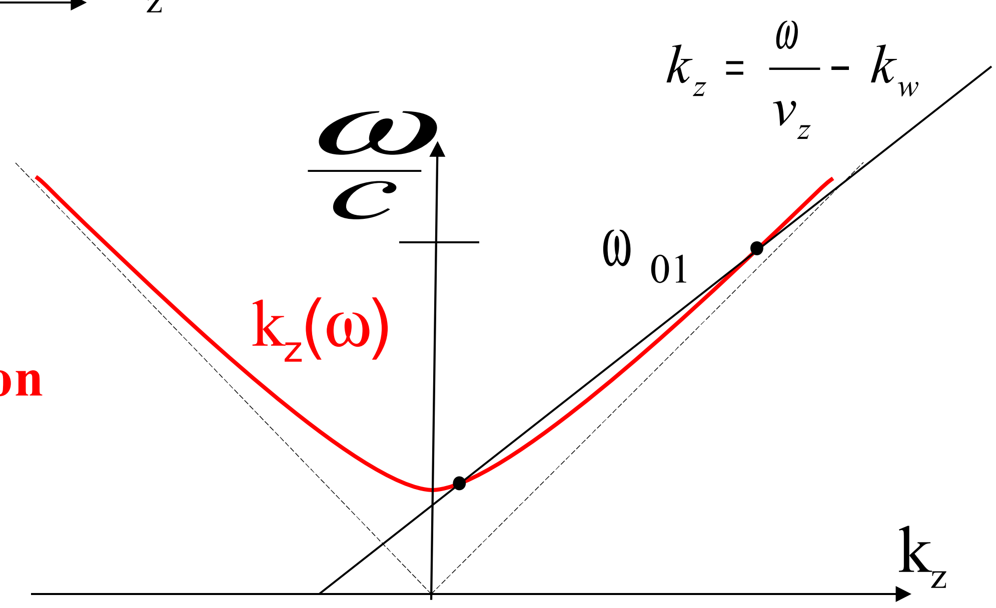
The Synchronizm Condition Solutions

$$\frac{\omega}{v_z} - k_z(\omega) - k_w = 0$$

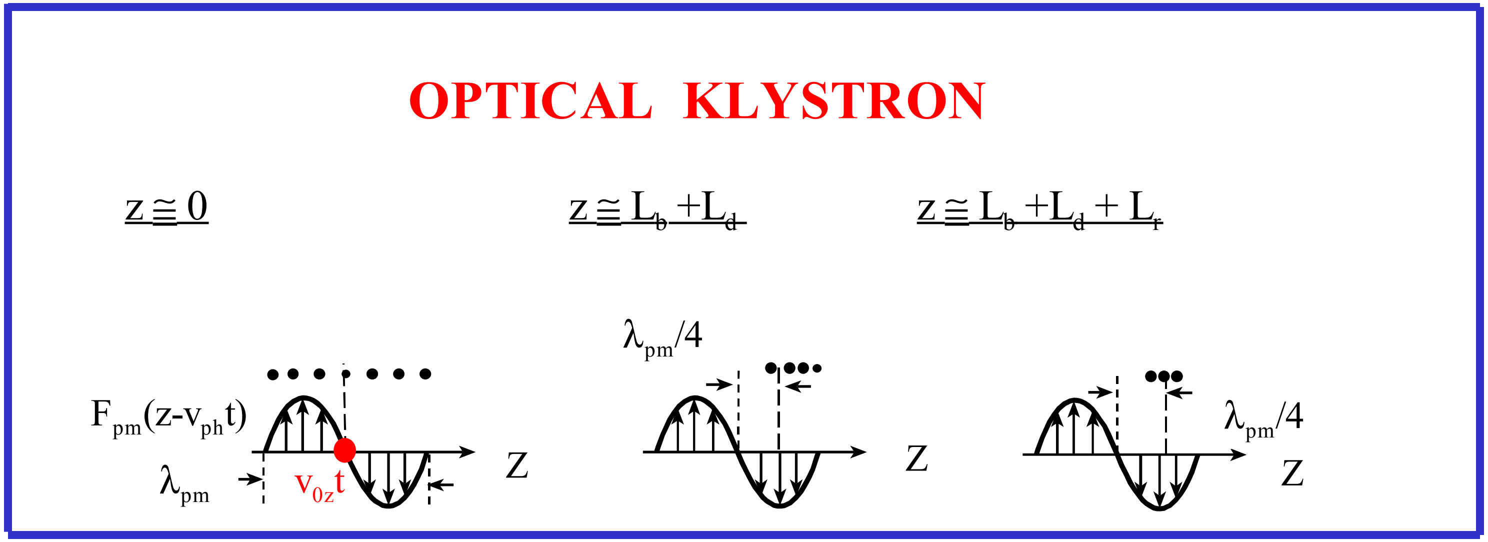
Free space dispersion

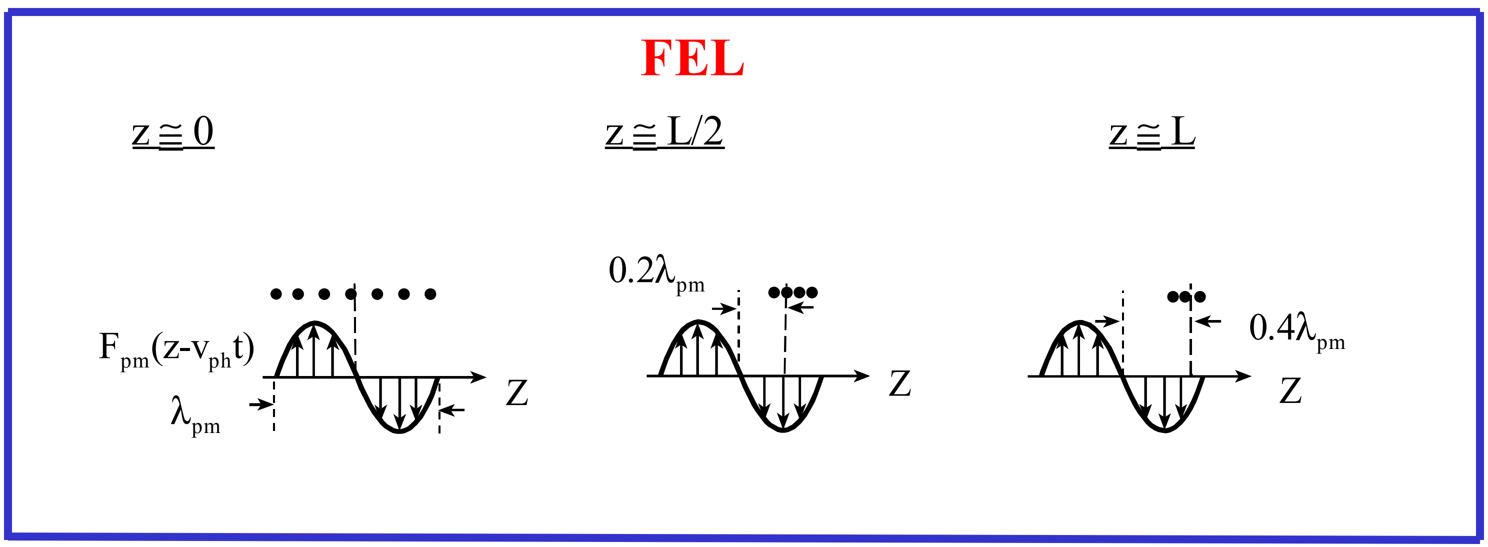


Waveguide dispersion



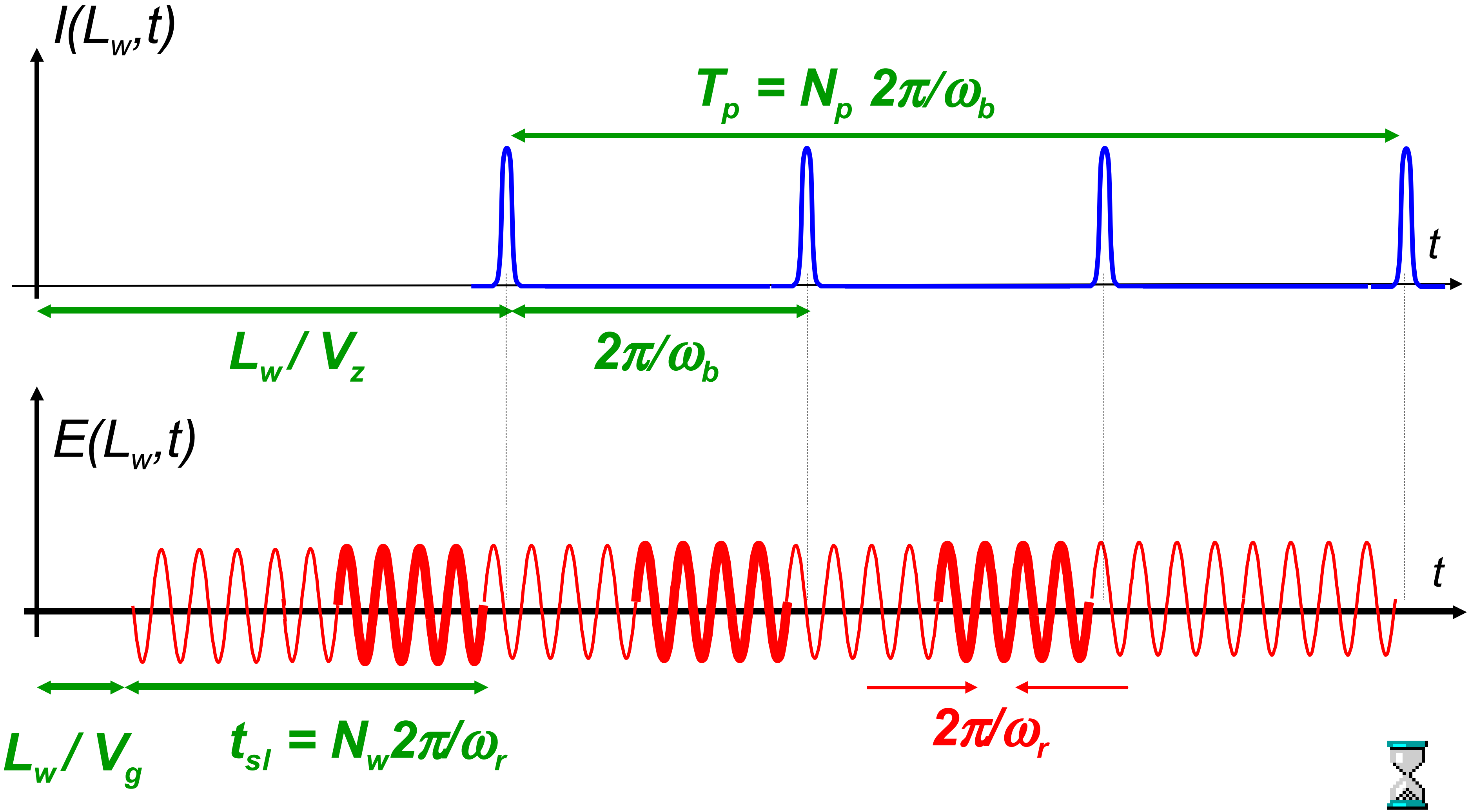
Stimulated Emission





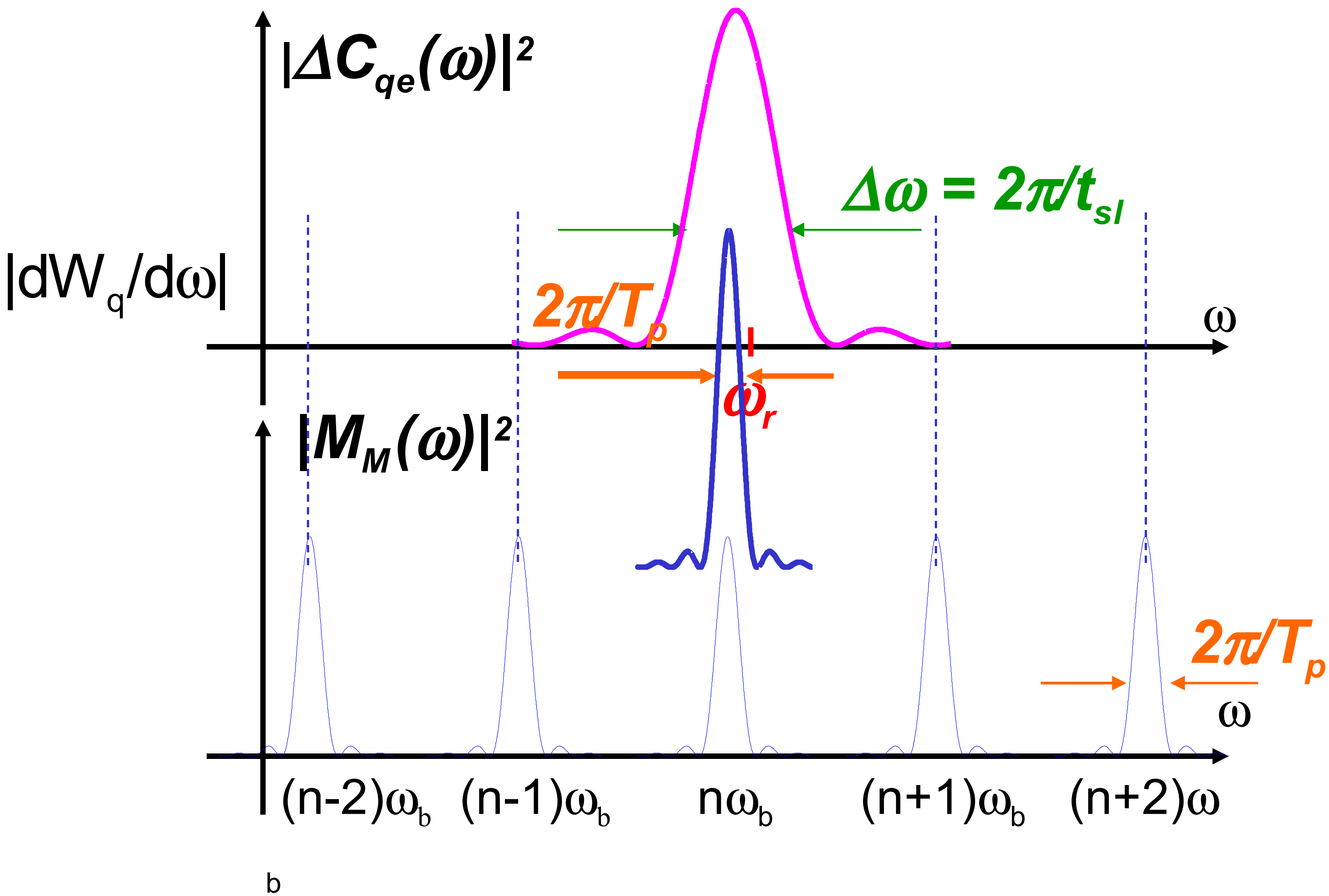
Superradiant Emission from a Pulse Composed of a Train of Bunches ($t_{sl} > 2\pi/\omega_b$)

Time Domain



Superradiant Emission from a Pulse Composed of a Train of Bunches ($t_{sl} > 2\pi/\omega_b$)

Frequency Domain



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- A. Gover, "Superradiant Spin-Flip Radiative Emission of a Spin-Polarized Free Electron Beam", Phys. Rev. Lett. **96**, 124801 (2006).