# AN EFFICIENT ALGORITHM OF THE RECONSTRUCTION OF SPATIAL FIELD WITH THE USE OF THE DATA DETERMINATED AT THE REGION BOUNDARY

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## 1 INTRODUCTION

Two approaches for solving of the problem of 3D magnetic field reconstruction can be considered:
— the immediate measurement of field in a whole given volume;
— the solution of the boundary problem for the equation $\nabla\vec{B} = 0$ with boundary conditions obtained by the straight measurement.

The distinctive properties of the last approach are: the accurate reconstruction of the magnetic field on a surface and a precise fit of the magnetic field at the volume boundary with the following 3D - field simulation. It provides fast manifold calculations of field components at arbitrary points of the given volume for a particle trajectory analysis. In a general way, to solve the problem a sufficiently dense regular 3D grid must be generated. Because of critical increase in a measurement point number of the magnetic field for the volume the alternative reconstruction of a complete field map by calculations of the field from its boundary values is considered. Obvious decrease in a point number for magnetic measurements on the surface makes it very attractive for measurement consumption.

Theoretically this problem can be considered as follows:
— in regular regions ( a rectangular box, a cylinder, a sphere ) rather simple analytical functions for the following magnetic field approximation can be used; these methods have been used with a great success in precision magnet systems [1, 2, 3];
— the direct numerical experiment using numerical sampling of a differential equation for the region with rather complex geometry lends support the possibility of the magnetic field fit with the required accuracy [4];
— for nonregular regions with sophisticated geometry such approach is rather difficult. However, the field components are harmonic functions

$$\bigtriangledown^2 B_i = 0 \quad in \quad \Omega \quad , \quad i = 1, 2, 3,$$

and $B_i$ are assumed to be known on the surface, thus the finite element technique [5] can be used to solve the problem effectively. Similar reasoning still stands for a scalar potential of the magnetic field.

Obviously, $B_i^{mes}$, measured at region boundaries, are not harmonic functions, but calculated by solving the inner Dirichle problem

$$\triangle\, B_i^{cal} = 0 \quad in \quad \Omega \quad , \quad B_i^{cal}|_{\partial\Omega} = B_i^{mes}$$

$B_i^{cal}$ are ones. Therefore, both the exact solution $B_i^{acc}$ of a field fit problem and the error ($B_i^{acc}$ - $B_i^{cal}$) are harmonic functions. Thus the error reaches its maximum at the boundary like any other harmonic function. In the case of 3–D magnetic field reconstruction this property of harmonic functions also permits to exclude the essential part of planar effect of Hall probes used in magnet measuring [6].

An analysis of computational errors can be done [5]. As a consequence of the analysis the requirements for a surface mesh width can be defined. It should be noted that the influence of calculational errors on the magnetic field reconstruction for such approach is much less as compared to the field measurement ones. The first stage of our approach is building up a calculational grid ( upper index "cal" ) and a measurement grid ( upper index "mes" ) to satisfy the fit requirements. At this preliminary stage both the results of an immediate measurement of field distribution in the volume and any 3D simulation can be used. That is extremely important for a complete testing of a software product.

To fit a magnetic field in a given region one can use two approaches. The first one includes the interpolation of measurement results for each field component to formulate boundary conditions and the following reconstruction of each component in the volume by solving Dirichle boundary problem for Poisson differential equation

$$\triangle\vec{B} = 0 \quad in \quad \Omega \quad , \vec{B}|_{\partial\Omega} = \vec{B}^{mes} \quad , \quad \vec{B} = B_i\vec{e}_i \quad ,$$

$\vec{e}_i$ - the orths of the Cartesian coordinate system.

The second approach suggests the similar work for a scalar potential of a vector field. Because of the absence of currents in the given region, a circled integral along any contour is zero, that is equal to the existence proof of a scalar potential. Besides, we have

$$\bigtriangledown \cdot \vec{B} = 0$$

Thus, the potential can be found by solving the problem:

$$\triangle\varphi = 0 \quad in \quad \Omega \quad , \quad \varphi|_{\partial\Omega} = \varphi^{mes} \quad , \quad \bigtriangledown\varphi = \vec{B}$$

Working out this problem one can easily fit the magnetic field by differentiating the potential with respect to orths' directions

$$B_i = \frac{\partial}{\partial x_i}\varphi$$

## 2 FINITE-ELEMENT PACKAGE FLRECON

This well-known approach was applied for developing the program package FLRECON by modernizing the KOMPOT package [7].

The finite element method with standard hexahedral elements (trilinear shape functions) is used for solving the boundary problem for Poisson equation. Thus, the system of linear algebraic equations is generated. As a technique of solving the linear systems of algebraic equations it has been chosen SSOR method with a polinomial acceleration of the convergence rate on the basis of Chebyshev procedures [8] and $B-T$ acceleration process [9].

To achieve an accuracy required for practical tasks 30-100 iteration were carried out.

Since the problem is reasonably simple, essentially the complete automatization of the program system has been made, which allows to use it in AT 386/387 12 MGz computers.

## 3 FIELD RECONSTRUCTION ACCURACY (NUMERICAL RESULTS)

If the differential methods are used for solving Dirichle problem for Laplace equation, the solving error is known to be maximum at the boundary like any other harmonic function. So the measurement result error obtained at the boundary is largely eliminated in the volume by the application of Laplace equation.

For illustration of this fact two models (2–D and 3–D ones) have been considered.

In the in axisymmetrical case (a 2–D model) in a rectangular area of $80 \cdot 80$ cm ( the internal boundary is defined by the radius R=30 cm) the two kinds of $B_r$ component disturbance at the internal boundary have been investigated: one related to a node and a high frequency distributed disturbance (integrated disturbance over the boundary is equal to zero ). The task has been solved by the use of the POISSON code. The grid step has been taken to equal 1 cm in every dimension. The error has been defined as a difference between solvings of the problem with disturbance and without one.

Error decrement along the normal to the internal boundary is shown in Fig. 1. The case of a node disturbance the error decreases by an order of magnitude at a distance of 3 grid steps (3 cm) and by two orders of magnitude at a distance of 20 grid steps. In the second case the error decreases by three orders of magnitude at a distance of 4 grid steps.

It is to be noted that the character of error decrement depends essentially on the grid step in the given region. In decreasing this step one can expect increasing the asymptotic rate of error decrement proportional to the relation of steps to some power.

Reconstruction of a magnetic field have been performed to illustrate the efficiency of the package for a region being close to the reality [10]. No consideration has been given to the ferromagnetic structural elements.

The 3–D model has the following details. The cross section is defined by a plane $\varphi$=const. Four corners of the cross section have the coordinates (0.2116,1.2), (0.9043,1.2), (4.7512,6.305), (1.1117,6.305) in meters along R-axis and Z-axis accordingly. The grid has the number of nodes $N_r \times N_\varphi \times N_z = 15 \times 180 \times 50 = 135000$, a grid step been uniform along appropriate dimension. Two coils have the same dimensions as the ones in the muon magnet. The cross section of the first coil is defined by the coordinates (4.07,1.6), (4.07,1.65), (4.77,1.65), (4.77,1.6). The cross section of the second coil is defined by the coordinates (4.77,1.87), (4.77,1.875), (5.47,1.875), (5.47,1.87). Each coil carries a current of 1 A. In the case of a high frequency distributed disturbance at the internal boundary error decrement is shown in Fig. 2.

3–D magnetic field reconstruction for Dirichle problem for the described model indicates that a relative accuracy

$$\frac{H_z^{cal} - H_z^{coil}}{H_z^{coil}},$$

were

$H_z^{coil}$ has been defined by a verified 3–D integral code as $H_z$ component for the indicated coils;
$H_z^{cal}$ the finite element solution.

does not exceed of (2 $\div$ 3) $10^{-6}$.

## 4 CONCLUSION

The numerical test demonstrates a hight efficiency of this program package. The rapid damping of a high frequency measurement error emerging at the surface of the region number consideration allows to increase an accuracy of field reconstruction over the whole region. This package provides the closest interpolation and the effective smoothing of field components for any subregion of an arbitrary shape.

The Hall probes for measuring distinct field components are spatially delivered, therefore a procedure of field data interpolation for a unique space grid is required. The ”boundary” method is free of that defects and permits to interpolate the data immediately on a unique inner grid, rather fine and regular, which is acceptable for operating the trajectory analysis program [1].

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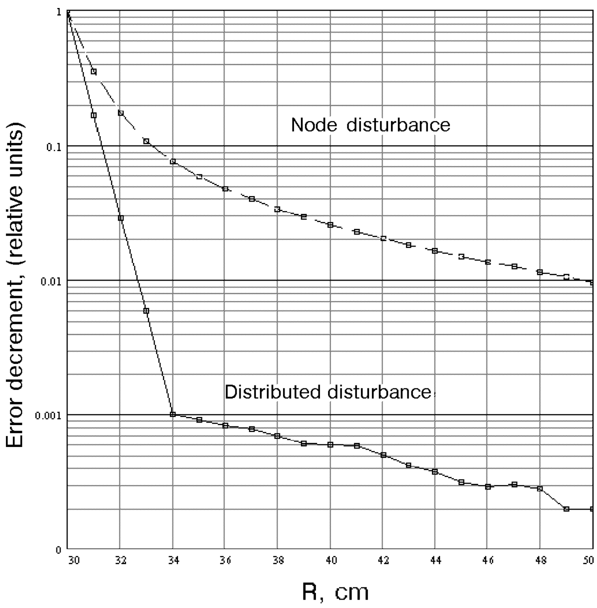


Figure 1: Error decrement in the given region (the 2-D model)

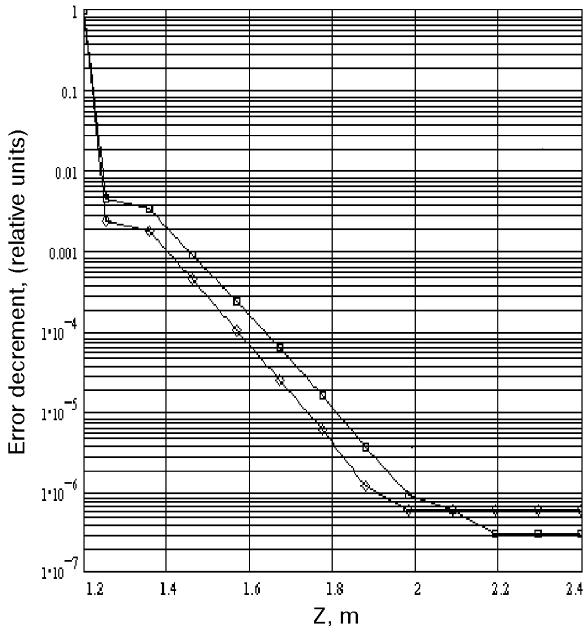


Figure 2: Error decrement in the given region (the 3-D model)