LOW DIMENSIONAL PHASE-LOCKING LASER PLASMA NONLINEAR INTERACTIONS

G.I. de Oliveira, L.P. de Oliveira and F.B. Rizzato
Instituto de Física, UFRGS, Porto Alegre, RS, Brasil

Abstract

In this paper we identify phase-locked states among the solutions of the Zakharov equations. Phase-locked states appear as resonant island chains in the appropriate Poincaré plots and the resonance separatrix appears to bring the first chaotic activity into the system.

It is of interest to see if low-dimensional effects play some relevant role in high-dimensional environments. The Central Manifold Theorem suggests that this kind of behavior could be typical.

In the present paper we study the so called Zakharov equations, a case where the effects of low-dimensionality are particularly clear.

The Zakharov equations describe a large class of nonlinear phenomena which can be modeled by the interaction of a high-frequency pump with slow fluctuations of a nonlinear media. Good examples of this kind of process comes from modern nonlinear laser-plasma physics, where the Zakharov equations describe the interaction of high frequency waves, like Langmuir or laser waves, with low frequency ion acoustic modes. [1]-[5].

In modulational regimes where wave energy is not yet dissipated in particle heating, the conservative Zakharov equations can be written in adimensional form

$$i\partial_t E + \partial_x^2 E = n\, E \tag{1}$$

and

$$\partial_t^2 n - \partial_x^2 n = \partial_x^2 |E|^2. \tag{2}$$

They couple the slowly varying amplitude of a high-frequency field $E(x,t)$, to a low-frequency sonic field $n(x,t)$. In the Langmuir turbulence, E is an electrostatic field and n is a density field.

In the adiabatic regime $\partial_t^2 n << \partial_x^2 n$, the governing set of equations can be approximated by a Nonlinear Schrödinger Equation (NLS) which is an integrable equation. Linearization of the NLS around an homogeneous steady-state produces growth rates for small perturbations with wave-vector k. It is known that the system becomes unstable via a pitchfork bifurcation [5],[6] if $k < k_u \equiv \sqrt{2\,\rho_*}$ where ρ_*, is proportional to the energy of the unperturbed state. Besides, if

$$\frac{k_u}{2} < k < k_u, \tag{3}$$

only a few modes are excited for in that case second and higher spatial harmonics of the basic perturbing wave vector are stable. We shall call the region defined by relation (3) the low-dimensional region. The growth rate, Γ, for the small perturbations is given by $\Gamma \sim k\sqrt{k_u^2 - k^2}$ from which is seen that the subsonic regime $\Gamma \ll k$ is valid if $k_u^2 - k^2 \ll 1$. If $k_u \ll 1$ this imposes no restriction on k besides the second inequality of relation (3), but if k_u is large, one must have

$$k \approx k_u. \tag{4}$$

If $k_u \ll 1$, the integrable NLS can be expected to furnish accurate results irrespective of the value of k. In other words, no noticeable transition to chaos is expected as the length scale of the system is varied. In the present work we shall focus on nonintegrable regimes with relatively larger values of k_u, $k_u \tilde{<} 1$; in that case, as soon as one abandons that region defined by condition (4) above, chaos is expected to set in. In a $k \times \rho_*$ parameter space, the $k_u \ll 1$ region is located at $\rho_* \ll 1$ and the $k \approx k_u$ region is a narrow band close to curve $k = k_u$. The low-dimensional region is contained between curves k_u and $k_u/2$.

As mentioned, at $k \approx k_u$ the dynamics is not only integrable but also low-dimensional. It can be shown that in this case the system can be reduced to a nonlinear Hamiltonian model with only one degree of freedom [5]. The nonlinear coupling is obtained by setting $\partial_t \to 0$ in Eq. (2), and substituting the resulting expression for $n(x,t)$ (we shall call it $n_s(x,t)$, "s" meaning "$static$") into Eq. (1) - this is the common procedure to obtain the NLS. Then, expanding the electric field as $E(x,t) = \sum_n E_p(t)\, e^{i\,p\,k\,x}$ and truncating the expansion to the triplet consisting of the homogeneous plus the unstable k and $-k$ modes, the reduced Hamiltonian H is obtained in the form $H = 2\,\rho_o\sqrt{\rho_+\rho_-}\,\cos\psi - k^2\,(\rho_+ + \rho_-) + \rho_+\rho_- + \rho_+\rho_o + \rho_-\rho_o$, where the relevant quantities are defined as $E_{o,+1,-1} \equiv \sqrt{\rho_{o,+,-}}e^{i\phi_{o,+,-}}$, with $\psi \equiv 2\phi_o - \phi_- - \phi_+$ and where ϕ's are conjugate to ρ's. In the present framework, $\rho_* \equiv \rho_o + \rho_+ + \rho_-$ is a constant of motion. The Hamiltonian is integrable because of the simple coordinate dependence and the appropriate phase-space plot is seen in Fig. (1); trapped and untrapped orbits move around the elliptic fixed point with trapped orbits describing closed orbits with a nonlinear frequency Ω_o.

According to the previous comments, as one starts to decrease the value of k for relatively large ρ_*, a transition to chaos is bound to happen. The purpose of this paper is to identify such a transition.

At this point, one should realize the fact that the transition must be induced by the presence of that component of the low-frequency density which was not included in the con-

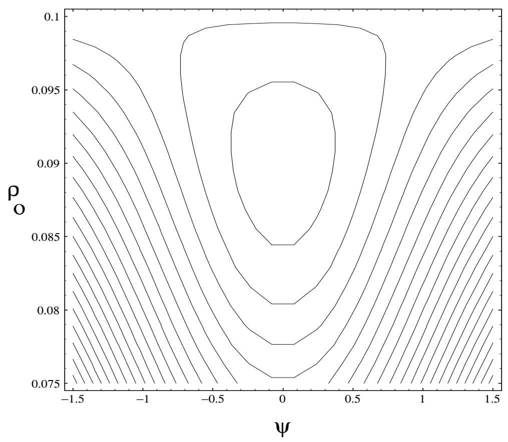

Figure 1: Low dimensional integrable trajectories on the (ρ_o, ψ) plane.

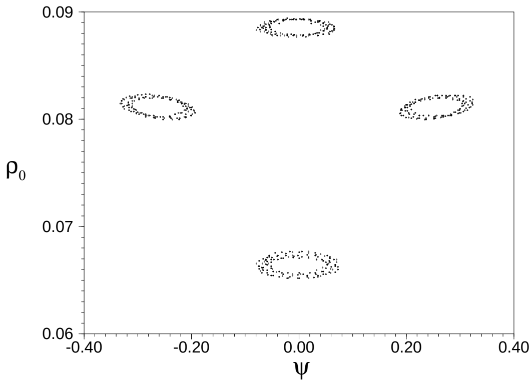

Figure 2: Poincare plot for $n = 4$. $t_{final} = 4644.51 \sim 300$ ion-acoustic cycles.

struction of the integrable NLS equation. We shall call this component n_f and consequently identify it as

$$n_f(x,t) = n(x,t) - n_s(x,t). \tag{5}$$

where "f" means "$fluctuations$". If one is within the low-dimensional region where the most salient modes are the ones with wave vectors k and $-k$, Eq. (2) readily reveals that n_f oscillates primarily with frequency $\Omega_f \sim k$, or period $T_f \sim 2\pi/k$; such a behavior shall be confirmed soon. This fluctuation, which is excited by the high-frequency modes, may be seen as a small perturbation to the integrable Hamiltonian model and if its amplitude is sufficiently high it might be possible that a 2-torus nonlinear resonance of the type $n\,\Omega_o = \Omega_f$ be detected on the phase-space. If this should be the case, one could say that the transition to chaos of the Zakharov equations contains as a relevant and noticeable feature, the presence of resonant islands, or, what is the same, phase-locked states, so characteristic of low-dimensional Hamiltonian systems.

Let us proceed to the numeric simulations to investigate the issue. We integrate Eqs. (1) and (2) with basis on a spectral method with $N = 256$ modes per dynamical variable. To search for the presence of nonlinear resonances we Poincaré plot variables ρ_o and ψ each time $\mathrm{Real}\{n_{f,k=1}\}$ attains maximum value; $d_t\mathrm{Real}\{n_{f,p=1}\} = 0$ with $d_t^2\mathrm{Real}\{n_{f,p=1}\} < 0$. Plotting the dynamical variables this way, we emulate the usual Poincaré plot of a two-degree-of-freedom system, where the plotting is performed each time one of the dynamical variables (here n_f) undergoes a complete cycle; other periodic plotting conditions produces figures similar to the ones to be shown here. In the simulations we take $\rho_* = 0.1$, $k = 0.9075\,k_u$, $\psi(t=0) = 0$ and $\rho_+(t=0) = \rho_-(t=0)$; the remaining variables are initially set to zero, a transient period is allowed such that n_s may be effectively enslaved, and the codes are run for about 500 - 1500 periods of n_f.

Among various resonances we have found, we show the ones seen next. From Fig. (2) one can see that under correct initial conditions a resonant island with $n = 4$ is found. A relatively regular power spectrum can be shown to associated with this resonant state. In low-dimensional periodically driven systems, chaotic separatrix may be found encircling resonant islands; if the initial condition is launched well within the island the orbit is relatively regular (and so is its power spectrum) and if the initial condition is launched close to the separatrix, the orbit and power spectrum becomes progressively more irregular. This is a manifestation of heteroclinic chaos.[7],[8]. We have found this behavior, as can be seen in Fig. (3). The emerging chaotic character of the separatrix was shown to be associated with a continuous power spectrum [6]. In Fig.(4) we plot the 2-torus winding number defined as the rate $lim_{t\to\infty} N_{n_f}/N_{\rho_o}$, the capital N representing the number of cycles of the respective subscripts, versus the initial condition $\rho_o(t=0)$. The winding grows as $\rho_o(t=0) \to \rho_*$ because Ω_o is smaller for outermost orbits and the phase-locked $n = 4$ state is clearly seen as a plateau. $n_f(x,t)$ can be shown to form a stationary pattern. In fact, it has been suggested [9] that nonintegrablity in this kind of system requires two counter-propagating ion-acoustic waves, which is equivalent to the requirement of the second time derivative of Eq. (2). We emphasize that the resonances detected here occur for the full simulation - no low-dimensional approximation has been used. We have varied the number of modes used to check for convergence.

After many cycles of n_f, $t >> 2\pi/\Omega_f$, the trajectories start to blur the resonance islands. In that case low dimensionality is only transient and higher dimensions eventually start to participate in the dynamics. This is seen in Fig. (5). However, if one takes a larger value of k such that higher harmonics $\pm 2\,k$ are farther from their instability region, the low dimensional locking is stabilized. This is what we see in Fig. (6) where a $n = 6$-resonance is shown to survive for much longer stretches of time. The resonance is obtained with $n = 6$ because if k is closer to k_u the nonlinearities are reduced, Ω_o diminishes, and the rotation number n increases so as to mantain resonance.

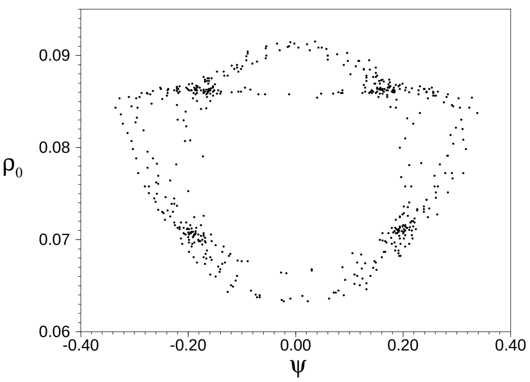

Figure 3: Separatrix of the $n = 4$ resonant island.

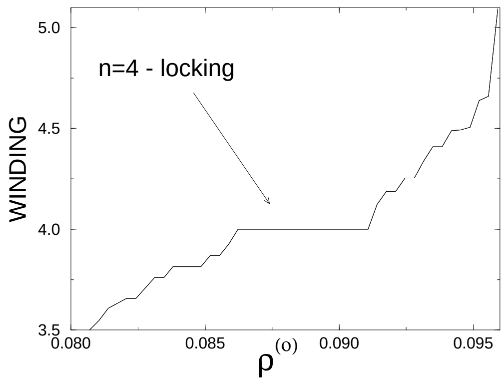

Figure 4: The winding number and the $n = 4$ locking.

1 ACKNOWLEDGMENTS

The authors wish to thank Iberê L. Caldas and Fernando Milman. This work was partially supported by Financiadora de Estudos e Projetos (FINEP) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Brazil. Numerical computing was performed on the CRAY Y-MP2E at the Universidade Federal do Rio Grande do Sul Supercomputing Center.

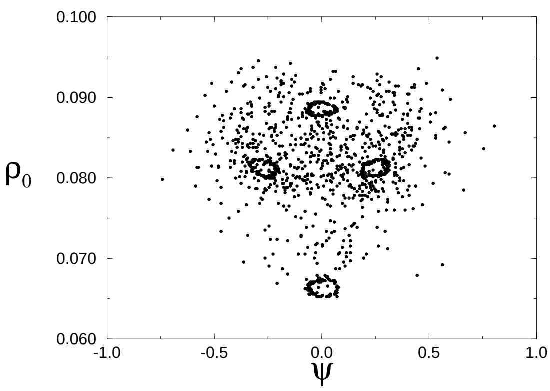

Figure 5: $n = 4$-locking. $t_{final} = 11611.3 \sim 750$ ion-acoustic cycles.

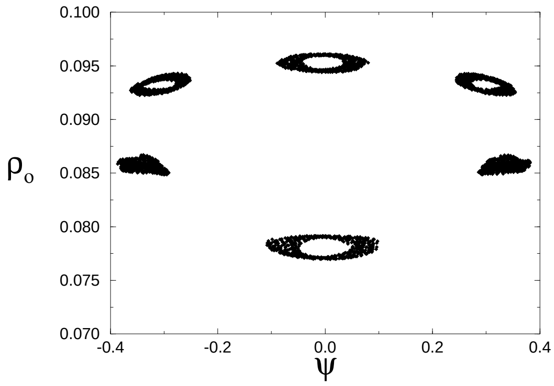

Figure 6: $n = 6$-locking. $t_{final} = 74533.8 \sim 5000$ ion-acoustic cycles.

2 REFERENCES

[1] S.G. Thornhill and D. ter Haar, Phys. Reports **43**, 43 (1978).

[2] P.A. Robinson and D.L. Newman, Phys. Fluids **B1**, 2319 (1989).

[3] H.T. Moon, Phys. Rev. Lett. **64**, 412 (1990).

[4] X. T. He and C. Y. Zheng, Phys. Rev. Lett. **74**, 78 (1995).

[5] G.I. de Oliveira, F.B. Rizzato, and A.C.-L. Chian, Phys. Rev. E **52**, 2025 (1995).

[6] G.I. de Oliveira and F.B. Rizzato, Phys. Lett. A, *accepted* (1996).

[7] A.J. Lichtenberg and M.A. Lieberman, *Regular and Stochastic Motion*, Springer (1983).

[8] G. Corso and F.B. Rizzato, Physica D **80**, 296 (1995).

[9] N. Yajima and M. Oikawa, Progr. Theor. Phys. **56**, 1719 (1976).