# COUPLED CALCULATION OF ELECTROMAGNETIC FIELDS AND STATIONARY TEMPERATURE DISTRIBUTIONS

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#### Abstract

Designing accelerator components often requires the study of its electro- or magneto-thermal behaviour. A consistent numerical method is presented for the coupled calculation of electromagnetic fields and the steady state heat distribution. The conversion of electric energy into heat plays an important role in many accelerator applications. As example we can think of rf-windows, cavity cooling or inductive soldering. The underlying mathematical problem for heat conduction in steady state equals Poisson's equation and is therefore formally identical to that one of electrostatics or stationary currents. Consequently, the same numerical methods can be adopted to the stationary temperature problem. The static module S of the program package MAFIA has been extented with the facility to compute stationary temperature distributions. As heat sources one may choose a material with a defined temperature, a heat source of defined heat density respectively heat emission. Especially heating by wall losses of resonant modes or heating by induced eddy currents can be computed easily.

## **1 INTRODUCTION**

A consistent numerical methods is presented for the coupled calculation of electromagnetic fields and the stationary temperature distribution. Many coupled temperature problems arise in the construction and during operation of an accelerator. MAFIA [?], [?], is a proven code for the computation of electromagnetic fields in accelerator components. The static module S of MAFIA has been extented about the computation of stationary temperature distributions. Now, heating by wall losses of modes or inductive heating can be computed completely within this code. The implementation includes 2D as well as 3D with cylindrical polars or cartesian coordinates.

## 2 THEORETICAL BACKGROUND

The governing equation for stationary heat transfer is deducted from Fourier's law

$$\operatorname{div}\left(\kappa \operatorname{grad} T\right) = -w. \tag{1}$$

 $\kappa$  [W/m·K] is the thermal conductivity, w [W/ $m^3$ ] the heat source, resulting from eddy-currents losses or Ohmic losses of resonant modes e.g., and T [K] the temperature.

Eddy-current losses are calculated using Poynting's theorem. The Ohmic losses of resonant modes in good but not

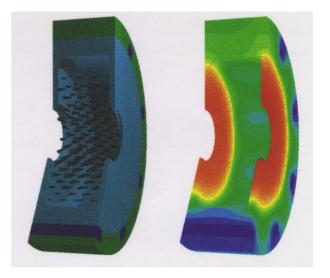


Figure 1: Vector plot of accelerating field and resulting temperature distribution in one of the 180 cells of the watercooled SBLC-structure. Part of the structure is cut away to look inside.

perfectly conducting cavities are computed by the power-loss method.

#### **3 NUMERICAL MODEL**

FIT yields the so-called Maxwell Grid Equations (MGE) on a non-regular grid doublet, corresponding *one-to-one* to the Maxwell's equations obeying very general definitions [?]. Resonant modes in loss free structures are determined with MAFIA's eigenmode solver E. Note that spurious modes are no problem with FIT [?]. Eddy currents are computed with the time harmonic module W3. Wall losses of resonant modes or Joule's energy caused by eddy currents may be calculated in the post-processor P. Losses are calculated via power-loss method resp. the Poynting-vector, rendering **w**.

For the steady state heat conduction problem the discretized Poisson's equation

$$\tilde{\mathbf{S}}\tilde{\mathbf{D}}_{\sigma}\tilde{\mathbf{S}}^{T}\mathbf{t}=\mathbf{w}$$
(2)

has been implemented and is solved in the static solver S. Note that the matrix  $\tilde{\mathbf{S}}\tilde{\mathbf{D}}_{\sigma}\tilde{\mathbf{S}}^{T}$  only differs by the diagonal material matrix  $\tilde{\mathbf{D}}_{\sigma}$  from the matrices for electrostatics resp. static currents.

#### **4 EXAMPLES**

Some examples where the heat originates in electrical energy dissipation will be considered.

## 4.1 Inductive Soldering of S-band Cups

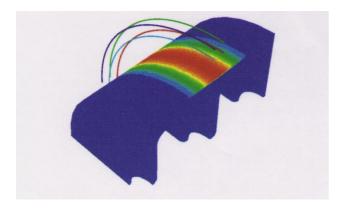


Figure 2: Energy density distribution produced by the driven coils in one quarter of the cylindrical structure.

The S-Band 2\*250 GeV Linear Collider Study SBLC, which is coordinated by DESY, is a planned 30 km linear collider for electrons and positrons. The SBLC Study forsees 2\*2452 constant-gradient structures with a loaded acceleration gradient of 17MV/m. This 6 m long discloaded waveguide structure consists of 180 tapered S-Band cells and runs at 3 GHz. For the disc-loaded structure single-cell shapes [?], so-called cups, are produced by conventional machining and then soldered together for 6 m structures. In this construction process an inductive oven will be used. A coil surrounds the actual location which has to be soldered. It is driven with a frequency of 10-15 kHz. A power of 20-80 kW is applied.

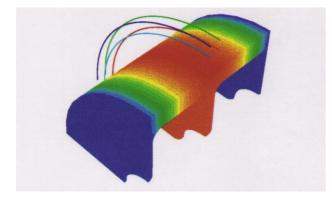


Figure 3: Temperature distribution: The area near the coils and the middle iris is heated.

First the eddy current problem is solved. Next the energy density is calculated. Now the temperature problem can be treated and gives the temperature distribution.

#### 4.2 Water-cooled Cavity

Normal conducting rf resonator cavities are traditionally used for the acceleration of charged particles. Usually they are made of copper. The rf resonator is driven by external power at a given resonance frequency. Wall losses occur due to wall current in the copper material. These wall currents heat up the structure and in most cases water cooling is necessary to keep the temperature constant.

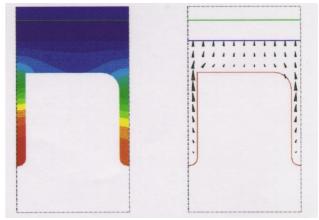


Figure 4: Temperature distribution and thermal gradient in a cut at one of the eight cooling channels. Only the upper half is shown. Compare also Fig. 1.

#### 4.3 RF-Window

Breaking rf-windows in the vacuum system, which is often caused by overheating, is a recurrent problem during the operation of accelerators. In order to reduce the heat load, such ceramic windows are often enlarged. Our example is a window for rectangular waveguides housed in a pill-box like short circular waveguide section. The ceramic disk is located in the middle of the circular section. The waveguide mode in use is the TE01 mode at a frequency of 3GHz.

The electromagnetic field problem is solved in time domain including the loss tangent of the ceramics. Such a time domain solution is very well suited for cw cases. The to-

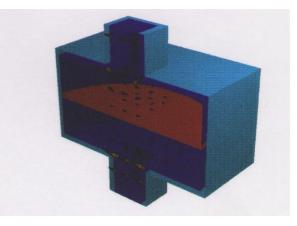


Figure 5: Vector plot of the electric field in an rf-window.

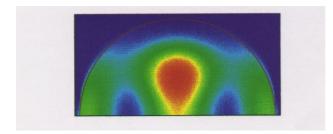


Figure 6: Energy density distribution in the ceramic pane of the rf-window.

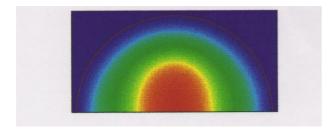


Figure 7: Temperature distribution in the ceramic pane of the rf-window.

tal power is injected through a special waveguide boundary conditions. The time harmonic waveguide excitation is simply monitored over time until steady state is reached. Then the field is stored and another quarter period is analyzed continuing the time domain process. This results in the real and imaginary part of the frequency domain solutions. The power density is a posteriori computed from the steady state electrical field vector and then used as heat source in the temperature analysis.

### 4.4 Waveguide with Load

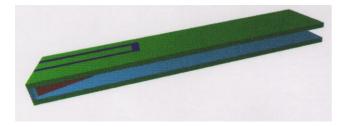


Figure 8: Geometry of a waveguide with load and attached cooling channel which can be seen on the outside.

Another rf-temperature combined problem occurs in high power rf applications, in which rf power has to be dissipated in lossy waveguide terminations. Such terminators are used for instance during testing of new power sources or in accelerating structures where higher order resonator modes have to be damped. The Standford/Berkeley/Livermore Bfactory is such a typical example. Resonators are used for particle acceleration and the particles in turn excite higher order cavity modes. These modes must be eliminated for beam stability reasons. Thus each resonator has substantial antenna built in, which extract rf energy. This energy is guided to nearby rf loads. These loads consist of lossy insertions in rectangular waveguides.

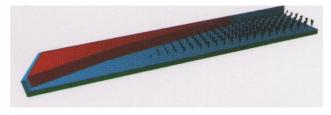


Figure 9: Vector plot of the electric field in a waveguide with load. The upper wall is cut away for this presentation.



Figure 10: Temperature distribution in the lossy load.

## **5 SUMMARY**

Heating by wall losses of travelling waves resp. resonant modes in accelerator structures or heating by Joule' energy caused by eddy currents can be computed completely within MAFIA. Several applications have been studied and presented in this paper.

## **6** ACKNOWLEDGEMENTS

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