# Set Up for Beam Energy Measurements at BESSY II\*

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### Abstract

The parameters necessary to dimension an equipment for the measurement of the beam energy of a storage ring via the resonance depolarisation method are collected. The lay out of the set up for BESSY II is given in comparison to the operational system at BESSY I.

#### **1 INTRODUCTION**

In order to be used as a primary radiation standard by the Physikalisch-Technische Bundesanstalt the electron energy E of the storage ring BESSY II, has to be known with a relative uncertainty of less than  $\Delta E/E = 10^{-4}$ . This high accuracy may be reached by determining the precession frequency  $\Omega_z$  or the spin tune  $\nu_s$  i.e. the number of precessions of the electron spinvector per revolution in the vertical (z) magnetic guide field [1]. For relativistic particles  $\Omega_z$  is given by:

$$\Omega_{\rm z} = (1 + \nu_{\rm s}) \,\omega_0 = (1 + \frac{E}{m_0 c^2} \,a) \,\omega_0. \tag{1}$$

The electron magnetic moment anomaly a, the electron rest mass  $m_0c^2$  and the revolution frequency  $\omega_0$  are known with an accuracy of better than  $10^{-6}$ . At energies of a few GeV and revolution frequencies of some MHz  $\Omega_z$  has to be determined with an uncertainty of  $\Delta\Omega_z \leq \pm 0.5$  kHz in order to reach the required accuracy of E.  $\Omega_z$  is measured by determining the resonance frequency  $\omega_{dep}$  at which the polarization of the stored electron beam is destroyed by a radial (or longitudinal) magnetic field  $H_x(t)$  applied in a short section of the storage ring.

## **2** POLARIZATION

The electron beam polarizes naturally due to the Sokolov-Ternov effect. For VUV light sources like BESSY II the polarisation time  $\tau_p$  is approx 1 h. at 1.7 GeV and 3 h for BESSY I at 800 MeV. Error fields coming from, for example, magnet misalignments lead to depolarization at resonances between the orbit and the spin motion. The strength of the error fields are proportional to *E* since the strength of all magnets scales with *E*. This means as a particle passes through the error field, its orbital angular deflection  $\Theta$  is independent of E but the spin is perturbed by an angle  $\nu_s \Theta$  which is proportional to E. In other words, the higher the beam energy, the more vulnerable is the beam polarization to the depolarizing resonances. Moreover the strength of some depolarization resonances is enhanced due to the presence of a noise source, the synchrotron radiation, which increases rapidely with E. For low energy machines the width of spin depolarizing resonances is small and can be avoided, thus a high polarization degree ( $\approx 92\%$ ) may be reached at BESSY II as shown by Monte-Carlo calculations with the computer code SITROS[2, 3].

### **3 DEPOLARIZATION**

After the polarization has build up, it may be destroied, if the radial field  $H_x(t)$  of the depolarizer is in resonance with  $\Omega_z$ . On resonance the spin is tilted successively around the applied field axis. The spin slowly moves up and down with the nutation frequency  $\Omega_x$ . The ensemble is depolarized by the diffusion of the spin components in the horizontal plane caused by the spin frequency spread and characterised by the spin coherence time  $\tau_{coh}$ . The resonance condition for depolarization is:

$$\omega_{\rm dep} = |\Omega_{\rm z} \pm n\omega_0| = |\nu_{\rm s} \pm k| \,\omega_0 \tag{2}$$

with  $n, k \in \mathbb{N}$ .

### 3.1 Depolarization Time

In order to hit the depolarization resonance at BESSY the frequency of the depolarizer is slowly swept (10 Hz) within a band of  $\Delta\omega_{dep} = \Delta\Omega_z$ . In an actual energy measurement the resonance is searched for within an overall frequency interval of  $20 \dots 30 \times \Delta\Omega_z$  by stepwise increasing the center frequency of the intervall  $\Delta\omega_{dep}$ . At each frequency step the depolarizer is switched on for a time  $> \tau_{dep}$ . Thus the overall time needed for depolarization is  $> 20 \dots 30 \times \tau_{dep}$  and the depolarization time  $\tau_{dep}$  is the criterion for dimensioning the depolarizer.

Since the frequency of the depolarizer is swept slowly, the effects of consecutive resonance crossings add up incoherently and the depolarization time  $\tau_{dep}$  is determined by the sweep interval  $\Delta \omega_{dep}$  and the nutation frequency  $\Omega_x$  as [4]:

$$\tau_{\rm dep} = \frac{\Delta\omega_{\rm dep}}{|\Omega_{\rm x}|^2} \tag{3}$$

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### 3.2 Nutation Frequency $\Omega_x$

The nutation frequency  $\Omega_x$  in a time variing radial magnetic field of peak value  $H_x$  and length  $l_{dep}$  is given by [4]:

$$\frac{\Omega_{\rm x}}{\Omega_{\rm z}} = \frac{1}{2} \frac{H_{\rm x} \ l_{\rm dep}}{\langle H_{\rm z} \rangle \ L} \left| F({\rm s}_{\rm dep}) \right| \tag{4}$$

with  $\Omega_z$  being the spin precession frequency in the average guide field  $\langle H_z \rangle$  of the storage ring of circumference *L*. The factor  $\frac{1}{2}$  stems from the rotating wave approximation. In this approximation the field  $H_x(t)$  is decomposed into two counterrotating components of half amplitude. On resonance one of them is in phase with the spin motion  $\Omega_z$ for each passage, whereas the effect of the out of phase component cancels on average.

### 3.3 Spin Response Function

The characteristic or "spin response function" F(s) of the ring optic covers the action of vertical  $\beta$ -oscillations on the spin motion forced by the depolarizer in resonance with  $\nu_s$  (spin orbit coupling). Depolarization may be supported  $(|F(s_{dep})| > 1)$  or hindered  $(|F(s_{dep})| < 1)$  by the lattice depending on the position  $s_{dep}$  of the depolarizer.  $F_{\nu}(s)$  grows at resonances with  $\nu = mq \pm \nu_z$ , where q is the number of superperiods of the storage ring lattice and  $\nu_z$  the vertical  $\beta$ -tune, as indicated by resonance denominators in the analytical expression for the spin response function [5].

 $F_{\nu}(s)$  may also be determined from linear or nonlinear tracking calculations comparing  $\Omega_x$  as calculated taking vertical orbit oscillations into account to  $\Omega_x$  as calculated switching orbit oscillations off:

$$|F(\mathbf{s})| = \frac{\Omega_{\mathbf{x}} \text{ with oscillations}}{\Omega_{\mathbf{x}} \text{ without oscillations}}.$$
 (5)

Fig. 1 shows the spin response function for one of sixteen cells of the BESSY II magnetic system as calculated by the analytical expresson [5] and by a simple linear spin tracking program. In the linear case only the horizontal fields as produced by quadrupoles are included in the calculation. Minor changes are observed in nonlinear calculations, when additional horizontal fields are taken into account, which arise primarily from sextupoles. Unfortunately the hardware of the vacuum and magnetic system does not allow to place the depolarizer striplines at the favourable positions with a high value of the spin response function. The depolarizer has to be placed in a straight section as close as possible to the achromat where a value of  $|F(s)| \approx 0.5$  may be reached.

## 3.4 Depolarizing Field

The radial field  $H_x(t)$  will be applied to the beam by a pair of striplines mounted at position  $s_{dep}$  in the storage ring vacuum chamber. The on axis radial magnetic field of the two current I(t) carrying striplines will be attenuated by a factor  $a \approx 2$  compared to the free space solution because of the nearby conducting walls of the vacuum chamber. This can be estimated using a simple model of image currents



Figure 1: Comparison of the spin response function  $F_{\nu}(s)$  as calculated by the analytical expression (line) and by spin tracking (squares).

Table 1: Parameters to calculate  $\tau_{dep}$  at 1.7GeV

	BESSY II
$\mu_0 \langle H_z \rangle L$	1.3 T 2π 4.359 m
$\omega_0$	$2\pi 1.25$ MHz
$\nu_{ m s}$	3.86
$\Omega_{\rm z}$	$2\pi$ 6.075 MHz
$\Delta \omega_{ m dep}$	$\pm 2\pi 0.5$ kHz
a	$\approx 2$
$l_{dep}$	50 cm
$ F(\mathbf{s}_{dep}) $	$\approx 0.5$
Z	50 Ω

and has been varified by numerical calculations for the field distribution inside the vacuum chamber.

### 3.5 Power

The power which has to be supplied by the amplifier in order to drive the current I(t) depends on the impedance Z of the striplines. It was decided to work in common 50 $\Omega$ -technique where vacuum feedthroughs, cables and amplifiers are comercially available. Collecting all terms, the depolarization time can be written as:

$$\tau_{\rm dep} = \Delta\omega_{\rm dep} \, \frac{Z}{2P} \cdot \left( \frac{\langle H_z \rangle 2L \ \pi r \ a}{\Omega_z \ |_{\rm dep} \ |F(s_{\rm dep})|} \right)^2, \qquad (6)$$

showing a linear dependency of  $\tau_{dep}$  on the amplifier power P. r is the distance between stripline and electron beam.

### 3.6 Dimension Figures for BESSY II

The result of plugging the numbers listed in table 1 into eq. 6 is:

$$\tau_{\rm dep} = \frac{22 \,\rm kW \,\rm s}{P} \tag{7}$$

i.e. with an amplifier power of 2 kW or a field of 0.9 Gm depolarization times of  $\approx 10$  s may be realized.

At BESSY I, where a depolarizing field of  $\approx 0.05$  Gm is installed and the values of the spin response function



Figure 2: Comparison of the change in the counting rate due to depolarisation outside of the ring chamber (upper curve) to inside (lower curve)

are 4.3 for the optics Metro resp. 1.3 for the WLS optics, the calculation gives  $\tau_{dep} = 4$  s resp. 47 s as compared to experimental values of  $\tau_{dep} \leq 20$  s resp.  $\approx 70$  s

# **4 POLARIMETER**

There are a number of spin dependent phenomena which can be used as a polarimeter [4]. As suggested by Baier the rate of electron-electron scattering within a bunch (Touschek scattering) is utilized since the Møller cross section of Coulomb scattering depends on the relative orientation of the spins of the scattering electrons. In the collision nonrelativistic transverse momentum from betatron oscillations is transfered to the longitudinal direction. The relativistic transformation to the laboratory system increases this effect by a factor of  $\gamma$ . In passing through bending magnets, the dispersion leads to the loss of the accelerated electrons to the outside and the decelerated electrons to the inside of the stored beam. Since other lifetime limiting processes like inelastic scattering on residual gas molecules generally lead to decelerated electrons, the Touschek detectors will be placed on the outside.

Up to  $\approx 50\%$  change of the longitudinal electron energy in a scattering event, the difference of the cross section from full polarization to depolarization increases, while the cross section itselfe decreases [6]. At BESSY II dispersive trajectories for energy deviations of > 8% due to scattering in a straight section will lead to the loss of the electron at the vacuum chamber after the following bending dipole. Thus, if the detectors are placed radially further away from the beam the counting rate of Touschek electrons will decrease whilst the change of the counting rate due to depolarization increases. Recently this could be shown at BESSY I (Fig. 2) where the change in the counting rate due to depolarisation is  $\approx 10\%$  if the detectors are placed outside of the storge ring chamber i.e.  $\approx 60$  mm off the beam as compared to  $\approx 4\%$  inside i.e.  $\approx 10$  mm off the beam. For low emittance storage rings like BESSY II the Touschek effect will be increased compared to BESSY I. Therefore the detectors at BESSY II will be placed outside of the vacuum chamber at a longitudinal position of high dispersion. Since secondary particles, mainly  $\gamma$ , will be detected, cristall scintillators (NaJ) will be used.

At BESSY I in 1994 it was shown that the increase of the Touschek scattering rate, due to depolarization, may as well be observed as a reduction of the beam lifetime [7]. The current/lifetime monitors at BESSY II will be used as an additional depolarisation detection system.

## **5 SUMMARY**

At BESSY II the resonant spin depolarization technique will be used for the high precision measurement of the stored electron beam. Due to the much smaller effect of the radial depolarizing field compared to BESSY I a high power amplifier has to be used in order to depolarize the beam in reasonable short times. At this high power level the depolarizer can be designed in conventional  $50\Omega$  technique. Depolarization will be detected by monitoring the countrate of Touschek-electrons with cristall scintillators mounted outside the storage ring vacuum chamber at a longitudinal position with high dispersion.

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