# REDUCTION OF LANDAU DAMPING IN HERA

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*Abstract*

Athough the nonlinear detuning in HERA-p should be large enough to cure all transversal instabilities at injection and the first part of the ramp strong excitation are frequently observed. Here it is shown, that for typical HERA sextupole distributions at injection the action of incoherent space charge tune shift must lead to a strong reduction of the effectivity of Landau damping. So weak instabilities occur.

## 1 TRANSVERSAL INSTABILITIES IN HERA – OBSERVATIONS

The operation of the HERA proton ring is affected by transversal instabilities which appear between 40 and 150 GeV and cause beam blow up and even beam loss.

There is a clear dependence on single bunch current as bunches with low charge densities (large emittance or low current) are not affected, while bunches with small emittances and high single bunch currents, loose stability and are blown up, showing strong transversal dipole oscillations. The observations related to this loss of stability hardly fit into a coherent picture due to a lack of classical threshold behaviour or long term reproducible energy or current dependence.

At injection energy an initially stable bunch can be destabilized by transversal scraping or longitudinal compressing leading to growth rates of the dipole amplitude in the order of a second. This indicates a very weak instability which therefore could affect the proton beam only in the nearly complete absence of Landau damping.

On the other hand there are no indications for a notable reduction of Landau damping. The decoherence time is in the order of milliseconds which proves a frequency spread large enough to damp any instability at the time scale of a second.

A large broad-band impedance as source of the instability can also be excluded. A broad-band impedance would lead to a notable shift of the coherent tune with beam current. In HERA such a dependence of the coherent tune on beam current could not be detected. The accuracy of tune measurement determines an upper bound of the transversal impedance $W_T$

$$|W_T| < 2\,M\Omega/\mathrm{m}\;. \qquad (1)$$

This value for the transversal impedance is consistent with a longitudinal broad-band impedance of $|W_L| \approx 1\,\Omega$ a value also confirmed by longitudinal stability considerations or by calculating the chamber impedance [1].

From typical decoherence times of some milliseconds (at 250 $\mu$A/bunch) we expect a transversal detuning $\delta Q$ in the range of

$$\delta Q = 5 \times 10^{-4} \cdots 1 \times 10^{-3}\;. \qquad (2)$$

From this we expect a stable proton beam for broad-band impedances below 13 $M\Omega/\mathrm{m}$ [1] – considerably higher then the measured upper limit in (1).

A narrow band impedance as source of the instability mechanism can be excluded, because up to 70 mA proton current can be stored without exciting notable multibunch modes.

To explain the loss of stability we first will show that the decoherence time has to be interpreted with more care.

## 2 DECOHERENCE TIME AND DETUNING

The effectivity of Landau-Damping relies on the amount of nonlinear detuning present.

The essential source of detuning in HERA are the sextupole components of the main dipole field (the persistent current contribution) and the sextupole correction coils necessary to adjust the global chromaticity.

The horizontal tune shift $\delta Q_\mathrm{x}$ is a sum of two parts $\Delta Q_\mathrm{xx}$, $\Delta Q_\mathrm{xz}$ each one depending only on the horizontal or vertical emittance. The same holds for the vertical tune shift $\delta Q_\mathrm{z}$. We define the respective detuning coefficients by

$$\begin{aligned}\delta Q_\mathrm{x} &= \frac{\partial Q_\mathrm{x}}{\partial \epsilon_\mathrm{x}} \cdot \epsilon_\mathrm{x} + \frac{\partial Q_\mathrm{x}}{\partial \epsilon_\mathrm{z}} \cdot \epsilon_\mathrm{z} = \Delta Q_\mathrm{xx} + \Delta Q_\mathrm{xz} \\ \delta Q_\mathrm{z} &= \frac{\partial Q_\mathrm{z}}{\partial \epsilon_\mathrm{x}} \cdot \epsilon_\mathrm{x} + \frac{\partial Q_\mathrm{z}}{\partial \epsilon_\mathrm{z}} \cdot \epsilon_\mathrm{z} = \Delta Q_\mathrm{zx} + \Delta Q_\mathrm{zz}\;.\end{aligned} \qquad (3)$$

The detuning in each plane depends on the betatron amplitude of the other plane. As long as the forces acting on beam are hamiltonian the cross-coefficients $\Delta Q_\mathrm{xz}$ and $\Delta Q_\mathrm{zx}$ coincide.

The coefficients can be calculated accurately from the persistent fields (measured) and the sextupole corrector settings using normal form analysis of the HERA one turn map [2]. It turns out that their values strongly depend on the distribution of the corrector strength around the ring. In practice the sextupole currents are not distributed according to the theoretic optimum, obtained by imposing the condition of local compensation of chromaticity. Instead, due to limitations of the control system, the sextupole circuits are power unbalanced. In Table 1 we give detuning coefficients calculated from different sextupole corrector setting used during the 95 run of HERA. The date of each set refers to the date when the respective file was stored. Often we find s situation where the main a terms ($\partial Q_{\rm x}/\partial \epsilon_{\rm x}$, $\partial Q_{\rm z}/\partial \epsilon_{\rm z}$) are large in comparison to the cross term $\partial Q_{\rm x}/\partial \epsilon_{\rm z}$.

Table 1: The chromaticity and detuning coefficients calculated from the sextupole currents of some injection files during the 95 HERA run.

| File | 5.7 | 24.7 | 11.9 | 29.11 |
|---|---|---|---|---|
| $\Delta Q_{\rm xx} \times 10^6$ | $-397$ | 210 | 79 | $-140$ |
| $\Delta Q_{\rm xz} \times 10^6$ | 435 | 93 | $-19$ | 103 |
| $\Delta Q_{\rm zz} \times 10^6$ | $-485$ | $-615$ | $-417$ | $-468$ |
| $\xi_x$ | 6.5 | $-6.0$ | 1.0 | 1.6 |
| $\xi_z$ | 6.6 | 4.0 | $-0.8$ | $-1.5$ |

# 3 DECOHERENCE TIME OF A ROUND BEAM

The decoherence time is a measure of the detuning strength, but its interpretation is not as unique as commonly supposed.

We assume a stationary distribution of parabolic form in both planes.

We emphasize that we have to consider initial distribution of finite size in phase space – any beam pipe has a finite aperture. Infinite distributions (without cut-off) may cause severe inconsistencies.

To simplify the notation we restrict to a round proton beam with equal width $\sigma$ in both lateral planes, and scale each length in units of $\sigma$. After a horizontal kick at time $t = 0$ the stationary distribution is displaced by a distance $\Delta$ in phase space. One has to distinguish the distribution which results with $\Delta < 1$) after a weak kick and with $\Delta > 1$ after a stronger kick. More important is the last case with its resulting distribution (see Fig. 1).

$$\rho(r,\, s,\, \theta_{\rm x},\, \theta_{\rm z}) =$$
$$\begin{cases} \frac{2}{\pi}(1 - r^2 - \Delta^2 + 2\Delta r\cos\theta_{\rm x}) \cdot \frac{2}{\pi}(1 - s^2) \\ \text{if } \theta_{\rm x} \in [-\theta_M,\, \theta_M] \quad \text{and} \quad r \in [r_{\rm min},\, r_{\rm max}] \\ 0 \text{ otherwise} \end{cases} \tag{4}$$

The horizontal, vertical amplitudes are $r$ and $s$ and the phases in their respective space are $\theta_{\rm x}$ and $\theta_{\rm z}$. We defined the maximal angel $\Theta_M$ by

$$\sin\Theta + M = \frac{1}{\Delta}$$

and the minimal and maximum amplitudes $r_{\rm min}$ and $r_{\rm max}$ for a given phase $\theta_{\rm x}$.

$$r_{\rm min} = \Delta\cos\theta_{\rm x} - 1 + \Delta^2\sin\theta_{\rm x}^2 \tag{5}$$

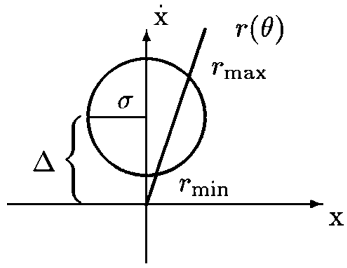


Figure 1: A horizontal kicked beam in phase space.

$$r_{\rm max} = \Delta\cos\theta_{\rm x} + 1 - \Delta^2\sin\theta_{\rm x}^2 \; . \tag{6}$$

The decoherence of the initial dipole amplitude $\Delta$ can be calculated as function of time by performing an integration of the initial distribution $\rho$ together with the transversal posit ions $\Phi(t)$ of each particle [3].

$$\begin{aligned} D(t) \;\; &= \;\; \int_0^\infty dr\, r \int_0^{2\pi} \frac{d\theta_{\rm x}}{2\pi} \int_0^\infty ds\, s \int_0^{2\pi} \frac{d\theta_{\rm z}}{2\pi} \\ & \quad \times \rho(r,\, s,\, \theta_{\rm x},\, \theta_{\rm z})\Phi_{\rm x}(r,\, s,\, \theta_{\rm x},\, \theta_{\rm z},\, t) \; . \end{aligned} \tag{7}$$

According to their amplitudes the particles oscillate with shifted tunes

$$\Phi_{\rm x}(t) = r\cos\{(Q_0 + \Delta Q_{\rm xx} r^2 + \Delta Q_{\rm xz} s^2)t + \theta_{\rm x}\} \tag{8}$$

$Q_0$ denotes the linear tune and $t$ the time in units of revolution times.

With Eq. (4) and Eq. (8) inserted into (7) we get the dipole moment as function of time (see [4]).

The kicked beam performs a rapid oscillation at the undisturbed tune. Its amplitude is modulated by a decoherence envelope, which changes at a time scale proportional to the inverse detuning frequency. To get information about this frequency we therefore have to separate the envelope $E(t)$ from the rapid betatron oscillations. Evaluating the expression (7) it results that $E(t)$ can be decomposed into two factors, one depending only on $\Delta Q_{\rm xx} t$ and the other only on $\Delta Q_{\rm xz} t$ [4]

$$E(t) = f(\Delta Q_{\rm xx} t) \cdot g(\Delta Q_{\rm xz} t) \; . \tag{9}$$

If the cross term $\Delta Q_{\rm xz}$ is small compared to $\Delta Q_{\rm xx}$ the larger detuning coefficient determines the decoherence time because the decoherence envelope then coincides with $f(\Delta Q_{\rm xx} t)$. The cross coefficient $\Delta Q_{\rm xz}$ can become very small or even zero – the decoherence time will not be affected. Now we will show that the space charge tune shift can destabilize the beam if one detuning coefficient becomes small.

# 4 STABILITY AND DETUNING

Generally speaking the dispersion integrals describe the collective response of a beam on an excitation. The real part is a measure of the non resonant response (in phase with excitation), the imaginary part is a measure of the resonant response ($\pi/2$ out of phase with excitation). The circulating beam together with its accompanying wakefields, acting back on the motion of the beam, form a potential unstable system. The inversed (complex) dispersion-integral determines directly the limits of the (complex) impedance which is still tolerable at a given beam current.

We follow the notation of [3]. The imaginary part of the dispersion-integral is denoted $\hat{D}\sin\hat{\Psi}$ and is given in case of a parabolic distribution by

$$\begin{aligned}\hat{D}\sin\hat{\Psi} &= 4\int_0^1 dr\, r(1-r^2)\int_0^1 ds\, s(1-s^2) \\ &\times\delta[\Omega-(Q_0+\Delta Q_{xx}r^2+\Delta Q_{xz}s^2)]\ .\end{aligned} \tag{10}$$

The free parameter $\Omega$ corresponds to the frequency of the excitation. Only when particle frequencies coincide with $\Omega$ resonant excitation is possible, and we get a contribution to the imaginary dispersion. Mathematically this is expressed by the delta function. We introduce the scaled frequency $f$, the fractional detuning $\delta$ and variables $x$ and $y$ by

$$f=\frac{\Omega-Q_0}{\Delta Q_{\mathrm{xz}}}\quad \delta=\frac{\Delta Q_{\mathrm{xx}}}{\Delta Q_{\mathrm{xz}}}\quad x=r^2\quad y=s^2\ .$$

Performing the $x$-integration we get

$$\hat{D}\sin\hat{\Psi}=\int_A^B dy(1-y)(1-f+y/\delta)\ . \tag{11}$$

The limits of integration $A$ and $B$ have to be chosen in such a way, that to each value of $y\in[A,\,B]$ the condition $x-f+y/\delta=0$ has a solution within the $x$ range [4]. This was not consequently done in previous works.

With the imaginary dispersion integral we determine destabilizing HERA impedance as function of $f$. In Fig. 2 we see the machine impedance that would be necessary to destabilize a proton bunch of 280 $\mu A$ (corresponding to 45 mA in 160 bunches) at the frequency $f$. We assume the typical detuning coefficients $\Delta_{\mathrm{xz}}=5\times10^{-5}$ and $\Delta_{\mathrm{xx}}=5\times10^{-4}$. The destabilizing impedance is considerably higher than the measured value of $2\ M\Omega$/m. But: the coherent space charge is rather strong in HERA $\Delta Q_{sc}\approx-10^{-3}$. Because the oscillating bunch is carrying its own fields the individual particle frequencies **inside** the bunch are **not** affected, but only the frequency of the oscillating bunch itself. Collective dipole instabilities can only occur at collective frequencies. These frequencies are shifted by an amount

$$f_{\mathrm{inc}}=\Delta Q_{sc}/\Delta Q_{\mathrm{xz}}\approx 20$$

in respect to the particle frequencies $f$.

The range of $f$ where the destabilizing impedance is strong (means where enough Landau damping is present) does not coincide with the frequency range where the dipole instability develops – it is shifted by the incoherent space charge tune shift and Landau damping is lost. In Fig. 2 the relationship between coherent tune shift incoherent space charge and the strength of Landau damping is sketched.

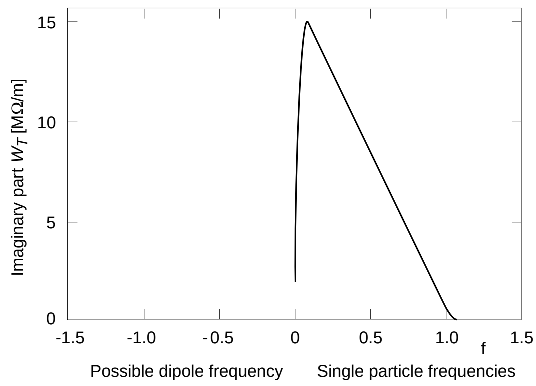


Figure 2: The imaginary part of the destabilizing impedances versus frequency. The intervals of the single particle and dipole frequencies do not coincide. This means: Landau damping is present, but not at the frequency of the dipole oscillation.

Any, even very weak, instability may then blow up the dipole amplitude, leading to the observed rise times in the order of seconds. The described effect of loss of Landau damping can be cured by changing the sextupole currents resulting in higher detuning coefficients (lowering the coefficient $f_{\mathrm{inc}}$). This is the empirical found method used frequently now in HERA.

# 5 REFERENCES


[1] ‘Instabilitäten in HERA-p und PETRA-p’, Klaus Balewski, in DESY-HERA 94–03, März 1993.

[2] ‘Efficient Computation of Fringe-Field Transfer Maps’, G.H. Hoffstätter, DESY 94–242, December 1994.

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[4] ‘Detuning and loss of transversal Landau damping in HERA-p’, J. Feikes, to be published.