# NON-DESTRUCTIVE BEAM MEASUREMENTS *

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*Abstract*

In high energy accelerators, especially storage rings, non-destructive beam measurements are highly desirable to minimize the impact on the beam quality. In principle, the non-destructive tools can be either passive detectors like Schottky, or active devices which excite either longitudinal or transverse beam motions for the corresponding measurements. An example of such a device is an ac dipole, a magnet with oscillating field, which can be used to achieve large coherent betatron oscillations. It has been demonstrated in the Brookhaven AGS that by adiabatically exciting the beam, the beam emittance growth due to the filamentation in the phase space can be avoided. This paper overviews both techniques in general. In particular, this paper also presents the beam tune measurement with a Schottky detector, phase advance measurements as well as non-linear resonance measurements with the ac dipoles in the Brookhaven RHIC.

## INTRODUCTION

Most of the non-destructive beam measurements are done by passively detecting the electro-magnetic signals generated by the beam when it passes by the detector, like beam position monitors (bpm),wall current monitors (WCM), beam loss monitors (blm), retarded field detector for detecting electron clouds and Schottky detectors. From these instruments, one can learn much about the beam closed orbit, bunch longitudinal distribution etc. The Schottky detector can provide additional information on the momentum spread of the beam, betatron tunes as well as chromaticities. However despite the rich information one can obtain from the passive detectors, it is still often required to manipulate the beam to gain more understanding about the machine optics properties, space charge effects, resonance structures and etc.

In order to minimize the impact on the beam motion with active measurements, the excitation has to be applied gradually. The adiabatic condition allows the beam to follow the external excitation and guarantees the beam conditions to be restored after the measurement is done and the excitation dies off. A typical example of this is the ac dipole, a magnet with an oscillating magnetic field.

* The work was performed under the auspices of the US Department of Energy

## NON-DESTRUCTIVE BEAM MANIPULATION TECHNIQUES

### *AC Dipole*

The ac dipole has been first used in electron machines to induce sizable betatron oscillations. Because one is less concerned with preserving beam emittance due to the synchrotron radiation damping, the ac dipole (shaker) is usually excited right at the beam betatron frequency. Such a device has been used in CESR [1] and LEP [2] for measuring coupling effects as well as twiss functions of the machine.

The technique of using an ac dipole to induce a long lasting coherence in a hadron machine was first developed in the Brookhaven AGS. It was demonstrated in the Brookhaven AGS [3] as well as the CERN SPS [4] that by slowly ramping up the amplitude of an ac dipole oscillating field at a frequency in the vicinity of the beam betatron frequency, the beam can be driven to a steady coherent oscillation. In the absence of non-linear effects, the driven coherent oscillation $z(s)(n)$ is given by

$$z(s)(n) = \frac{B_m L}{4\pi B\rho\delta}\sqrt{\beta(s)\beta_0}\sin(2\pi\nu_m n + \phi(s)). \quad (1)$$

Here $B\rho$ is the magnetic rigidity, $B_m L$ is the oscillating amplitude of the integrated AC dipole field strength. $\beta(s)$ and $\beta_0$ are the beta functions at the observation point $s$ and the ac dipole location. $n$ is the number of revolution turns. $\nu_m$ is the ac dipole tune defined as the ratio between the ac dipole frequency and beam revolution frequency. $\delta = |\nu_m - \nu_z|$ is the resonance proximity factor and $\nu_z$ is the betatron tune.

As long as the driven oscillation is built up slow enough to allow the particles to follow, the beam distribution should be restored once the external driving force is gradually turned off. Fig. 1 shows a simulation of 1000 particles in Gaussian distribution before and after the adiabatic ac dipole excitation. The fact that the two distributions match each other confirms the preservation of the beam emittance. This was also experimentally confirmed in the Brookhaven AGS [3]. However, the adiabatic condition can be broken if the ac dipole frequency is inside of the beam tune spread even if the excitation is energized/de-energized slowly. In this case, part of the beam then becomes unstable and emittance growth is then inevitable. The effect of the emittance growth dependence on the chromaticity was also studied at the CERN SPS, and the data clearly shows the larger the chromaticity, the more emittance growth [4].

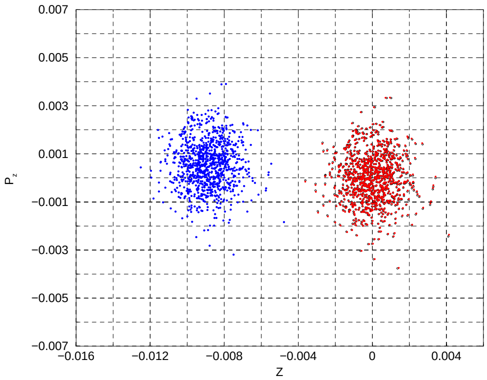


Figure 1: Simulation of beam distribution before, during and after the ac dipole excitation. The beam distribution with a steady coherent oscillation is shown with blue dots. All the distributions are shown in the frame which rotates resonantly with the ac dipole frequency.

The long lasting coherence excited by the ac dipole can have many applications. It was first applied in the Brookhaven AGS polarized proton operation to avoid the beam polarization loss when crossing a strong intrinsic spin depolarization resonance [5]. Furthermore, with sizable long lasting driven oscillation at all the bpms distributed around the machine, the amplitude and phase of the oscillation can be precisely determined by linearly fitting the turn by turn beam position data. Below is the list of ac dipole applications.

**Linear optics measurement** The driven oscillations between two locations is uniquely defined by the magnetic fields in between provided the ac dipole is outside the region between the two locations. The phase advances and beta functions can be determined by the phase and amplitude of the driven oscillations [6]. Assuming the betatron oscillation amplitudes are measured at three beam position monitors 1, 2 and 3 as well as the phase advances in between the BPMs, the beta function at BPM 1 is given by

$$\beta_1 = \beta_1^m \sqrt{\frac{\beta_2/\beta_1}{\beta_2^m/\beta_1^m}\frac{\sin\phi_{12}^m}{\sin\phi_{12}}} \qquad (2)$$

where $\beta_i$ is the measured beta function at BPM i, $\beta_i^m$ is the model beta function at BPM i, $\phi_{ij}$ is the measured phase advance between BPM i and BPM j where $i, j = 1, 2, 3$, and $\phi_{ij}^m$ is the model phase advance between BPM i and BPM j.

In RHIC, a pair of ac dipoles were employed for measuring the beta functions and phase advances [6]. Fig. 2 is a typical example of the measured beta functions and phase advances at storage energy in the RHIC Yellow ring.

**Non-linear resonance driving term measurement** It has been theoretically and experimentally proved that a particular spectral line of a particle's complex normalized coordinates as shown in Eq. 4 corresponds to a particular resonance whose driving term is proportional to the amplitude of the spectral line [7, 8].

$$\begin{aligned} x(n) - iP_x(n) = {} & \sqrt{2J_x}e^{i(2\pi\nu_x n+\psi_x)} - \\ & 2i\sum j f_{jklm}(2J_x)^{\frac{j+k-1}{2}}(2J_y)^{\frac{l+m}{2}} \\ & \times e^{i[(1-j+k)(2\pi\nu_x n+\psi_x)+(m-l)(2\pi\nu_y n+\psi_y)]}. \end{aligned} \qquad (3)$$

The coherent oscillation adiabatically driven by an ac dipole lasts at a fixed amplitude for a very long time. Furthermore, the driven oscillations between any two locations without an ac dipole in between are uniquely determined by the magnetic fields in the region. Hence, a particular spectral line in the Fourier spectrum should also correspond to a particular non-linear resonance. Similar to the case of the free oscillation excited by a one-turn kicker, the amplitude of the spectral line is proportional to the driving term of the corresponding resonance [9]. These factors make the ac dipole a promising alternative tool for exploring the non-linear beam dynamics.

A prove-of-principle beam experiment was conducted during the RHIC Au 2004 Run [10]. Fig. 3 shows the measured $3^{rd}$ resonance driving term at RHIC Au injection energy. The horizontal tune was placed close to the $3^{rd}$ order resonance for the experiment. The complex coordinates were constructed using the turn-by-turn oscillation data recorded at all the available bpms. The good agreement between the measured driving term and the model prediction based on an known sextupole components in the magnets shows this method is very promising. From the measured driving term, the local sextupole strength can also be obtained [10]

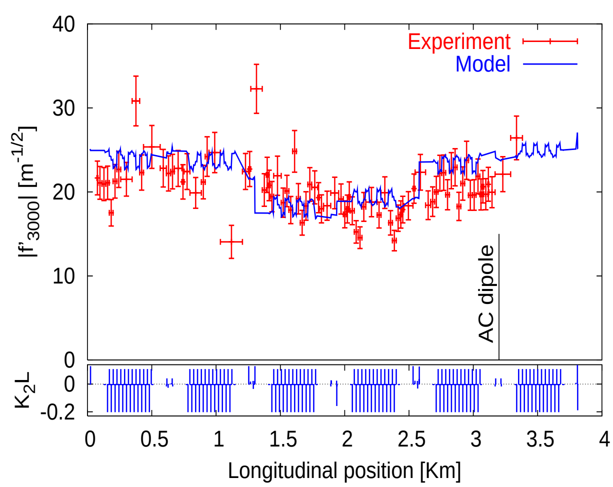


Figure 3: First measurement of resonance driving terms with an ac dipole.

**Linear coupling measurement** In an accelerator with non-zero linear coupling, the driven oscillation excited by an ac dipole in the horizontal plane gets coupled into the vertical plane and vice versa. The ratio of the driven oscillation amplitude between the two planes is determined by

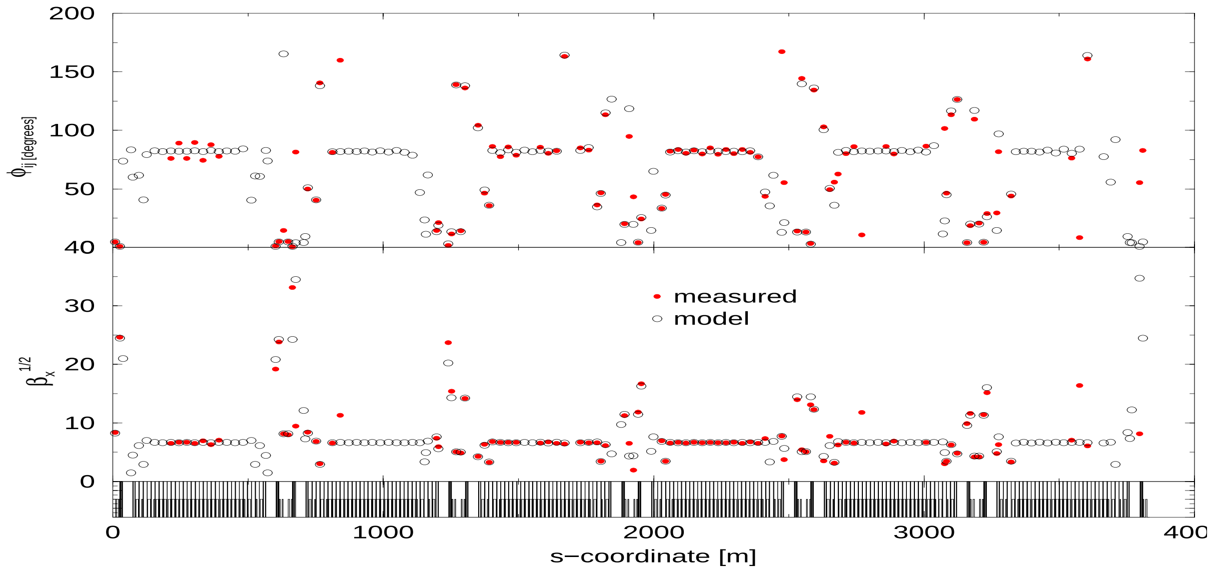


Figure 2: RHIC linear optics measured by the RHIC ac dipoles

the amount of coupling. In the case of weak linear coupling with two eigen tunes well separated, the driven coherences are given by [11]

$$x(s) = \gamma\frac{\sqrt{\beta_x}}{4\pi\delta_x}(\gamma\frac{B_m L}{B\rho}) \quad (4)$$

$$y(s) = \frac{\sqrt{\beta_y}}{4\pi\delta_x}(\gamma\frac{B_m L}{B\rho})(\bar{c}_{22} + \bar{c}_{12}\sin(Q_m\theta+\chi)). \quad (5)$$

with a horizontal ac dipole. Similarly, the driven oscillations in the two transverse planes with a vertical ac dipole is

$$y(s) = \gamma\frac{\sqrt{\beta_y}}{4\pi\delta_x}(\gamma\frac{B_m L}{B\rho}) \quad (6)$$

$$x(s) = \frac{\sqrt{\beta_x}}{4\pi\delta_x}(\gamma\frac{B_m L}{B\rho})(\bar{c}_{11} + \bar{c}_{12}\sin(Q_m\theta+\chi)). \quad (7)$$

Here $\gamma$ and $\bar{c}_{ij}$ with $i, j = 1, 2$ are the elements of the 4x4 matrix $R$ which transforms the $(x, x', y, y')$ to the decoupled coordinates $(u, u'v, v')$ [12]

Fig. 4 shows the measured coupling strength. The bottom plot is the measured ratio between the horizontal coherent oscillation amplitude and vertical coherent oscillation amplitude with the vertical ac dipole and the top plot shows the measured ratio between the vertical coherent oscillation amplitude and horizontal coherent oscillation amplitude with the horizontal ac dipole. Both measurements consistently show a linear dependence on the skew quadrupole family strength. Both data also show that the minimum of the measured coupling strength occurs at zero trim strength in skew quadrupole family 1. This is consistent with the fact that the coupling was well compensated at injection during normal operations.

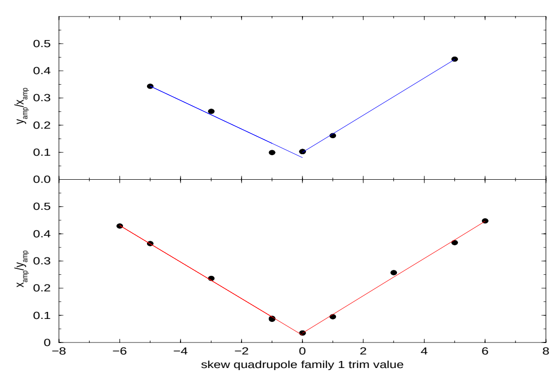


Figure 4: Linear coupling measurement with the RHIC ac dipoles.

## *AC quadrupole*

There have been rising interests in measuring the quadrupole mode transfer function for studying the beam envelope evolutions as well as the incoherent tune shift under the influence of space charge, intra-beam scattering and etc [13]. So far, the technique of measuring quadrupole mode transfer function either relys on the injection mismatch or uses a quadrupole kicker to induce a sizable quadrupole mode oscillation. For electron machines, because of the strong damping, the quadrupole mode oscillation due to either mismatch or a quadrupole kick often decays fast. Hence, a high frequency quadrupole operating at a frequency of two times the beam betatron frequency was successfully tested in the KEK Photon Factory storage ring to induce an quadrupole mode oscillation [14]. A bunch shape oscillation was confirmed with the beam profile measured by the streak camera.

The case of exciting quadrupole mode oscillation with an ac quadrupole in a hadron machine has been carefully

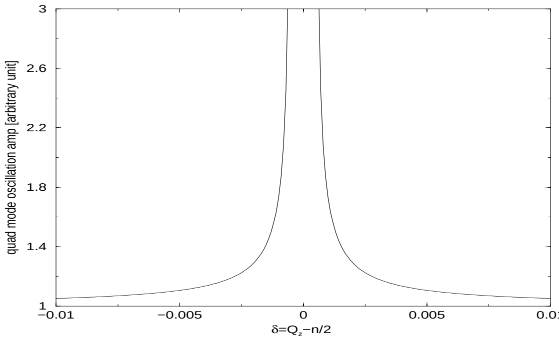


Figure 5: The amplitude of a driven quadrupole mode oscillation driven by an ac quadrupole as a function of the resonance proximity factor $\delta$.

studied by S. Y. Lee and Weiming Guo [15]. Their theoretical work shows that a stable quadruple mode oscillation can be achieved by driving the beam at a frequency nearby two times the betatron frequency. Fig. 5 shows the size of the quadrupole mode oscillation as a function of the resonant proximity factor $\delta = |\nu_z - \frac{1}{2}(n - \nu_m)|$.

The applications of the driven quadrupole mode oscillation are also studies in the same paper [15]. A simple example is to apply this technique together with a quadrupole pick-up to measure the beam emittance non-destructively. The quadrupole pick up, first proposed by R. H. Miller [16], measures the quadrupole moment $Q$ of the beam from the signals of the four electrodes: Eq. 9

$$Q = \frac{(R+L)-(U+D)}{L+R+U+D} \propto \sigma_x^2 - \sigma_y^2 + (\bar{x}^2 - \bar{y}^2). \quad (8)$$

Here, L, R, U and D stand for the signals from the four electrodes at left, right, up and down positions respectively. $\sigma_x$ and $\sigma_y$ are the horizontal and vertical rms beam sizes. $\bar{x}$ and $\bar{y}$ are the beam offsets in the horizontal and vertical planes. With the driven quadrupole mode oscillation, the quadrupole moment becomes

$$Q \propto \frac{1}{2}(Q_0 + Q_m sin(2\pi\nu_m n) \quad (9)$$

and $Q_m = \sigma_x^2 + \sigma_{P_x}^2 \propto \beta_x\epsilon_x$, and $\epsilon_x$ is the beam emittance. By mapping out $Q_m$, the beam emittance can then be calculated by measuring the quad-mode oscillation transfer function $Q_m$ [15].

### *Schottky*

Schottky detector is a typical example of passive non-destructive diagnostic tool. The Schottky spectrum is the statistical noise of the finite number of particles in a beam [17]. It was first observed in the CERN ISR [18, 19] with unbunched beam. Since then this technique has been widely adopted by the hadron accelerators to measure betatron tunes, momentum distributions etc.

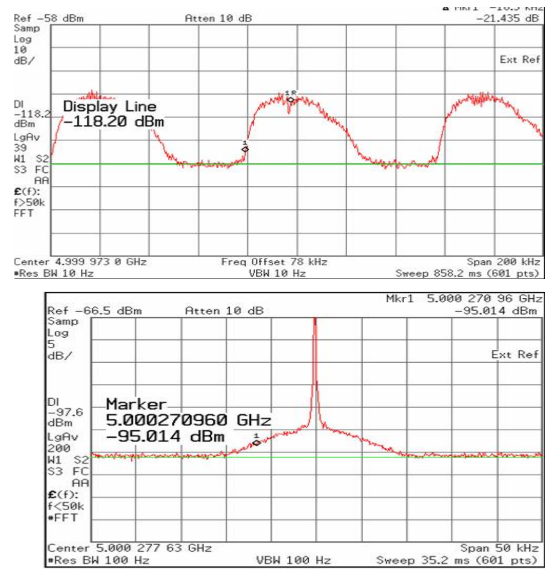


Figure 6: Longitudinal Schottky spectrum of Au beam and proton beam.

In general, there are two types of Schottky spectra. The longitudinal Schottky spectrum of an unbunched beam consists of spectral lines at the harmonics of the revolution frequency [17]. Becaue of the momentum spread across the beam, each spectral line has a finite width. For a bunched beam, because of the synchrotron oscillation, each revolution spectral line is then surrounded by sidebands at an interval of the synchrotron frequency. This technique of measuring the synchrotron frequency has been used in the Brookhaven RHIC for adjusting the phase of all the rf cavities [20]. Furthermore, the structure of the longitudinal Schottky spectrum also reflects the bunch's longitudinal distributions. Fig. 6 shows the Schottky spectrum of polarized proton as well as Au beam. The fact that the Au beam Schottky shows no coherent spikes is due of Intra Beam Scattering which damped out any longitudinal coherence. The proton spectrum, on the other hand, shows a strong coherent spike. Detail studies [21] showed that this coherent spike is due to the over-densed spots in the longitudinal phase space of proton beam which can last for a long time because of the lack of filamentation.

The Schottky spectrum of a particle's transverse betatron oscillation consists of pairs of spectral lines around each revolution harmonics. Each spectral line is located at $n \pm \nu_z$. Here, $n$ is the harmonic of the revolution frequency and $\nu_z$ is the betatron frequency of the beam. If the beam is not perfectly centered in the middle of the Schottky pickup, an additional spectral line at the revolution harmonics will also appear. Because of the tune spread across the beam, each corresponding spectral line of the Schottky spectrum of a beam has a width, which is given by

$$\Delta f = f_0((n \pm \nu_z)\eta \pm \Delta\nu_z). \quad (10)$$

Here, $f_0$ is the revolution frequency. The tune spread of

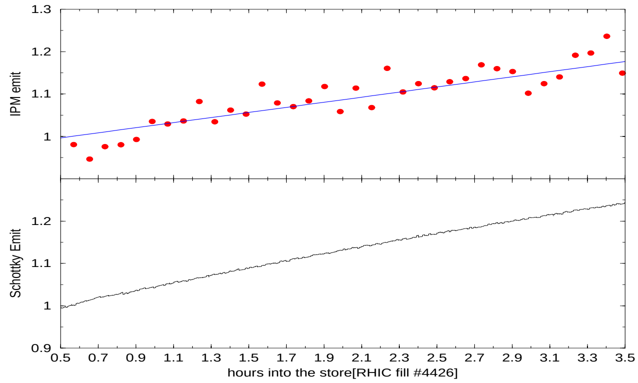


Figure 7: Horizontal emittance measured with RHIC IPM and RHIC HF Schottky In both cases, the measured emittance is normalized by the emittance measured at time $t = 0.5$ shours.

the beam can come from non-zero chromaticities, nonlinear effects as well as beam-beam effect. Furthermore, the power of each spectral line at $n \pm \nu_z$ is proportional to the rms size of the beam. The top plot of Fig. **??** shows the horizontal beam emittance in the yellow ring measured by the RHIC High Frequency Schottky as well as the RHIC IPM (Ionization Profile Monitor) during a typical store of Au beam. Both Schottky and IPM measurements show a growth of 16% of emittance within 3 hours due to Intra-Beam Scattering.

## CONCLUSION

Three examples of non-destructive beam measurements are discussed. The ac dipole technique has been successfully applied in the Brookhaven RHIC to measure the linear optics and couplings. The first non-linear driving term measurement using an ac dipole also yields very promising results.

The possibility of using an ac quadrupole to induce a quadrupole mode oscillation has been investigated by S. Y. Lee and Weiming Guo. The success of using an ac quadrupole to induce quadrupole mode oscillation in KEK also showsthat the ac quadrupole technique has a potential in beam measurements as well as beam manipulations.

The Schottky spectrum measurement has been widely adopted in the hadron accelerators ever since its first discovery in the CERN ISR. It has demonstrated to be a very powerful beam diagnostic tool for monitoring betatron tunes, emittances, etc in high energy hadron machines.

## ACKNOWLEDGMENT

The author would like to thank M. Blacksiwitz, M. Brennan, U. Iriso-Ariz, S. Y. Lee, W. W. Mackay, T. Roser, R. Tomas, K. Vetter, S. Y. Zhang and many others for the fruitfull discussions and stimulating suggestions. I would like especially to thank T. Roser, R. Tomas and W. W. Mackay for taking time to proof-read the paper.

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