Relativistic Effects in the Particle Acceleration by Large Amplitude Waves*

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Abstract

In this work, we reconsider the problem of particle acceleration by large amplitude electromagnetic waves. We make use of a fully relativistic Hamiltonian formalism to show that, as opposed to nonelativistic results recently obtained [1], acceleration in unmagnetized systems is severely arrested when the phase-velocity of the electromagnetic mode approaches the speed of light. For subluminous waves, however, acceleration is shown to be still effective.

1 INTRODUCTION

With the advent of powerful radiation-generation systems such as free-electron lasers, cyclotron autoresonance masers, gyrotrons, and ion-channel lasers, a good deal of effort has been directed to the study of the interaction of low-energy particles and large-amplitude electromagnetic waves [1-7]. Whenever wave-particle exchange is likely to occur particles can be highly accelerated, a feature of importance not only for particle acceleration itself, but also for the current-drive techniques of controlled thermonuclear research.

Theoretical models have been developed in order to investigate the dynamics of charged particles, either magnetized or not, in intense electromagnetic fields. In particular, Kuo & Lee [1] have analyzed the interaction of accelerating single-particles with a strong circularly polarized electromagnetic wave in the absence of a background magnetic field. Based on a Lorentz-force nonrelativistic formalism, they have derived a nonlinear Schrödinger equation governing the time evolution of a single-particle parallel (with respect to the wave propagation vector) velocity. This equation has either periodic or soliton-like solutions, depending on the initial conditions. In the periodic case particles draw energy from the wave in a reversible way, accelerating and decelerating periodically. On the other hand, soliton-like acceleration has been found to be more efficient, with the velocity monotonically increasing up to extremely large values.

In the present work we perform a fully relativistic analysis of the problem. It is found that soliton-like solutions with finite wave amplitudes may only appear in those cases where the phase-velocity of the wave, v_ϕ, is smaller than c, the speed of light; in deep contrast to the nonrelativistic results obtained in [1], as $v_\phi \to c$ the wave amplitude necessary to drive soliton-like acceleration tends to infinity. At any rate, for $v_\phi < c$ the soliton-like acceleration is shown to be of relevance. In fact, it is shown that for a given and finite amplitude of the wave, there exists a unique value of $v_\phi < c$, which produces maximum acceleration, and that this unique value of v_ϕ is closely related to the one respective to the soliton-like process.

In view of the necessary condition $v_\phi < c$, one needs a medium that decelerates the relevant electromagnetic modes. As experimentally shown [8], the deceleration can be obtained if one places the system in a dielectric medium whose refractive index is larger than unity. The resulting energization process could be loosely seen as an inverse Čherenkov effect where particles gain energy from a resonant wave propagating with phase-velocity smaller than the speed of light.

2 THE MODEL

In the model we consider charged particles interacting with an intense right-hand circularly polarized electromagnetic wave of frequency ω and wave-vector $\mathbf{k} = k\,\hat{\mathbf{z}}$. The vector potential of the wave, $\mathbf{A}$, is written as

$$\mathbf{A} = \frac{2\epsilon c}{\omega}\left[\sin(kz-\omega t)\hat{\mathbf{x}} + \cos(kz-\omega t)\hat{\mathbf{y}}\right], \qquad (1)$$

where ϵ is the wave eletric field amplitude and c is the speed of light; the phase-velocity of the wave, v_ϕ, is obtained as $v_\phi \equiv \omega/k$.

In the present paper we are mainly interested in the wave-particle interaction leading to particle acceleration. Therefore, as typical in accelerating configurations, we consider particles with very low initial energies, $\mathbf{p}_\perp = 0$. Scaling time and distance to ω and ω/c, respectively, and performing a time removal canonical transformation, $P_z \to P_z$, $(z - ft) \to \phi$ and $H \to \mathcal{H} \equiv H - fP_z$, the dimensionless Hamiltonian system corresponding to this situation is

$$\begin{aligned}\mathcal{H} &= \sqrt{1 + 2\epsilon^2\left[1-\cos(\psi)\right] + (P_z)^2} - f\,P_z \\ &= \gamma - f\,P_z,\end{aligned} \qquad (2)$$

with

$$\dot{P}_z = -\partial\mathcal{H}/\partial\phi = -\epsilon^2\sin(\psi)/(f\gamma), \qquad (3)$$

$$\dot{\phi} = \partial\mathcal{H}/\partial P_z = P_z/\gamma - f, \qquad (4)$$

where the dot stands for time derivative, and $(\phi - \phi_o)/f \equiv \psi$.

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3 ACCELERATION PROCESS

3.1 Soliton-like acceleration

Before we start the investigation of the relativistic dynamics dictated by Hamiltonian (2), let us briefly analyze the soliton-like solutions obtained within the nonrelativistic regime studied in [1]. Expanding (2) for ϵ, $P_z \ll 1$ one writes down an expression for the nonrelativistic Hamiltonian $\mathcal{H}_{nr}$ as a function of the coordinates ψ_{nr} and $v_{z,nr} = P_{z,nr}$:

$$\mathcal{H}_{nr} = (v_{z,nr} - f)^2/2 - \epsilon^2 \cos(\psi_{nr}). \qquad (5)$$

One readily notes that the above Hamiltonian is a pendulum-like one with unstable fixed points located at $v^u_{z,nr} =$ and $\psi^u_{nr} = \pm\pi$. The separatrix orbit joining these unstable fixed points and touching the $v_{z,nr} = 0$ -axis is precisely the curve along which the solitary solution found by Kuo & Lee [1] evolves. Along the separatrix particles are monotonically accelerated from $v_{z,nr,o} = 0$ and $\psi_{nr,o} = 0$ up to $v_{z,nr} = 1$ and $\psi_{nr} = -\pi$ for $f = 1$ and $t \to \infty$ (in the generic case, the maximum velocity satisfies $v_{z,nr} = f$). A major drawback of this kind of solution is that the smallness conditions imposed on P_z and ϵ are not verified when $v_z \sim 1$. We shall proceed to cure this failure with the appropriate relativistic analysis. Before that, however, we remark that the numerical value of the Hamiltonian along the separatrix is ϵ^2. With that one can compute the wave amplitude ϵ_{sep} for which the lower separatrix is just tangent to the $v_{z,nr} = 0$ axis - this results in the soliton-like acceleration for particles launched with $v_{z,nr,o} = 0$ and $\psi_{nr} = 0$; in agreement with previous calculations [1] we find $\epsilon_{sep,nr} = f/2$.

Let us now turn to the relativistic case, starting with an initial look at the corresponding phase-space. In order to make the phase-space analysis clearer and more similar to the nonrelativistic case, it is worthwhile to rewrite Hamiltonian (2) as a function of v_z instead of P_z. From the relation $P_z = \gamma v_z$ we have

$$P_z = v_z \sqrt{\frac{1 + 2\epsilon^2[1 - \cos(\psi)]}{1 - (v_z)^2}}. \qquad (6)$$

Using the above equation in (2) one finds the desired function. In Fig. (1), we show a contour-plot of $\mathcal{H} = \mathcal{H}(v_z, \psi)$ for $\epsilon = 0.5$ and $f = 0.8$. Although the phase-space differs substantially from the nonrelativistic one, the presence of trapped and untrapped orbits separated by a (highlighted) separatrix trajectory can be still appreciated.

In the fully relativistic case one can obtain an expression equivalent to $\epsilon_{sep,nr}$ - it reads:

$$\epsilon_{sep} = f/(2\sqrt{1 - f^2}). \qquad (7)$$

In Fig. (2), we plot ϵ_{sep} and $\epsilon_{sep,nr}$ vs. f. Solid line represents values obtained from the relativistic formalism and dashed line those obtained from the nonrelativistic one. As could be expected, in the relativistic case ϵ_{sep} diverges as $f \to 1$. In this limit, the maximum longitudinal momentum rapidly grows with $v_{z,max} \to 1$ and infinite wave amplitudes are required.

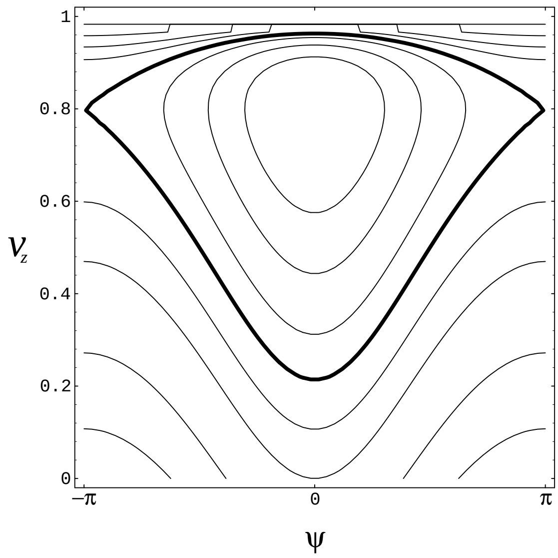

Figure 1: Phase-space portray of the relativistic Hamiltonian for $\epsilon = 0.5$ and $f = 0.8$. The separatrix is highlighted.

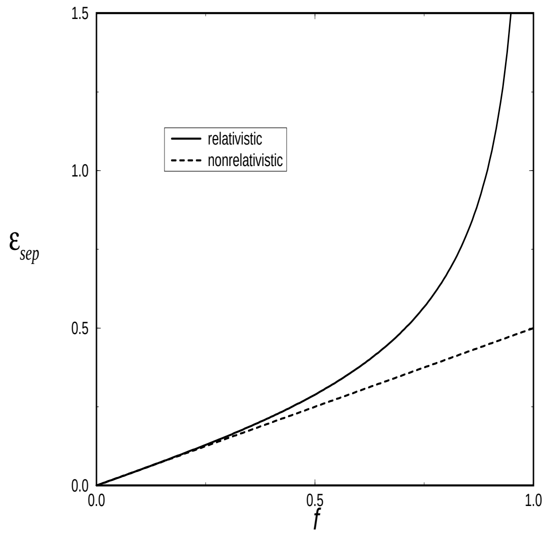

Figure 2: Wave amplitude ϵ_{sep} vs. the normalized wave phase-velocity f. Solid (dashed) line represents the relativistic (nonrelativistic) case.

3.2 Generic case

Given that one has to work with finite fields we immediately rule out the soliton-like acceleration for $f = 1$. It is fairly intuitive, as can be checked with little calculation, that larger wave amplitudes imply larger acceleration; therefore we shall not analyze acceleration efficiency versus wave amplitude. However, a subtler question yet to be answered is the following: for a fixed and finite value of the wave amplitude, what is the value of f creating the largest acceleration? Or in other words: what is the optimal value of f? We shall address this issue from now on.

Recalling that for finite wave amplitudes the $f = 1$-separatrix cannot touch the $v_z = 0$-axis, particles launched with that value of f, at $v_z = 0$ and $\psi = 0$, will describe untrapped orbits. If one decreases f, the separatrix also starts to lower down and get closer to $v_z = 0$ until, at $f = f_{sep}$, it touches that axis. At this moment, initially low-energy particles undergo soliton-like acceleration similarly as described in the previous sub-section. Further decrease of f ultimately causes these low-energy particles to get trapped in the wave field.

Let us proceed to an explicit calculation of the maximum velocity excursions allowed in the trapped and untrapped cases, recalling that particles with $v_{z,o} = 0$ and $\psi_o = 0$ always evolve in time preserving $\mathcal{H} = 1$.

As can be seen from Fig. (1), untrapped particles undergo maximum excursion at $\psi = -\pi$. At this location, using both (2) and (6), and the fact that $\mathcal{H} = 1$, we find a quadratic equation for v_z

$$(1 + Af^2)\, v_z^2 - 2Afv_z + A - 1 = 0, \qquad (8)$$

where $A \equiv 1 + 4\epsilon^2$. The existence of two roots for v_z is natural because the system posses two orbits with $\mathcal{H} = 1$, one below the lowest separatrix and the other above the highest. The solution describing the velocity excursion we are interested in is, of course, the one with the lowest speed:

$$v_{z,max}^{untr} = \frac{Af - \sqrt{A^2f^2 - (1 + f^2A)(A - 1)}}{1 + f^2A}. \qquad (9)$$

When the discriminant of Eq. (8) vanishes, both roots coalesce at $v_{z,max}^{untr} = f_{sep} = v_z^u$, where f_{sep} is obtainable as a function of ϵ as in Eq. (7). At this point, particles with $v_{z,o} = 0$ and $\psi_o = 0$ find themselves orbiting along the separatrix of the system. For $f < f_{sep}$, solutions of Eq. (8) are imaginary; this means that particles become trapped, no longer reaching $\psi = -\pi$.

Let us proceed with the investigation, considering the trapped case for $f < f_{sep}$. From Fig. (1), one readily notes that trapped particles undergo maximum acceleration at $\psi = 0$. Then, on using (2) and (6) with $\psi = 0$, and the fact that $\mathcal{H} = 1$, we find

$$v_{z,max}^{tr} = 2f/(1 + f^2), \qquad (10)$$

which is the maximum trapped velocity excursion for initially low-energy particles.

In Fig. (3) we gather our results Eq. (9) and (10), and plot the maximum velocity excursion, $v_{z,max}$, vs. f for $\epsilon = 0.5$. The discontinuity in the curve appears at $f = f_{sep}$ where trajectories change their topological character from trapped to untrapped. It is apparent that the curve is not monotonic, presenting just to the left of the discontinuity an optimal f leading to the largest possible v_z for a fixed wave amplitude ϵ. It is thus seen that effective acceleration depends on a judicious choice of f. In other words, contrarily to what could be thought initially, for finite wave fields the most effective acceleration does occur for subluminous waves, $f < 1$, and not at $f = 1$.

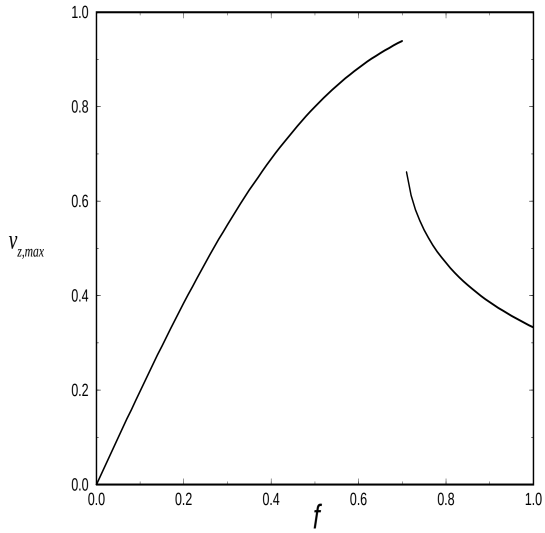

Figure 3: Maximum fully relativistic longitudinal speed $v_{z,max}$ vs. f for $\epsilon = 0.5$.

For $\epsilon \sim 0.5$ and $\omega \sim 10$ GHz, which means electric fields of the order of 10^6 - 10^7 V m^{-1} - typical of moderate laboratory waves - and $f = f_{opt}$, one finds for the maximum longitudinal speed allowed $v_{z,max} \sim 0.93c$.

4 REFERENCES

[1] S. P. Kuo & M. C. Lee 1994 J. Plasma Phys. **52**, 339.

[2] P. Sprangle, L. Vlahos & C.M. Tang 1983 IEEE Trans. Nucl. Sci. **NS-30**, 3177.

[3] R.C. Davidson, Y.-T. Yang & R.E. Aamodt 1989 J. Plasma Phys. **41**, 405.

[4] C. Chen 1992 Phys. Rev. A **46**, 6654.

[5] R. Pakter, R. S. Schneider & F. B. Rizzato 1993 Phys. Rev. E **47**, 3787; *ibid* 1994 Phys. Rev. E **49**, 1594.

[6] L. Friedland 1994 Phys. of Plasmas **1**, 421.

[7] R. Pakter, G. Corso, T.S. Caetano, D. Dillenburg & F.B. Rizzato 1994 Phys. Plasmas **1**, 4099.

[8] S. Von Laven, J. Brancum, J. Golub, R. Layman & J. Walsh 1982 Appl. Phys. Lett. **41**, 408.