VEPP-5 POSITRON SOURCE YIELD SEMI–ANALYTICAL ESTIMATIONS

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1 INTRODUCTION

To determine maximum yield of a conventional positron source for different parameters of matching system with adiabatically decreasing magnetic field, limitations obtained from analytical treatment of particle dynamics in matching device and accelerating sections are applied to distributions of positrons after conversion target, provided by GEANT[1] runs. Analysis is carried out in a framework of requirements and limitations connected with design of positron source for injector complex for electron-positron factories VEPP–5 which are under construction in Budker Institute of Nuclear Physics, Novosibirsk [2, 3]. Primary electron bunches of 300 MeV, $\pm 1\%$ energy spread, $2\sigma_z = 6\ mm$ are focused by triplet into 1 mm beam spot size at conversion target. Adiabatic matching device is followed by first 3 m accelerator section which is placed in solenoid with focusing magnetic field of a 0.5 T. Iris radius of accelerating cavities is $12 mm$, operating frequency is $2856 MHz$, accelerating ratio is $25 MeV/m$. Positron bunches accelerated upto energy $510 MeV$ are injected into storage ring which energy acceptance is $\pm 3\%$.

2 POSITRON DISTRIBUTIONS AFTER TARGET

The first subject of interest in converter design is a dependence of total positron conversion ratio after target upon thickness of a material[4, 5]. This curve for tungsten target reach a maximum value of about 70% and slow vary at range $2.0 \div 2.5$ radiation lengths. Total positron conversion ratio is an integral value and may differ in orders from best positron yields of experimental setups (a few percents). Details of positron distributions over energies, angles and dynamics of particles in matching device and during further acceleration have to be taking into account.

In Fig. 1 an example of positron distribution over energy E and angle Θ with longitudinal axis is presented by lines of equal density. Numbers correspond to contour lines values for density per MeV and per one degree. The positron density has a maximum at energy around 10 MeV and angles above $15°$. This distribution is considered as basis for further analysis.

Another important factor which influence final conversion ratio is bunch lengthening. For 'zero' length initial bunch and tungsten target in 2.5 radiation lengths accumulating GEANT statistics shows that more then 67% of positrons after target are in less then $0.17\,mm$ behind a reference particle which is moving straight ahead with a speed of light. That corresponds to $0.6°$ of RF phase and may be neglected.

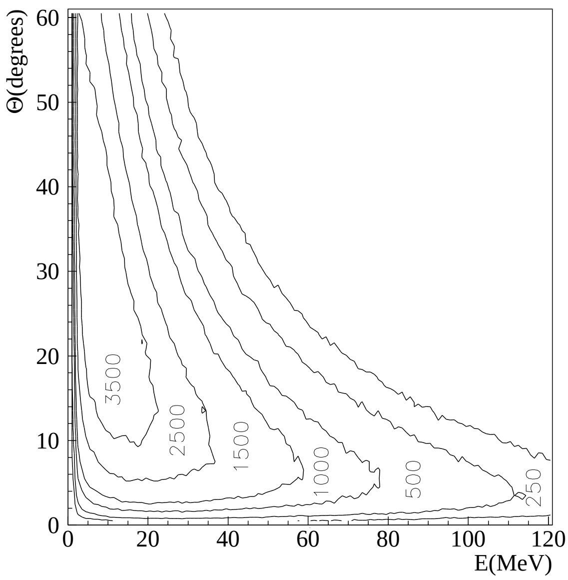

Figure 1: *Lines of equal positron density. Tungsten, 2.5 radiation lengths.* 10^7 *incident electrons.*

The multiple scattering processes in target increase a transverse size of bunch especially for low energy particles. For initial bunch with zero radius spots with radii 0.5, 0.75, 1.0 and 1.5 mm cover correspondently 42%, 65%, 75%, 90% of positrons. Therefore, transverse size of positron bunch after target is comparable with initial radius of the incident electron bunch.

3 ANGULAR LIMITATION

One of the main characteristics describing matching system is a dependence of positron maximum angle captured by system with positron energy. Detailed analytical treatment of particle motion in magnetic field of adiabatic device within some assumptions may be found in [5].

For particle which starts from axis, i.e. with $r = 0$,

$$\Theta^0_{max} = \frac{e\sqrt{B_0 B_s}a}{2P} \ ,$$

is asymptotic solution for maximum angle, a is a radius of accelerator section iris, P is a positron momentum, B_0, B_s are magnitudes of top (initial) magnetic field in matching

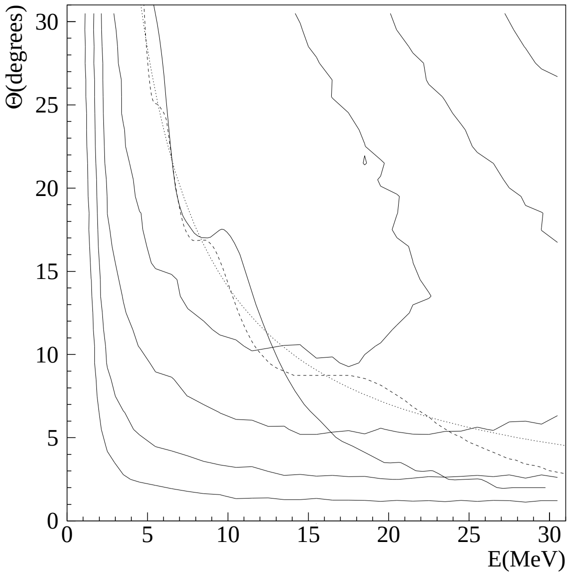

Figure 2: $\Theta_{max}(r = 0)$ *for matching device field lengths of 15 cm (solid), 30 cm (dashed) and asymptotical curve (dotted).*

device and field of solenoid. In fig.2 this dependence represented by dotted curve. Two other curves are results of numerical integration of motion equations. Analysis which takes into consideration second order terms provides similar results. Final formula and details are not presented in this paper.

For positrons starting not from axis maximum angle is a function of r and ϕ, angle between radial and transverse momentum. Transforming formulae for an acceptance hyperellipsoid [5] a quadratic form may be written for $\vartheta = \Theta_{max}(r,\phi)/\Theta^0_{max}$.

$$\begin{aligned}\left(\rho^2-1\right)^2 &= \vartheta^2\left(\cos^2\phi+(1-\rho^2)\sin^2\phi\right)+\\ &\quad \vartheta\cdot 2\left(\rho-\rho^3\right)\sin\phi\end{aligned}$$

where ρ is a initial positron radius normalized by $r_{max} = a\sqrt{B_s/B_0}$, maximal radius at which positrons are captured. Dependences of normalized angle ϑ over ϕ for fixed ρ are illustrated by fig. 3. For particles starting with $r = r_{max}$ the maximum captured angle is twice larger then for positrons which starts from the axis, but their contribution in final conversion ratio have to be rather small.

Estimations of conversion ratio based on Θ_{max} for $\rho = 0$ seems to be quiet reasonable. For example, for $B_0 = 3.5\,T$ and $B_s = 0.5T$ r_{max} is $4.5mm$ and spots with ρ of 0.16 and 0.33 covers 65% and 90% of positrons for zero radius initial bunch. About half of all positrons have ϕ within $\pm 60°$.

4 BUNCH LENGTHENING

Analysis of angular limitation basically originated from necessity to keep particles within aperture of accelerator section does not take into account accelerating field and the longitudinal positron motion.

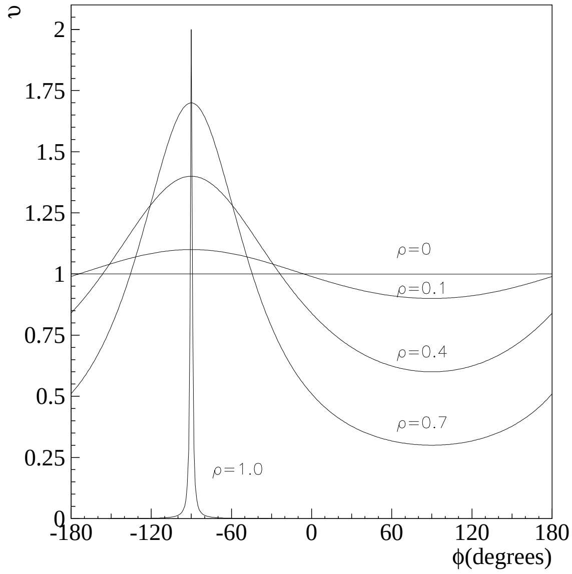

Figure 3: *Normalized maximum positron angle via ρ and ϕ values.*

Due to different time of flight in matching device positron bunch became longer. This delay and phase slippage during acceleration impose additional restrictions on maximal initial angle Θ.

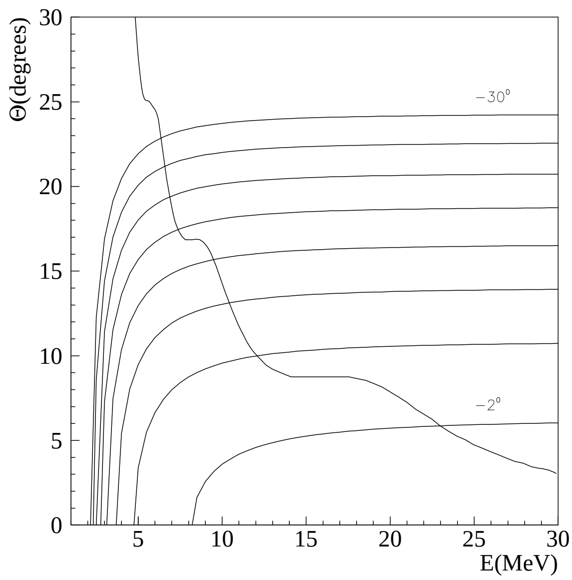

Figure 4: *Lines of equal RF-phase difference at first section entrance.*

These effects are illustrated by fig.4 and fig.5. Time delay relative to particle moving straight ahead with a speed of light is converted into a RF-phase difference. Lines of equal difference are plotted on the plane of initial energies and angles for $B_0 = 3.5\,T$, $B_s = 0.5\,T$, adiabatic field length of 30 cm, from $-2°$ to $-30°$ with step $-4°$. Particles after target start from axis. Utilized analytical expression for RF-phase difference and its derivation are omitted

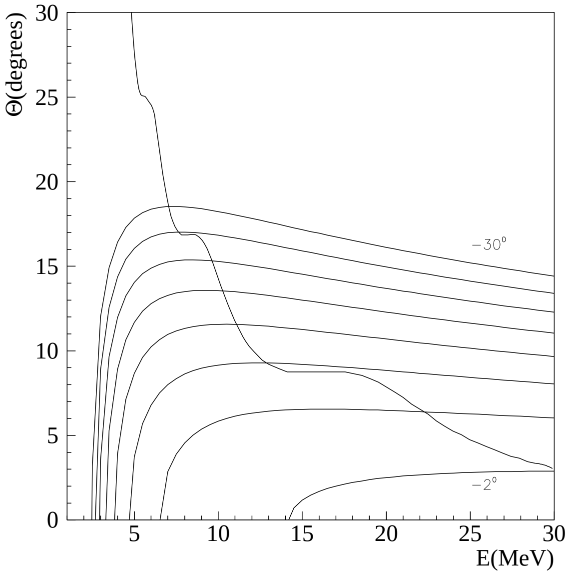

Figure 5: *Lines of equal RF-phase difference after first accelerator section for fixed phase of accelerating field.*

in this paper.

For longer adiabatic field maximum angle for the same RF-phase difference decreases. In the following accelerator sections a bunch phase slippage practically may be neglected.

Finally, limitations derived from the $\pm 3\%$ energy spread requirement at the end of the linac have to be taken into account. A reasonable restrictions may be obtained in assumption that in following accelerating sections RF-phase is tuned in a way that the center of a bunch is always at the maximum of accelerating field. In this case the following relation

$$\triangle E = \triangle E_0 + 2AL\sin^2(\varphi_b/4)$$

determine the region of RF-phase length φ_b and energy deviation after first section $\triangle E_0$, within which particles at the linac exit have energy deviation less then $\triangle E$. A is accelerating rate, L is a length of acceleration. For fixed φ_b one can find corresponding $\triangle E_0$ and obtain output positron yield.

5 POSITRON YIELD

In table 1 results of integration over E and Θ distribution taking into account limitations which are illustrated by fig.5 are presented. Tungsten target have thickness of 2.5 radiation length.

Although presented analysis does not include some details of positrons distribution after target, it involves into consideration basic factors which determine final yield of positron source. It fill the gap between tens and hundreds percents of total positron yield and few percents yields of experimental setups. Results obtained by particles tracing in fields of matching device and accelerating field of first section, utilizing statistics which was obtained by GEANT runs [6], are comparable with presented above.

Table 1: *Positron yield for different lengths and maximum magnetic field strength of the AD matching device. Field of solenoid* $0.5\,T$.

Adiabatic device field length	$2\,T$	$3.5\,T$	$5\,T$
15 cm	3.6%	5.3%	7.0%
30 cm	3.3%	4.8%	6.7%

6 REFERENCES

[1] GEANT—Detector Description and simulation Tool. CERN, Geneva 1993.

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