# MEASUREMENT OF BETA AND DISPERSION FUNCTIONS IN THE PLS STORAGE RING \*

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#### Abstract

During commissioning of the PLS storage ring, beta and dispersion functions were measured in the PLS storage ring. Two methods were used to measure beta functions; a quadrupole tweaking method and a sensitivity matrix method. Results are compared with the designed beta functions and shown to be well in agreement. Using sensitivity matrix method has a by-product so that fractional part of the betatron tune can be obtained. Dispersion functions are obtained by measuring the orbit change with RF frequencies. It is shown that the measured dispersion functions agree with the design values and spurious rms vertical dispersions are less than 1 cm.

### **1 INTRODUCTION**

Beta ( $\beta$ ), dispersion ( $\eta$ ) functions and tunes are the major paramters that determine the performance of storage ring. In the PLS, beta and dispersion functions are routinely measured and corrected if desired while continuously monitoring the fractional tune values in the main control room.

In order to measure  $\beta$ -functions, two complementary methods have been adapted in the PLS. One is the usual quadrupole tweaking method and another is the sensitivity matrix method. The quadrupole tweaking method is simple and straightforward; by measuring tune change with quadrupole magnet strength variation, the  $\beta$ -function can be calculated based on simplified formula. This method, however, has a drawback so that it merely gives an average  $\beta$  if two or more quadrupoles are connected in series by the same power supply. This is the case for the PLS storage ring. In PLS, the lattice employs a triple bend achromat (TBA) structure with 12 super-periods. One noteworthy feature of the PLS is such that the bending magnets are separate functioned in order to ease the fabrication. This feature necessitates the adaptation of additional quadrupoles for vertical focusing. As a result, there are total 12 quadrupoles per one super-period and they are mirror symmetrically distributed, leading to total 144 quadrupoles. These 144 quadrupoles are powered by total 39 magnet power supplies. Thus, quadrupole tweaking method in PLS gives an information on  $\beta$  functions at 39 places. If the magnet lattice shows a 12-fold symmetry, however, this method can give a reasonably accurate information on  $\beta$  functions. Whether the lattice possesses

a symmetry or not can be confirmed by measuring dispersion functions before measuring  $\beta$  functions by quadrupole tweaking method.

Sensitivity matrix method, on the other hand, has a very nice feature that it gives an information on  $\beta$  functions at the location of all corrector magnets and beam position monitors (hereafter BPMs). As a by-product, this method also provides an accurate value of the fractional tune values and phase advances between correctors and BPMs. We will describe a detailed procedure for measuring  $\beta$  functions using sensitivity matrix later in this paper.

## 2 β FUNCTION MEASUREMENT BY QUADRUPOLE TWEAKING METHOD

The method of quadrupole tweaking is well-known and straightforward. It is based on the measurement of tune variation with respect to the strength change of quadrupole magnet [1];

$$\Delta \nu = \frac{1}{4\pi} \int \beta \Delta k \, ds = \frac{1}{4\pi} \sum_{i} \beta_i \frac{\Delta I_i}{I_i} k_i l_i \quad , \quad (1)$$

where  $\nu$  is the tune,  $I_i$  the main current of the quadrupole magnet power supply,  $l_i$  is the effective length of the  $i^{th}$ quadrupole,  $k_i$  is the strength of the quadrupole magnet (ie,  $k_i = B'/B\rho$ ). Note that the summation in eq. (1) runs for all quadrupoles powered in series by the same power supply. In Fig. 1 is shown the result of  $\beta$  measurement for both horizontal and vertical planes for one super-period only. For comparison, design values are also described in this figure. It can be seen from this figure that the measured  $\beta$  is well in agreement with the design value. As mentioned in the above, however, in PLS there are only 39 quadrupole power supplies whereas the number of quadrupole magnets is 144. Therefore, the method of quadrupole tweaking in PLS gives only the averaged  $\beta$  functions. If the symmetry of the lattice is broken, the measurement error would become large.

## 3 DISPERSION FUNCTION MEASUREMENT

Whether or not the measured  $\beta$  function possesses a symmetry around a ring can be checked by first measuring dispersion functions. If the measured dispersion function shows a symmetry one would then expect that the measured  $\beta$  would also possess a symmetry. Dispersion function measurement can be done by measuring orbit change

<sup>\*</sup> Work supported by Ministry of Science and Technology and Pohang Iron and Steel Companny



Figure 1: Measured  $\beta$  functions by quadrupole tweaking method in the PLS storage ring

at the position of beam position monitors with respect to the RF frequency variation, because change in RF frequency has the same effect as change in beam energy. The dispersion function ( $\eta$ ) is given by

$$\eta_z = \frac{\Delta z}{\Delta P/P} = -\alpha_c \frac{\Delta z f_{RF}}{\Delta f_{RF}},\tag{2}$$

where z denotes either x or y,  $\alpha_c$  is the momentum compaction factor, and  $f_{RF}$  is the RF frequency. Fig. 2 shows the measured horizontal dispersion function in PLS as compared with the designed values. This figure shows that the measured dispersion functions are close to the design values. It also shows a near 12-fold symmetry. Therefore, one can naturally expect that the measured  $\beta$  in Fig. 1 also possesses a 12-fold symmetry.



Figure 2: Measured horizontal dispersion functions in the PLS storage ring

By the same way, the vertical dispersion function can be measured and the result for PLS is shown in Fig. 3. This figure shows the vertical dispersion function at the position of BPMs. Ideally of course the vertical dispersion should be zero. However, closed orbit distortion at sextupoles and magnet misalignment can result in the appearance of the spurious vertical dispersion. In Fig. 3 the rms vertical dispersion is calculated to be 0.93 cm which is small. If this value is used in the following approximate formula, the vertical emittance  $\epsilon_y$  can be estimated;

$$\epsilon_y = C_q \frac{\gamma^2 < \frac{\mathcal{H}_y}{\rho^3} >}{J_y < 1/\rho^2 >},\tag{3}$$

where  $C_q = 3.84 \times 10^{-13}$  m,  $\rho$  is the bending radius,  $\gamma$  is the usual relativistic factor,  $J_y$  is the vertical damping partition number, and

$$\mathcal{H}_y = \beta_y \eta_y'^2 + 2\alpha_y \eta_y \eta_y' + \gamma_y \eta_y^2. \tag{4}$$

Assuming  $\eta'_y \approx 0$  and  $\alpha_y \approx 0$ , we can write eq. (3) in simplified form;

$$\epsilon_y \approx \frac{c_q \gamma^2 < \eta_y^2 / \beta_y >}{J_y \rho}.$$
 (5)

Using

$$2\pi\nu_y = \int \frac{ds}{\beta_y} = \frac{2\pi R}{\bar{\beta}_y} \tag{6}$$

and inserting corresponding values for the PLS (ie,  $\bar{\beta}_y \approx 8.8 \text{ m}$ ,  $\gamma \approx 4000$ ,  $\rho \approx 6.3 \text{ m}$ ,  $J_y \approx 1$ , and  $2\pi R$ =280.56 m), we get  $\epsilon_y \approx 1.8 \times 10^{-11} \text{ m}$ .



Figure 3: Measured spurious vertical dispersion functions in the PLS storage ring

## 4 β FUNCTION MEASUREMENT USING SENSITIVITY MATRIX

As was mentioned in the above,  $\beta$  function measurement using quadrupole tweaking gives the  $\beta$  values at the position of quadrupole power supplies. Therefore, it has a drawback in the sense that it can only give the averaged  $\beta$ if two or more quadrupoles are powered by the same power supply. On the other hand, measured sensitivity matrix can be used to extract  $\beta$  functions and phases at the position of corrector magnet and beam position monitor respectively [2]

To measure the sensitivity matrix, first each corrector is excited one by one. Then the resulting orbit change at all BPMs is gathered to get  $\Delta z_i / \Delta I_j$  where  $\Delta z_i$  denotes the orbit change at the *i*<sup>th</sup> BPM in the *z* plane and  $\Delta I_j$  is the current change of the  $j^{th}$  corrector magnet. As a result, an  $m \times n$  sensitivity matrix is constructed where m is the number of BPMs and n is the number of corrector magnets respectively. Note that the sensitivity matrix is based on the linear theory, and the effect due to the presence of sextupole magnets is ignored. Therefore, in measuring the sensitivity matrix it is desired to change the corrector magnets by a small amount to reduce the effect of sextupoles. In PLS, the amount of the current change of correctors is limited to within 10% (corresponding to about 0.1 mrad) of the maximum values.

The result applied to the PLS storage ring is shown in Fig. 4. This figure describes the measured horizontal  $\beta$  functions at the position of all correctors and BPMs as compared with the design  $\beta$  functions. As the figure clearly indicates, the measured  $\beta$  functions are very close to the design values.



Figure 4: Measured with sensitivity matrix and designed horizontal  $\beta$  functions in the PLS storage ring

As mentioned, using sensitivity matrix one can provide an information on the fractional tune because correct tune minimizes  $\Delta S$ . In fig. 5 is displayed the  $\Delta S$  as a function of the fractional tune. It is seen from this figure that  $\Delta S$  is indeed minimized exactly at the design fractional tune, which is 0.28 in the case of the PLS storage ring. In PLS, the fractional tune is continuously monitored in the main control room through the spectrum analyzer. At the time of the  $\beta$  function measurement by sensitivity matrix, the fractional tune was exactly set to be the design value. Therefore, Fig. 5 confirms the validity of our measurement method.

#### **5 SUMMARY**

Beta and dispersion measurements on the PLS storage ring have been described. Measured dispersion function was found to be well in agreement with the design values and spurious vertical dispersion function was less than 0.9 cm in rms. Beta function measurement was carried out by two different methods; a quadrupole twaking method and a method using measured sensitivity matrix. Both methods confirmed that the beta functions were close to the design values in the PLS storage ring.



Figure 5: Horizontal fractional tune obtained by minimizing  $\Delta S$  in the PLS storage ring. This figure clearly illustrates that  $\Delta S$  is minimized exactly at the design tune which is 14.28

#### **6 REFERENCES**

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