SUPPRESSION OF BEAM-BEAM RESONANCES IN CRAB WAIST COLLISIONS

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Abstract

The recently proposed Crab Waist scheme of beam-beam collisions can substantially increase the collider luminosity since it combines several potentially advantageous ideas. One of the basic ingredients of the scheme is the use of dedicated sextupoles in the interaction region for the vertical beta function waist rotation at the interaction point. In this paper we show how this nonlinear focusing helps to suppress betatron and synchrobetatron resonances arising in beam-beam collisions due to particles' vertical motion modulation by their horizontal oscillations.

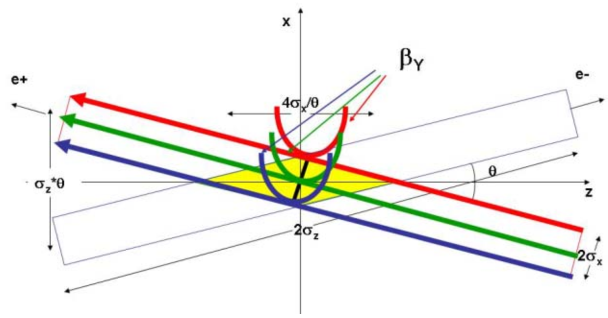
INTRODUCTION

In high luminosity colliders with standard collision schemes the key requirements to increase the luminosity are: very small vertical beta function β_y at the interaction point (IP), high beam intensity and large horizontal emittance ε_x and beam size σ_x. However, β_y can not be much smaller than the bunch length σ_z without incurring in the "hour-glass" effect. It is, unfortunately, very difficult to shorten the bunch in a high current ring without exciting instabilities. In turn, the beam current increase may result in high beam power losses, beam instabilities and a remarkable enhancement of the wall-plug power. These problems can be overcome with the recently proposed Crab Waist (CW) scheme of beam-beam collisions [1] where a substantial luminosity increase can be achieved without bunch length reduction and with moderate beam currents.

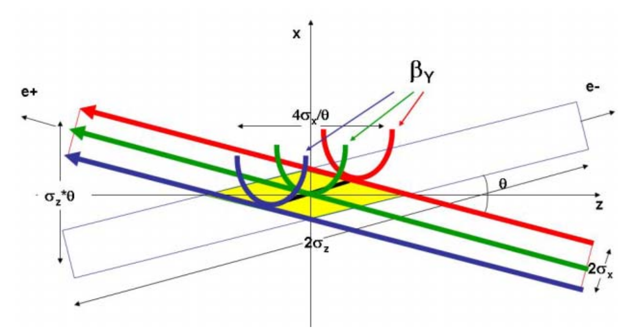
In the following we briefly describe the Crab Waist collision concept and discuss in detail one of the basic mechanizms that allows increasing the luminosity in crab waist collisions, namely, the suppression of beam-beam resonances induced due to modulation of particles' vertical motion by their horizontal oscillations. Numerical examples demonstrating the effect of crab waist sextupoles are also shown.

CRAB WAIST CONCEPT

The Crab Waist scheme of beam-beam collisions can substantially increase collider luminosity since it combines several potentially advantageous ideas. Let us consider two bunches colliding under a horizontal crossing angle θ (as shown in Fig. 1a). Then, the CW principle can be explained, somewhat artificially, in the three basic steps. The **first one** is large Piwinski angle. For collisions under a crossing angle θ the luminosity L



a) Crab sextupoles OFF.



b) Crab sextupoles ON.

Figure 1: Crab Waist collision scheme.

and the beam-beam tune shifts scale as (see, for example, [2]):

$$L \propto \frac{N\xi_y}{\beta_y^*}; \quad \xi_y \propto \frac{N\sqrt{\beta_y^*/\varepsilon_y}}{\sigma_z\theta}; \quad \xi_x \propto \frac{N}{(\sigma_z\theta)^2} \tag{1}$$

with N being the number of particles per bunch. Here we consider the case of flat beams, small horizontal crossing angle $\theta \ll 1$ and large Piwinski angle $\phi \gg 1$, where Piwinski angle is defined as:

$$\phi = \frac{\sigma_z}{\sigma_x} tg\left(\frac{\theta}{2}\right) \approx \frac{\sigma_z}{\sigma_x}\frac{\theta}{2} \tag{2}$$

The idea of colliding with a large Piwinski angle is not a new one (see, for example, [3]). It has been also proposed for hadron colliders [4, 5] to increase the bunch length and the crossing angle. In such a case, if it were possible to increase N proportionally to $\sigma_z\theta$, the vertical tune shift ξ_y would remain constant, while the luminosity would grow proportionally to $\sigma_z\theta$, see (1). Moreover, the horizontal tune shift ξ_x drops like $1/\sigma_z\theta$. However, differently from [4, 5], in the crab waist scheme described here the Piwinski angle is increased by decreasing the horizontal beam size and increasing the crossing angle. In this way we can gain in luminosity as well, and the horizontal tune shift decreases. Moreover, parasitic collisions (PC) become negligible since with higher crossing angle and smaller horizontal beam size the beam separation at the PC is large in terms of σ_x. But the most important effect is that the overlap area of the colliding bunches is reduced, since it is proportional to σ_x/θ (see Fig. 1).

Then, as the **second step**, the vertical beta function β_y can be made comparable to the overlap area size (i.e. much smaller than the bunch length):

$$\beta_y^* \approx \frac{\sigma_x}{\theta} << \sigma_z \tag{3}$$

It worth to note that usually it is assumed that ξ_y (see the expression for L in (1)) always reaches the maximum allowed value, so called "beam-beam limit". So, reducing β_y at the IP gives us several advantages:

- Luminosity increase with the same bunch current.
- Possibility of the bunch current increase (if it is limited by ξ_y), thus farther increasing the luminosity.
- Suppression of the vertical synchrobetatron resonances [6].
- Reduction of the vertical tune shift with the synchrotron oscillation amplitude [6].

Besides, there are additional advantages in such a collision scheme: there is no need in decreasing the bunch length to increase the luminosity as proposed in standard upgrade plans for B- and Φ-factories. This will certainly helps solving the problems of HOM heating, coherent synchrotron radiation of short bunches, excessive power consumption, etc.

However, large Piwinski angle itself introduces new beam-beam resonances which may strongly limit the maximum achievable tune shifts (see [7], for example). At this point the crab waist transformation enters the game boosting the luminosity. This is the **third step**. As it is seen in Fig. 1b, the beta function waist of one beam is oriented along the central trajectory of the other one. In practice the CW vertical beta function rotation is provided by sextupole magnets placed on both sides of the IP in phase with the IP in the horizontal plane and at π/2 in the vertical one (as shown in Fig. 2).

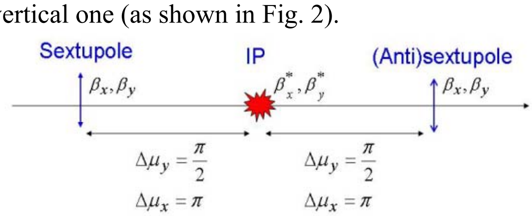


Figure 2: Crab sextupole locations.

The crab sextupole strength should satisfy the following condition depending on the crossing angle and the beta functions at the IP and the sextupole locations:

$$K = \frac{1}{\theta}\frac{1}{\beta_y^*\beta_y}\sqrt{\frac{\beta_x^*}{\beta_x}} \tag{4}$$

The crab waist transformation gives a small geometric luminosity gain due to the vertical beta function redistribution along the overlap area. It is estimated to be of the order of several percent [8]. However, the dominating effect comes from the suppression of betatron (and synchrobetatron) resonances arising (in collisions without CW) due to the vertical motion modulation by the horizontal betatron oscillations [9, 10].

In the following we explain the mechanism of beam-beam resonances suppression.

SUPPRESSION OF RESONANCES

First of all, for large Piwinski angles ϕ>>1 we need to change the concept of Collision Point (CP). Indeed, for large horizontal separations (in units of σ_x) the vertical beam-beam kick drops as $1/R^2$, while the horizontal one drops as 1/R. It means that for the vertical kick the center of the opposite bunch becomes not so important and can be not seen at all by the particles with large longitudinal displacements due to large horizontal separation. Thus CP has to be defined in a different way: it is *the point where a test particle crosses the longitudinal axis of the opposite beam*. In particular it means that the X-coordinate of CP in the "strong" frame is always zero, by the definition.

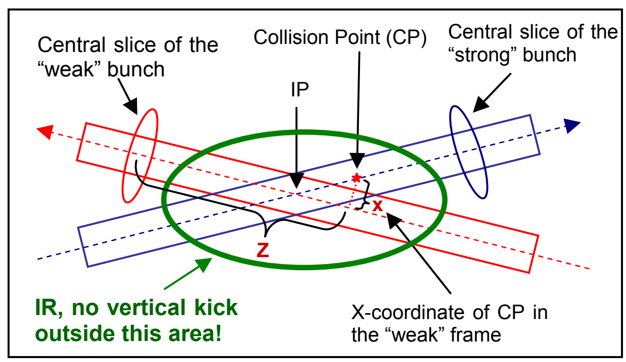


Figure 3: Collision with large Piwinski angle.

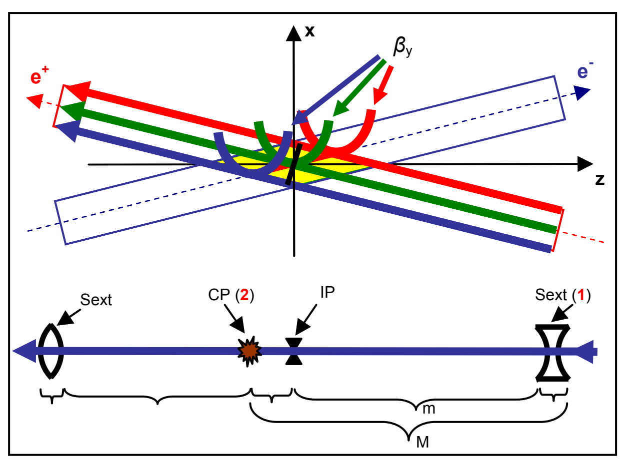


Figure 4: Crab Waist scheme.

Now let us show that the CW transformation kills the vertical betatron phase modulation. According to [9] the transport matrix ***M*** (see Fig. 4) from the entrance of the first sextupole (point 1) to the CP (point 2), vertical betatron motion only, can be written as:

$$M = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ V & 1 \end{pmatrix} \tag{5}$$

where the first matrix corresponds to the drift space from IP to CP, L being the drift length, the last matrix corresponds to the sextupole, considered here as a thin linear lens, and in the middle we have the unperturbed matrix ***m*** from the sextupole location to the IP. For this unperturbed matrix we have $m_{22} = 0$, since $\alpha_y = 0$ at the IP and $\Delta\mu_y = \pi/2$. As a result we get $M_{22} = 0$ as well. On the other hand, considering the "new" lattice (sextupoles included) we can write the standard formula for M_{22}:

$$M_{22} = \sqrt{\beta_y / \beta_{1y}} \cdot \left(\cos(\Delta\mu_{1y}) - \alpha_{1y} \cdot \sin(\Delta\mu_{1y})\right) \tag{6}$$

where β_{1y} and α_{1y} are the beta- and alpha-functions at the CP. Since it is the waist at the CP, α_{1y} must be equal to zero, so we get $\cos(\Delta\mu_{1y}) = 0$, resulting in $\Delta\mu_{1y} = \pi/2$, that is exactly what we wanted. In the other words, the vertical betatron phase advance from the first sextupole to CP and then from CP to the second sextupole remains to be $\pi/2$ for all the particles independently on their X-coordinate. This feature allows substantial increase of the beam-beam tune shift ξ_y.

We performed a number of beam-beam simulations which confirmed advantages of the Crab Waist scheme. In Fig. 6 one can see two luminosity tune scans performed for the SuperB set of parameters [11]. The "geographical map" colors were used: red corresponds to the maximum luminosity, blue – to the minimum. One can see a clear resonance suppression and "good" areas expansion when the Crab sextupoles are switched on. It worth to note that the bunch current in Fig. 5b is higher by a factor of 2.5! One more example is given in Fig. 6, where the beam tails and vertical blowup (r.m.s. beam size which affects the luminosity) for one of the good working points are shown versus the bunch current.

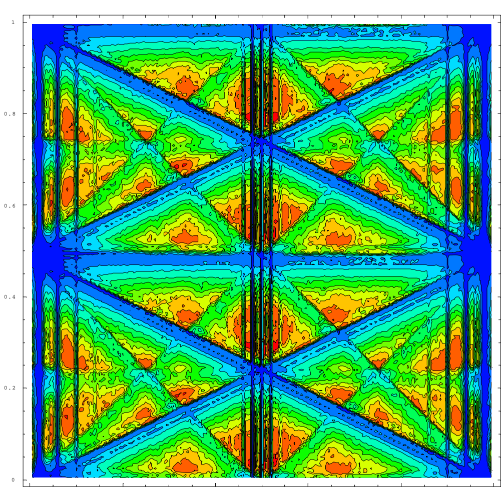


Figure 5a: Luminosity tune scan, CW=0.

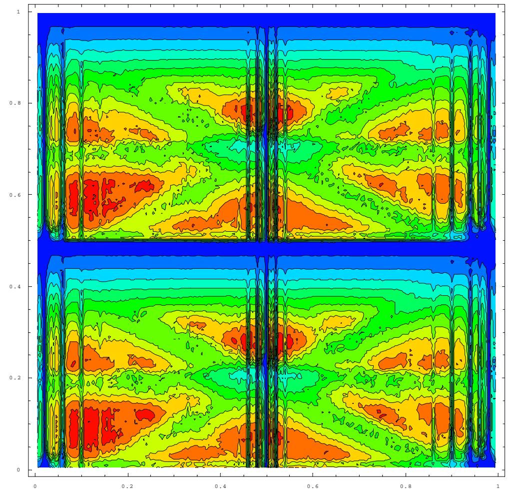


Figure 5b: Luminosity tune scan, CW=1.

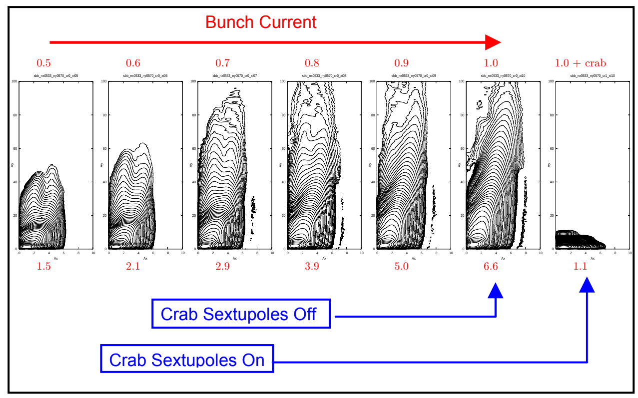


Figure 6: Beam tails and vertical blowup (numbers at the bottom) vs. bunch current and Crab Waist.

CONCLUSIONS

Collision scheme with large Piwinski angle and Crab Waist looks very attractive, since it strongly suppress the beam-beam resonances, thus allowing significant increase of the beam-beam tune shift ξ_y and the collider luminosity. This has been confirmed by the recent experimental results obtained on DAΦNE [12].

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