# SYNCHROTRON RADIATION SPATIAL STRUCTURES INSIDE THE BUNCH 

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## 1. INTRODUCTION

Synchrotron radiation inside the bunch has an asymmetric spatial structure with respect to radial coordinate. Particles scattering on the synchrotron radiation depends on this coordinate. The microstructure of synchrotron type beamstrahlung is also considered.

## 2. SINGLE PARTICLE FIELD

We define the area of synchrotron radiation hard part spatial region ( $\gamma$-region) as localization of light signals emitted in a cone with apex angle $\gamma$ with respect to the particle velocity at all previous moments.


Fig. 1
The surface of $\gamma$-region is represented in fig. 1 by crosssections $z=$ const for $\gamma=100$ and $R=100 \mathrm{~cm}$. In [1] is shown that $\gamma$-region width coincides with synchrotron radiation characteristic wavelength $R \gamma^{-3}$ and remains the same at any distance from particle.

In fig. 1 some parameters of cross-section $\mathrm{z}=$ const of the $\gamma$-region are presented. The size along $x$ axis $h_{x}$ is of the order of $R \gamma^{-3} / \sqrt{x / 2 R}$. Vertical size $h_{z}$ is $2 \sqrt{2 x R} \gamma^{-1}$. Note that the area of cross-section is $\approx 4 R^{2} \gamma^{-4}$ and is independent of $x$.

Fields inside and outside of $\gamma$-region are studied in authors' previous papers (see, e.g. [2-7]). Ignoring the details of calculations note, that if one changes the viewpoint along the $z$ axis the field inside $\gamma$-region fulfills three oscillations. The field magnitude is $\sim 2 \sqrt{2} e \gamma^{4} / \sqrt{R^{3} x}$, the electric component of the field is directed along the radial axis. The field analysis is valid for distances $x \geq R \gamma^{-2}\left(\varphi \geq \gamma^{-1}\right)$. Note the paper [8], where the equivalence of traditional spectral approach and formulas of spatial distribution is shown.

In [9] the loss of the energy at the scattering of on-coming particle is obtained by integrating the radiational friction force $f_{R A D}$ inside $\gamma$-region. If $\lambda \gamma \leq \hbar / m c$ the classical value is multiplied by the ratio of Compton scattering cross-section in ultrarelativistic case to the Thomson cross-section. The particle energy loss $\Delta E$ inside the $\gamma$-region is
$\frac{\Delta E}{m c^{2}}=2 \alpha \gamma^{3} \frac{r_{0}^{2}}{R x}\left(\ln q+\frac{1}{2}\right)$,
where $r_{0}=e^{2} / m c^{2}, \alpha=e^{2} / \hbar c$.

## 3. RADIATION INSIDE THE BUNCH

The analysis of synchrotron radiation structure inside the bunch is important in problems of beam dynamics in accelerator physics. In [2] a parameter of synchrotron radiation packing density inside the bunch is introduced as a ratio of total volume of $\gamma$-regions included in bunch to the bunch volume. To determine dense and dilute bunches an analogous criterion was used for beams of particles with rectilinear trajectories [10,11].

Synchrotron radiation packing on internal and external peripherals of the bunch differ essentially. Let us present some figures of $z=$ const cross-sections for the bunch model with parameters $N=10^{9}, \quad \gamma=6 * 10^{4}$, $R=5^{*} 10^{4} \mathrm{~cm}, \quad R_{x}=0.1 \mathrm{~cm}, \quad R_{y}=0.075 \mathrm{~cm}, \quad R_{z}=1 \mathrm{~cm}$. Distribution of particles in transverse cross-section is uniform. In fig. 2 the "spots" are the cross-sections of particles $\gamma$-regions. In this figure $N_{\gamma}=59662$ is the number of such spots.It seems that a "total" intersection of $\gamma$-regions near the external peripheral of the bunch takes place. However, it is an effect of


Fig. 2
visualization only. Indeed, for given parameters of the bunch the spots sizes in $x$ direction are less than the screen pixel size, and the intersection of $\gamma$-regions is
fictive. For a realistic representation of the pattern fig. 3 is presented, where the picture size in horizontal direction is $0.005 R_{x}$, in vertical $-0.1 R_{y}$.


Fig. 3
The center of the figure corresponds to the $x=0.9025 R_{x}$, $y=0.05 R_{y}$. It is seen that there is no "total" intersection of spots even on the external peripheral of the bunch. In fig. 4 is presented the same cell of cross-section, however, the number of particles is increased: $N=10^{10}$. The packing density in this cell $\sim 10^{-2}$.
The next fig. 5 shows the density packing $d_{\gamma}$ in different parts of bunch cross-section, which is divided into cells, values of $d_{\gamma}$ are set vertically. A particle of on-coming bunch is scattered depending on number of $\gamma$-regions it crosses and fields magnitude in them. The energy total loss of on-coming particle $\Delta E$ is estimated by the formula:

$$
\begin{equation*}
\frac{\Delta E_{T O T}}{m c^{2}}=\overline{\Delta E_{x}} d_{\gamma}\left(2 R_{z} /\left(R / \gamma^{3}\right)\right) \tag{2}
\end{equation*}
$$



Fig. 4
where the averaging $\overline{\Delta E}$ is done with respect to value $x$. The result of this procedure is presented in fig. 6. Since the averaging was done by only one cross-section of the bunch there is some statistical uncertainty, corresponding to the rather less number of particles in sample. It is seen that the total energy losses are less than $10^{-7} \mathrm{mc}^{2}$ even for the peripheral particles.


Fig. 5
The scattering process for the cyclic colliders is to be considered especially, taking into account the considerable number of collisions.


Fig. 6
A similar pattern is obtained for Gaussian distribution of particles in transverse cross-section.

## 4. BEAMSTRAHLUNG STRUCTURE

At the interaction point of linear colliders the electromagnetic field of on-coming bunch serves as an external magnetic field. The trajectories before collision are rectilinear and the field can be calculated in rectilinear approximation. The field with such parameters causes bending of on-coming bunch particles trajectories and the radiation of synchrotron character beamstrahlung. Since the colliding beams have equal intensity, the approximation of such field is valid for bunch frontal particles only. The curvature radius of a particle is determined by its location in the bunch crosssection and distribution of particles in the bunch.

In present paper we confine ourselves to the round beam of radius $R_{r}$. For the bunch with uniform distribution and particle radial coordinate $r$ the radius of curvatureis determined by the formula:
$R=\frac{\gamma}{2 N} \frac{R_{z} R_{r}^{2}}{r_{0} r}$.
For the Gaussian distribution of the particles in transverse cross-section the curvature radius can be obtained from [10]. Further calculations are presented for Gaussian distribution.


Fig. 7
For parameters $\quad N=7 * 10^{10}, \quad \gamma=10^{5}, \quad R_{z}=0.1 \mathrm{~cm}$, $R_{r}=1.6 * 10^{-4} \mathrm{~cm}$ (SLC type beam) are presented in fig. 7.


Fig. 8. Density.
The synchrotron radiation packing density is presented in fig. 8. The total energy loss of a particle intersecting the bunch is:
$\frac{\Delta E_{T O T}}{m c^{2}}=\overline{\Delta E_{x, R}} d_{\gamma}\left(2 R_{z} /\left(\bar{R} / \gamma^{3}\right)\right)$.


Fig. 9. Energy losses.
The results of averaging are presented in fig. 9 .
For next linear colliders we select the following parameters: $\quad N=10^{10}, \quad \gamma=5^{*} 10^{5}, \quad R_{r}=10^{-5} \mathrm{~cm}$, $R_{z}=0.005 \mathrm{~cm}$. Number of spots in fig. 10 is $N_{\gamma}=2751$.

The packing is dilute, density maximum value does not succeed $3^{*} 10^{-6}$ (see. fig 11). More interesting results are obtained for total energy losses of colliding particles (see fig.12). The value of loss


Fig. 10
is of the order of few percents from initial energy of the


Fig. 11. Density.
particle. As a result changes in disruption processes and


Fig. 12. Energy losses.
beamstrahlung during the bunch collision take place.

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