

# SIMULATION OF THE LONGITUDINAL MICROWAVE INSTABILITIES IN THE BEPC STORAGE RING

Wang Guangwei Li Yun

Accelerator Lab, Dept. of Engineering Physics, Tsinghua Univ., Beijing 100084, China

Guo Zhiyuan

Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100039, China

(supported by National Natural Science Fund and Demonstration Subject of BEPC National Lab)

## ABSTRACT

The bunch lengthening behavior in The BEPC is investigated by using numerical simulation approach. The simulation results agree well with mesurment. So our simulation technique and impedance model used in this paper are reliable.

## 1 INTRODUCTION

A lot of data was accumulated in the measurement of BEPC bunch length<sup>[7]</sup>. The objection of our study is to understand the effect of single component to bunch lengthening. Because it is difficult to get quantitative result from theory of microwave instabilities, numerical simulation is a useful method, and it can give some inspiration to the theory.

The parameter of impedance is from the computer code URMELT and measurement result<sup>[6]</sup>. Most part of impedance come from cavity, bellows and kickers.

BEPC is undergoing a project to enhance the luminosity,  $Mini-\beta$ , in which bunch length is one of the essential require. Simulation is also a helpful tool to this project.

## 2 SIMULATION TECHNIQUE

We use a now standard tracking method for simulating the wakefield on the longitudinal phase space of the beam<sup>[5]</sup>. In a typical simulation program, particles are represented by many "macro-particles". Eq.(1),(2) are the equations of simulation, where the energy error is  $\varepsilon_i^n$  and the arrival time is  $\tau_i^n$ , i labels the macro-particles number and n labels the turn number.

$$\varepsilon_i^{n+1} = \varepsilon_i^n + eV \sin(\phi + \omega t_i^n) - eU_0 - \frac{2T_0}{\tau_e} \varepsilon_i^n + 2\sigma_e \left( \frac{T_0}{\tau_e} \right)^{1/2} R_i - eW_i^{n+1} \quad (1)$$

$$t_i^{n+1} = t_i^n + \frac{\alpha T_0}{E_0} \varepsilon_i^{n+1} \quad (2)$$

$eW_i^{n+1}$  is the wake field that acts on the macro-particles.  $eV \times \sin(\phi + \omega t_i^n)$  is the action of RF cavity. The

average effect of radiation is  $-\frac{eU_0 - \frac{2T_0}{\tau_e} \varepsilon_i}{\tau_e}$ . The quantum excitation effect can be gotten from random number  $R_i$

(its average is 0, rms. is 1):  $2\sigma_e \sqrt{\frac{T_0}{\tau_e}} R_i$

The most interesting and difficult part of the simulation is in the term  $eW_i^{n+1}$ . The bunch was separated into 50 bins every  $\sigma_z$ . By Fourier Transformation, we get the spectrum of the bunch. Then the wake field is the revers Fourier transformation of the product of impedance and the bunch spectrum. In simulation, we use longitudinal broadband impedance model, and get impedance from measurement. We choose 8 main impedance modes to superimpose.

Table 1 parameters of 8 modes

N	f(GHz)	R/Q	Q
1	0.165	197.9	5.10
2	1.770	2826.0	9.00
3	1.580	1816.0	1.25
4	2.050	370.0	9.50
5	3.440	1058.0	2.80
6	4.700	330.0	37.00
7	5.250	290.0	40.00
8	6.750	590.0	4.50

$$Z_{//}(\omega) \approx \sum_{i=1}^8 Z_i(\omega) \quad (3)$$

$$Z_i(\omega) = \frac{R_i}{1 + j \left( \frac{\omega_i}{\omega} - \frac{\omega}{\omega_i} \right) Q} \quad (4)$$

Particle number in a bunch is in order of  $10^{11}$ . It takes several damping times to get stable distribution. The longitudinal damping time of 2.015 Gev is 11.3ms, Revolution time of particle is  $0.802 \mu s$ . If we use five damping times, it corresponds to 70000 turns. Limited by speed and volume of computer, particle number and

turn number cannot be too large. On the other hand, accuracy is reversely proportional to the square of N, that is:  $\propto \frac{1}{\sqrt{N}}$ . So we should use particle number as large

as possible. The practical number we use is list below.

We take 100000 macroparticles. The charge of each macroparticle is decided by the bunch charge. Random error is 0.3. We take the turn number equal to the turns in 5 damping time: 70266. In addition, original distribution in phase space does not influence other parameters, so we can choose original distribution casually. For example, we can use  $\delta$  function as the original bunch distribution of simulation, or use the stable distribution of last simulation as the original bunch distribution.

Using macroparticles whose number is much less than the number of real particles, may cause some high frequency noise. It is not serious in our simulation. The reason is that the couple of impedance in high frequency and the bunch spectrum is weak.

### 3 SOME RESULTS AND ANALYSES

Particle motion was simulated for 5 damping times. Every turn, center position of bunch, length of bunch, and energy spread were calculated. At the end of simulation, we get stable bunch length and energy spread by averaging the last many turns.

The curve of stable phase visa beam current I is in Figure 1. The relationship of bunch length and energy spread visas beam current I are in Figure 2 and Figure 3 respectively. The threshold current of microwave instability is around 10mA. The solid line in Fig2 is the measurement results with a scaling law on BEPC. We can draw the conclusion that the simulation results agree with the measurement results very well.

When there is only RF field acting on electrons, longitudinal distribution of bunch is approximately Gaussian. When wake field is added, the potential well is distorted, and the longitudinal distribution will deviate Gaussian distribution. Neglecting radiation damping and otherfactor, the longitudinal distribution of bunch can be gotten<sup>[6]</sup>:

$$I_{beam}(\tau) = I_{beam}(0) \exp\left(\frac{-\omega_s^2 \tau^2}{2\alpha^2 \sigma_\epsilon^2} + \frac{e}{\alpha \sigma_\epsilon^2 E_0 T_0} \times \int_0^\tau d\tau' \int_{\tau'}^\infty I_{beam}(\tau') W(\tau' - \tau') d\tau'\right) \quad (5)$$

If wake field is known, this integral equation can be solved by using eq.(5). We use the same wake field as in the simulation to get the longitudinal distribution of bunch according to eq.(5). As beam current increase, the properties of longitudinal distribution of bunch are as follows: First, the distribution deviates from the Gaussian distribution much more. This is because the distortion of potential well increases with beam current. Second, the stable phase of bunch is decreased, it is

because energy lost caused by real part of impedance must be compensated through decreasing the phase.

The bunch lengthening is caused by two factors: potential well distortion and microwave instability. These two factors are not interrelated, so bunch length  $\sigma_l$  can be written as :

$$\sigma_z^2 = \sigma_{z0}^2 + \sigma_{zp}^2 + \left(\frac{\sigma_{z0}}{\sigma_{\epsilon 0}}\right)^2 (\sigma_\epsilon^2 - \sigma_{\epsilon 0}^2) \quad (6)$$

or written in unitless form:

$$\left(\frac{\sigma_z}{\sigma_{z0}}\right)^2 = \left(\frac{\sigma_p}{\sigma_{z0}}\right)^2 + \left(\frac{\sigma_\epsilon}{\sigma_{\epsilon 0}}\right)^2 - 1 \quad (7)$$

$$\sigma_p^2 = \sigma_{z0}^2 + \sigma_{zp}^2 \quad (8)$$

Where  $\sigma_z$  is total bunch length, it is the results of simulation.  $\sigma_p$  is calculating results of potential distribution.  $\sigma_{zp}$  is the bunch length caused purely by potential distortion.

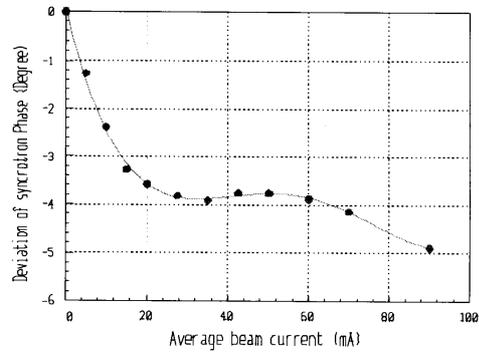


Fig 1. Phase deviation of 2.015GeV

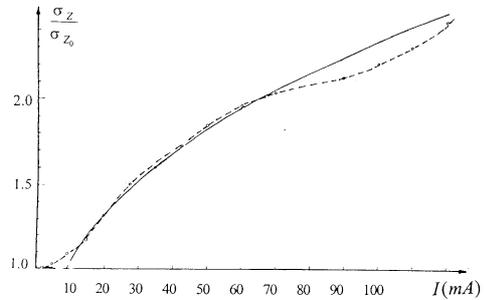


Fig 2. Bunch Lengthening of 2.015GeV

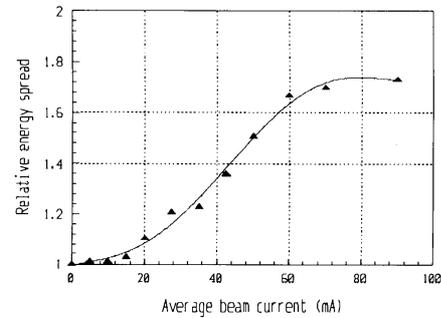


Fig 3 Energy spread increasing of 2.015GeV

When the beam current is less than the threshold, energy spread of bunch remain unchanged, and bunch lengthening related to potential well distortion only. When the beam current is stronger than threshold, the total bunch length is the sum of two factors. The percentage caused by potential well distortion and caused by energy spread is listed in Table 2. We can see from the table that the percentage of calculation agree well with eq.(7). As current increasing, deviation of eq.(7) from calculation increases. The percentage of natural bunch length in total length decreases rapidly, the percentage of bunch length caused by energy spread increases quickly and the percentage of bunch length caused by potential well distortion change slowly.

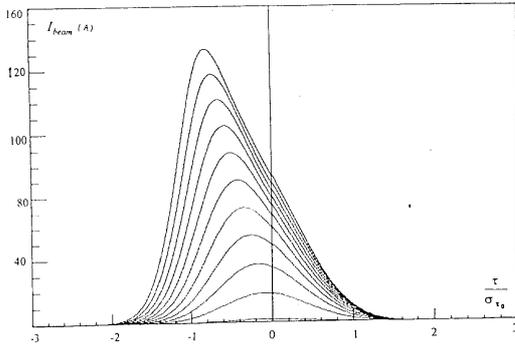


Fig 4. Change of bunch longitudinal distribution due to potential well distortion

Table 2 Comparison of influence of potential well distribution and energy spread on total bunch length

Single beam current (mA)	deviation from eq.(7)	percentage of factors in total bunch lengthening		
		Energy spread $(\frac{\sigma_z^2}{\sigma_{z0}^2} - 1) \times \frac{\sigma_{z0}^2}{\sigma_z^2}$	Potential Well Distortion $\frac{\sigma_{zP}^2}{\sigma_z^2}$	Natural Energy Spread $\frac{\sigma_{z0}^2}{\sigma_z^2}$
45	0.1%	0.6%	7.0%	92.6%
50	1.4%	23.7%	6.6%	71.1%
55	2.0%	29.1%	7.2%	65.75
60	2.6%	29.0%	8.35	64.9%
65	1.9%	28.7%	9.45	64.1%
70	3.0%	31.9%	10.2%	60.8%
80	5.8%	37.1%	11.6%	55.2%
90	5.8%	39.3%	13.4%	52.55
100	5.8%	43.4%	14.6%	48.0%

## 4 SUMMARY

Simulation of BEPC storage ring by macroparticle method gives out preliminary properties of longitudinal instability. The results agree well with measurement, so the simulation is reliable on the whole. BEPC is undergoing some improvement of decreasing impedance. The data we use in the code is the old one of BEPC, we plan to change them to new one to get more information.

## REFERENCES

- [1] 'Physics of Collective Beam Instability in High Energy Accelerator', by Alex Chao, 1993.
- [2] E. Keil, and W.Schnell, CERN-TH-RF/69-48 (1969)
- [3] 'Simulations of the Longitudinal Instability in the Damping Ring', by K.L.F.Bane and K.Oide, SLAC-PUB-6216 (1993)
- [4] 'The Physics of Electron Storage Ring', SLAC-121, UC-28 (1979)
- [5] 'Physics of High Energy Particle Accelerators', SLAC Summer School(1982)
- [6] 'Measuring the Longitudinal Impedance of Components in the BEPC Storage Ring', by G.W.Wang, Doctoral Thesis, IHEP, China (1989)
- [7] 'The Studies of Current Dependent Phenomena at the BEPC', by Z.Y.Guo et al, Proceedings of the Workshop on Beam Instabilities in Storage Riings, China 1994.

## APPENDIX A: BEPC PARAMETERS USED IN PROGRAM

Particle Energy	E=2.015GeV
Peak RF Voltage	$V_{rf}=2.0MV$
Revolutionary Frequency	$f_0=1.2576MHz$
RF Frequency	$f_{rf}=201.2MHz$
Radiation lost per turn of synchrotron particle	$U_0=141.0KeV$
momentum compaction factor	$\alpha=0.04$
damping time	$\tau_e=11.3ms$
stable phase	$\phi_0=176^\circ$
natural length of bunch	$\sigma_{z0}=2.58cm$
natural energy spread	$\sigma_{\epsilon 0}=1.080MeV$
simulation time	$t=5\tau_e$
number of superparticles	N=100000