COHERENT MODE ANALYSIS OF HIGH INTENSITY BEAMS IN SYNCHROTRONS

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Abstract

According to F. Sacherer's analytical study long time ago, a resonance crossing of incoherent tune which is depressed by space charge force does not cause emittance growth nor impose intensity limit in a synchrotron. By employing a multi-particle tracking simulation, we have looked at coherent modes of a whole beam; not only envelope modes (that is a quadrupole mode, which Sacherer studied in detail) but also other modes such as sextupole and octupole one. That mode analysis shows that a crossing of depressed coherent modes at the resonance of the same order, which is excited by error fields, is a source of emittance growth and beam loss. Beam halos induced by coherent mode oscillations of the core are also discussed.

1 INTRODUCTION

Space charge effects in a ring are usually measured by an incoherent tune shift, namely Laslett tune shift, including effects of image charge. In order to avoid crossing of lower order resonances by individual particle, it is believed that the tune shift should be less than -0.25, or -0.5 at most.

Synchrotrons in operation, however, show that the crossing of half-integer resonances is not a problem[1]. Simulation study confirms that rms emittance growth or beam loss does not occur when the largest tune shift estimated with the Laslett formula hits a half-integer resonance[2]. Instead, a beam becomes unstable if the beam intensity is further increased and the largest tune shift becomes almost as twice as much of a distance between bare tune and the resonance. L. Smith[3] and F. Sacherer[4], in fact, had studied in 1960s using envelope equations and found that the resonance affects a beam when the incoherent tune is further down by a factor of 8/5 than the distance in the tune diagram. They studied the K-V beam to start with, but later found the same equations are valid for the rms quantity of a beam[5].

Employing a multi-particle simulation, we extend the study to the following areas. We first look at the coherent quadrupole mode and its resonance for more general particle distributions: K-V, waterbag, parabolic, and gaussian. Although Sacherer had already showed that the envelope equations are valid also for the rms beam size, that does not necessarily mean the resonance occurs in the same way for any distributions. In order to compare different kinds of distributions, we adopt the concept of *equivalent beams*, which means that the rms emittance is the same no matter what a distribution is.

Secondly, coherent oscillations of higher order modes: sextupole and octupole, are measured and their resonances with external error fields are studied. Recently an analytical derivation of higher order mode frequency is discussed by I. Hofmann[6]. We would like to understand a coupling of those modes to external error fields.

Finally, we propose a possible mechanism of halo formation in a synchrotron, where the particle-core mode, which is familiar in linac beam dynamics, is adopted. As a source of time dependent core oscillations, the coherent modes, which is the main subject of this paper, is employed.

2 COHERENT MODE ANALYSIS

Let us define the coherent mode frequency (or tune). In a multi-particle simulation, a moment of any order is defined turn by turn in the following way[7].

$$M_{coh}^{klm}(n) = \frac{1}{N}\sum_{j}^{N} x_j^k(n) y_j^l(n) z_j^m(n)$$

After tracking as many turns as initially specified, the moment values are Fourier transformed as

$$M_{coh}^{klm}(\nu) = \frac{1}{n_{\max}}\sum_{n}^{n_{\max}} M_{coh}^{klm}(n)\exp(-2\pi i \nu n)$$

That gives a frequency spectrum of the moments. We define coherent mode tune as the characteristic frequency of the moment.

2.1 Model Lattice and Incoherent Tune Shift

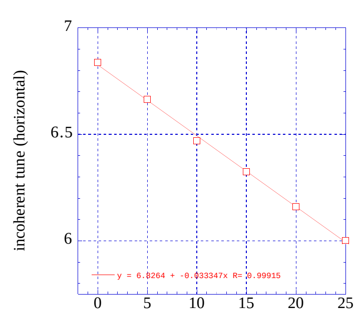

Figure 1: Incoherent tune shift vs. beam intensity.

As a model lattice, JHF booster ring is assumed[8]. The booster ring has four-fold symmetry and its nominal bare tune is (6.84,5.81). With normalized rms emittance of 25 π mm-mrad at the kinetic energy of 200 MeV, intensity dependence of incoherent tune is plotted in Fig. 1. Since the K-V beam is taken, all the particles have the same tune.

2.2 Quadrupole Mode

Figure 2 shows a spectrum of the quadrupole mode calculated with the moment of x^2. The distribution is K-V. When the intensity is zero (left figure), there is only one peak that corresponds to twice of the bare tune of 6.84. Once the space charge force is increased, the mode frequency is decreased (right figure).

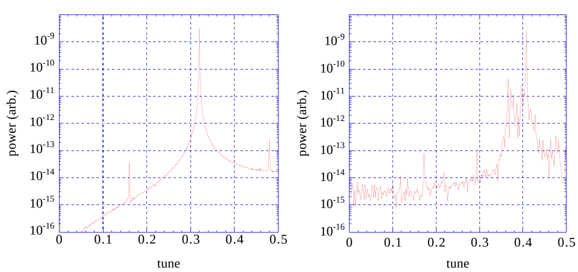

Figure 2: Spectrum of coherent quadrupole mode with zero intensity (left) and at 2A (right). The peak in the left corresponds to the tune of 13.68 (twice of the bare tune). A move towards the right means decreasing of tune.

By picking up the position of the peak, intensity dependence of the coherent quadrupole tune is shown in Fig. 3. The slope is about 120% of that of Fig. 1, meaning that the coherent tune moves less than twice of the incoherent one. The quadrupole mode tune shift does not depends on the initial distribution as long as the beam is *equivalent*.

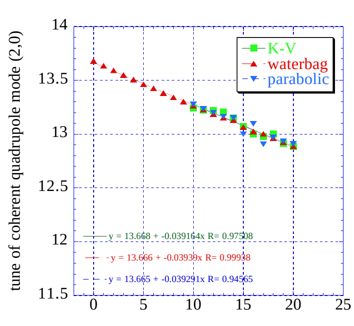

Figure 3: Coherent quadrupole tune vs. intensity. The tune shift is independent of the initial distribution for equivalent beams.

Once the coherent tune hits an integer at about 17A, rms emittance growth is observed as in Fig. 4. Figure 4 also shows that the intensity at which the emittance growth occurs is the same for any particle distributions, except a gaussian beam, which has little resonance behavior.

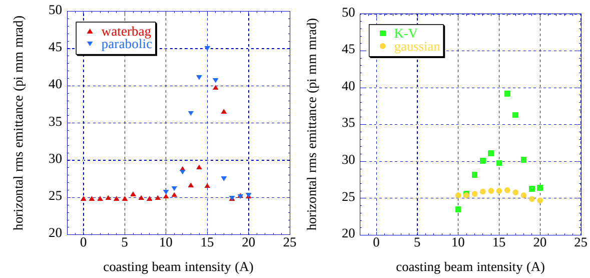

Figure 4: RMS emittance vs intensity. The rms emittance growth occurs when the coherent quadrupole tune hits an integer.

2.3 Sextupole and Octupole Modes

Now, let us look at higher order coherent modes and the resonance of them with external error fields. We introduce sextupole errors which are supposed to excite a third-integer resonance at 20/3.

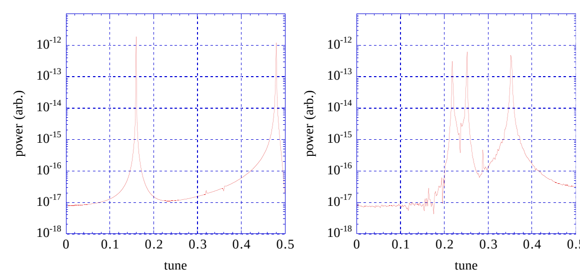

Figure 5: Spectrum of coherent sextupole mode with zero intensity (left) and at 2A (right). In the left figure, a peak on the left corresponds to the tune of 6.84 (same as the bare tune). A peak on the right does 20.52 (three times of the bare tune). The left peak splits into two when the intensity is increased.

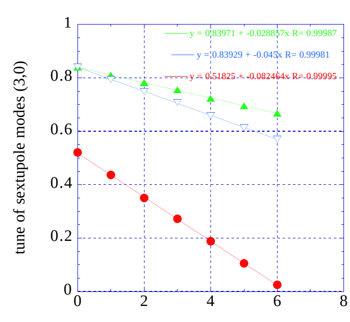

Figure 6: Slope of one of split modes is about one third of the other isolated mode.

A spectrum of the coherent sextupole mode, which is calculated with the moment of x^3, is shown in Fig. 5. At zero intensity, there are two peaks corresponding to the bare tune and three times of it. When the intensity is increased, all the tunes are lowered. At the same time, one peak splits into two. The tune shift of three modes near zero intensity is shown in Fig. 6.

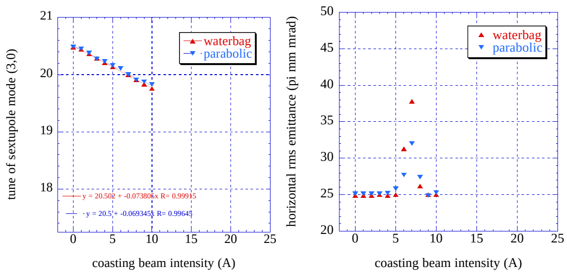

Figure 7: Coherent sextupole tune vs intensity (left) and rms emittance vs intensity (right). One peak of the tune corresponds to almost three times of the bare tune is depicted. The rms emittance growth occurs when the coherent sextupole tune hits an integer.

As the coherent quadrupole mode, intensity dependence of the sextupole tune is smaller if one compares it with the three times of the incoherent one. It is about 72% as shown in Fig. 7 (left). When the coherent tune is down to an integer, rms emittance growth occurs (Fig. 7 (right)). The slope of the tune shift is the same for any initial distributions, at least for waterbag and parabolic, and so the intensity where growth occurs.

The coherent octupole mode and its resonance with external error fields behave in a similar way.

2.4 *Quadrupole Mode Excited by COD*

So far, we assume external error fields and discuss the resonance of the coherent modes with them. It seems that nothing happens if there is no external error fields. It is found by simulation and confirmed analytically[9], however, that image charge induced by a closed orbit distortion (C.O.D.) plays a role of error fields. For example, the image charge of a circular beam pipe can be a driving term of quadrupole errors. Figure 8 shows the quadrupole mode and rms emittance growth. Although there is no explicit magnetic error fields in that case, emittance growth occurs when the mode frequency becomes an integer. Needless to say, if there is no error fields nor C.O.D., the emittance growth is not observed.

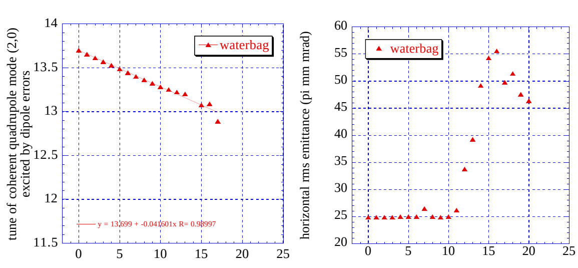

Figure 8: When a C.O.D. exists, a coherent mode resonates with image charge and emittance growth occurs.

3 HALO FORMATION IN A SYNCHROTRON

A formation mechanism of beam halos based on the particle-core model is investigated extensively in a linac for last several years. We have applied the similar model to a synchrotron and study halos in a synchrotron. The particle-core model assumes time dependent core oscillations and its induced parametric resonance of a particle at tail. In a linac, the major source of core oscillations is introduced by an initial mismatch at injection and transitions between two different structures.

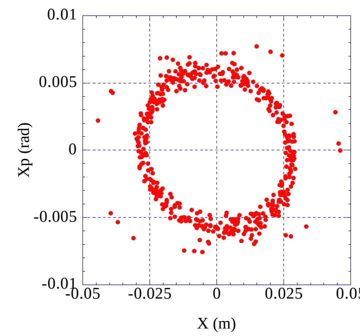

Figure 9: Halo particle found in the multi-particle tracking. Parametric resonance excited by coherent quadrupole mode is the source of halo formation. Resonance structure is clearly seen.

In a synchrotron, the initial mismatch may or may not the source of core oscillations. We have investigated halos induced by coherent mode oscillations. First, beam intensity is determined in such a way that the coherent quadrupole tune becomes 13. Then, a particle near the edge of the core whose tune is about 6.5 satisfies a parametric resonance and makes a large excursion in the phase space as in Fig. 9. That particle becomes a beam halo.

We would like to acknowledge R. Baartman for valuable suggestions and discussions on the coherent mode analysis.

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