ROTATING ELECROMAGNETIC FIELD TRAP FOR HIGH TEMPERATURE PLASMA AND CHARGE CONFINEMENT

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Abstract

This paper demonstrates that there exists a special combination of oscillating electromagnetic fields capable of trapping ultra high charge densities. Contrary to conventional electromagnetic traps, the motion in this dynamic trap is stable for arbitrarily high electromagnetic field amplitudes. This, in turn, leads to the possibility of using enormous electric and magnetic fields from RF or laser sources to confine dense ultrahigh temperature plasmas and particle beams.

INTRODUCTION

In this paper we consider primarily the linear stability of nonrelativistic particles. Consequently, we focus on electric fields having linear dependence on the coordinates (i.e., quadrupole fields) and on constant magnetic fields. There are combinations of electric quadrupole and constant magnetic fields for which trapped particle motion is stable – these correspond to the Penning trap fields (see, e.g., [1]). The fields, which obey stationary Maxwell equations, are:

$$\begin{aligned} &\mathrm{B_Z} = \Delta, \mathrm{B_X} = 0, \mathrm{B_Y} = 0, \\ &\mathrm{E_Z} = \mathrm{A}\cdot \mathrm{Z}, \mathrm{E_X} = -\mathrm{A}\cdot\mathrm{X}/2, \mathrm{E_Y} = -\mathrm{A}\cdot\mathrm{Y}/2, \end{aligned} \tag{1}$$

where A>0 gives the electric field gradient and Δ is the magnetic field. Now let e>0 be the modulus of the electron charge and M_e its mass. For nonrelativistic electrons the oscillation frequency in the z direction is $\omega_Z = \sqrt{e\mathrm{A}/\mathrm{M}_e}$. The squares of the remaining two frequencies are:

$$\omega^2 = \frac{e^2\Delta^2 - e\mathrm{AM}_e \pm \sqrt{(e^2\Delta^2 - e\mathrm{AM}_e)^2 - \mathrm{M}_e{}^2 e^2 \mathrm{A}^2}}{2\mathrm{M}_e{}^2}$$

In order for these to be real and positive, the following inequality must hold: $\Delta^2 > 2\mathrm{AM}_e / e$.

Field combination (1) is not the only one that provides stable motion. If the magnetic field is at an angle with the z axis, stable motion is still obtained when the magnetic field along the z axis is sufficiently strong. For example, if $e\Delta^2 / \mathrm{AM}_e = 3$ the maximum acceptable angle of the magnetic field with z axis is 38.4°.

The practical problem is that it is not possible to generate strong stationary electric fields. RF or laser electric fields are orders of magnitude larger than stationary fields. In the next section we will show how to combine oscillating electromagnetic fields, assuming zero stationary electric field, such that in some rotating frame the fields resemble the Penning fields of Eq. (1). The idea is motivated by electromagnetic traps based on alternating gradient focusing (e.g. Paul traps), but the principle of this trap is different in the sense that if the rotation frequency is much less than the particle oscillation frequencies, the Hamiltonian of the motion will be nearly time independent and the motion is stable in the limit of arbitrary large electromagnetic fields. This leads to the possibility to use compression of the electromagnetic field to achieve extraordinary plasma temperatures and densities (for alternating gradient traps particle frequencies of motion can not significantly exceed those of the oscillating fields – the corresponding Mathieu equation gives unstable solutions).

IROTATING ELECTROMAGNETIC FIELDS WITH STABLE MOTION

Our goal is to find a combination of oscillating fields (with zero static electric field) such that in some periodically rotating system the fields take the form of Eq. (1). We consider only leading terms in the Taylor expansion of the oscillating field near the centre of the trap and treat independently modes with uniform nonzero magnetic field (magnetic dipole modes) and linear electric field (electric quadrupole modes). If the rotation frequency is much smaller than the particle oscillation frequencies we will have fully stable motion (in a frame rotating with frequency ω, the centrifugal and the Coriolis force are of the order of $\mathrm{M}_e\omega^2 r$. These should be much smaller than the field force, with corresponding oscillation frequencies ω_i (i=1,2,3): $\mathrm{M}_e\omega^2 r << \mathrm{M}_e\omega_i{}^2 r$ or $\omega << \omega_i$.). In the nonrotating (laboratory) system, the quadrupole electric field can be written as $\vec{\mathrm{E}} = \mathrm{M}\cdot\vec{\mathrm{X}}$, where $\vec{\mathrm{X}} = \{\mathrm{X},\mathrm{Y},\mathrm{Z}\}$ is the coordinate vector and *M* is a matrix with Tr M=0 (as a consequence of $\nabla\cdot\vec{\mathrm{E}} = 0$). Also, for electric quadrupole modes, the magnetic field at the origin is equal to zero, and the matrix M is symmetric. This follows from $\nabla\times\vec{\mathrm{E}} = -\dfrac{1}{c}\dfrac{\partial\mathrm{H}}{\partial\tau} \approx 0$. Let O_3 be the time dependent matrix coupling the laboratory to the rotating coordinate system. The magnetic field vector $\vec{\mathrm{B}}_N$ in the rotating system is: $\vec{\mathrm{B}}_N = \mathrm{O}_3\vec{\mathrm{B}}$. The quadrupole electric field $\vec{\mathrm{E}}_\mathrm{N}$ in the rotating system is:

$$\vec{\mathrm{E}}_\mathrm{N} = \mathrm{O}_3\,\mathrm{M}\,\mathrm{O}_3{}^T\vec{\mathrm{X}}_\mathrm{N}, \tag{2}$$

where $\vec{\mathrm{X}}_\mathrm{N} = \{\mathrm{X_N},\mathrm{Y_N},\mathrm{Z_N}\}$ is the coordinate vector in the new rotated frame. In the rotating system the fields have to be of the (1) form:

$$\vec{B}_N = \begin{bmatrix}\Delta\\0\\0\end{bmatrix}, \vec{E}_N = M_1\cdot\vec{X}_N = A\begin{bmatrix}1&0&0\\0&-1/2&0\\0&0&-1/2\end{bmatrix}\begin{bmatrix}X_N\\Y_N\\Z_N\end{bmatrix}, \qquad (3)$$

where M_1 is the quadrupole field matrix in the rotated frame.

Now let us find a matrix O_3 such that the quadrupole field matrix M contains only elements that are periodic harmonic functions without constant terms:

$$M_{ij} = \sum_{k=1}^{l} m_{ijk}\sin(\omega_{ijk}\tau), \qquad (4)$$

where l is a finite integer, $m_{ijk} = m_{jik}$, and $\sum_{i=1}^{3} m_{iik} = 0$.

There are infinitely many solutions for the matrix O_3 that satisfy relations (3). Before explaining the general approach, we give one example:

$$O_3 = \begin{bmatrix}\frac{\sqrt{2}}{\sqrt{3}}\sin\omega\tau & \frac{\sqrt{2}}{\sqrt{3}}\cos\omega\tau & \frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}}\sin\omega\tau & \frac{1}{\sqrt{3}}\cos\omega\tau & -\frac{\sqrt{2}}{\sqrt{3}}\\ \cos\omega\tau & -\sin\omega\tau & 0\end{bmatrix}. \qquad (5)$$

From (2) and (3) one can calculate the matrix M of the electric quadrupole field in the laboratory frame:

$$M = O_3^T M_1 O_3 = A\begin{bmatrix}\frac{-\cos 2\omega\tau}{2} & \frac{\sin 2\omega\tau}{2} & \frac{\sin\omega\tau}{\sqrt{2}}\\ \frac{\sin 2\omega\tau}{2} & \frac{\cos 2\omega\tau}{2} & \frac{\cos\omega\tau}{\sqrt{2}}\\ \frac{\sin\omega\tau}{\sqrt{2}} & \frac{\cos\omega\tau}{\sqrt{2}} & 0\end{bmatrix} \qquad (6)$$

This means that in the lab frame the electric field is a combination of 4 quadrupole fields:

$$\begin{aligned}
&1.E_x = -\frac{AX}{2}\cos 2\omega\tau, E_y = \frac{AY}{2}\cos 2\omega\tau\\
&2.E_x = \frac{AY}{2}\sin 2\omega\tau, E_y = \frac{AX}{2}\sin 2\omega\tau\\
&3.E_x = \frac{AZ}{\sqrt{2}}\sin\omega\tau, E_z = \frac{AX}{\sqrt{2}}\sin\omega\tau\\
&4.E_y = \frac{AZ}{\sqrt{2}}\cos\omega\tau, E_z = \frac{AY}{\sqrt{2}}\cos\omega\tau
\end{aligned} \qquad (7)$$

The magnetic field is:

$$B = O_3^T B_N = \Delta\{\frac{\sqrt{2}}{\sqrt{3}}\sin\omega\tau, \frac{\sqrt{2}}{\sqrt{3}}\cos\omega\tau, \frac{1}{\sqrt{3}}\}. \qquad (8)$$

Therefore, the combination (7-8) leads to a rotating coordinate system in which the field orientation is of the type (1). The general relation between the matrices O_3 and M is:

$$M = O_3^T M_1 O_3 = \frac{3}{2}A\begin{bmatrix}O_{11}^2-1/3 & O_{12}O_{11} & O_{13}O_{11}\\ O_{12}O_{11} & O_{12}^2-1/3 & O_{13}O_{11}\\ O_{13}O_{11} & O_{13}O_{11} & O_{13}^2-1/3\end{bmatrix}. \qquad (9)$$

One can see that in order to have zero static electric field, the products of the first row of O_3 matrix elements should have time averages equal to 1/3 for the squared elements and zero for the cross terms. Additionally, the sum of these three matrix elements squared should be equal to 1 to satisfy the orthogonality of the matrix. Of course, there are an infinite number of periodic solutions for elements of O_3, since we have infinite number of harmonics and only few relations to satisfy. We limit ourselves with the example (5) as a proof of the existence of such dynamic systems and consider in the next section achievable electron densities.

ACHIEVABLE CHARGE DENSITIES

There are various ways to obtain fields (7-8). One possibility is to use open systems. This is applicable to RF and to laser fields as well. One needs to form a Hermite-Gaussian beam in an open resonator. Another option is to build a cavity with fields (7-8). Because the fields obey Maxwell equations, the creation of the fields is a matter of design and we concentrate on physically achievable densities with RF and with laser fields. First, we consider the well developed region of RF frequencies around 10 GHZ. We assume all 3 frequencies of the system (1) are of the order of the magnetic field rotation frequency. For the nonrelativistic electron the rotation frequency is:

$$\nu_e = eB/2\pi M_e \approx 2.86\cdot 10^{10}B(T). \qquad (10)$$

For a field of 1 Tesla, the oscillation frequency is about 3 times larger than 10 GHz. Magnetic field amplitude of 1 Tesla corresponds to an electric field of 300 MV/meter, which is about that achieved at a copper surface at this frequency. But here we must mention one of the most pronounced advantages of radiation fields: **contrary to a stationary magnetic field**, **the electromagnetic field density can be increased orders of magnitude in free space as compared to the density at the reflective surfaces if it is formed by choosing a special design for the optical system**. Figure 1 shows the trajectory of a particle in 10GHz fields with B=1T, electric field gradient $A = 2.25\cdot 10^{10} V/m$ and initial conditions x_0 = y_0 = 1mm, and the remaining coordinates equal to zero).

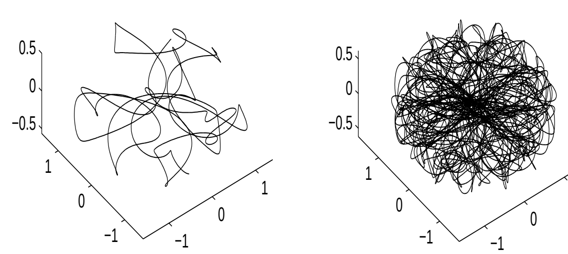

Figure 1 Sample trajectory for the rotating field trap

The plot shows the trajectory after 10 (left) and 100 (right) field oscillation periods. For this particular case the motion becomes unstable when the frequency of the magnetic field exceeds approximately 12.6 GHz.

Consider a sphere of electrons trapped in these fields. A conventional "accelerator" criterion of its stability is that the space charge tuneshift due to self fields should not exceed about 10% of the unperturbed single-particle frequency:

$$n_e \approx 0.2(2\pi \nu_e)^2 / r_e c^2 , \qquad (11)$$

where r_e is the classical electron radius. Equation (11) gives $n_e \approx 2.56\cdot 10^{11}$cm^{-3} for a moderate 1 Tesla dipole magnetic field of the form given in Eq. (8). This field has one static and two oscillating components. For this type of trap electromagnetic fields can be squeezed to obtain very large gradients –the system size, though, would be extremely large. However, the static magnetic component provides a limitation – its maximum achievable value is about 20 T (achieved for superconducting magnets). For this limiting value, Eq. (11) gives $n_e \approx 10^{14}$cm^{-3}. In fact, the static magnetic field is not necessary - it only enhances the region of stability in the parameter space (see explanations after (1)). If we take the radius of the electron sphere to be 6 cm, an electron cloud of density $n_e \approx 10^{14}$cm^{-3} would have an overall charge of about 1 coulomb. However, the size of the system would be of the order of 30 m in radius)! To estimate the size of the trap with some given gradient a we take Eqs (10-11). Having in mind that $z_0 >> \lambda$ we take $z_0 \approx 3\lambda$. The full size squared (equal to twice the rms size) of the radiation spot near the trap centre is $W_0^2 = (\lambda z_0/\pi)$. The maximum field at the rms size is $W_0 A/2$. The distance z of the mirror from the trap center is determined by the maximum achievable field Emax: $z = z_0 W_0 A/2E_{max}$. For example, one may assume that the achievable field for superconducting mirror is about 100 MV/m for 10 GHz. This gives z=16.9 m for the full size of the radiation spot at the mirror,5.5 meters. The mirror radius should be at least 3 times larger (i.e. 16.5 meters) to reduce field radiation from the system. In total we need 6 pairs of mirrors to generate the fields (7-8). This shows that the open system may be not an optimal choice for such a trap, but it also shows that there is, in principle, no physical obstacle to its construction.

Of course, for such sizes and intensities electrons become moderately relativistic, but we anticipate that the motion will be stable in this case also. Since the trap size should be less than the wavelength, one can estimate the maximum electron energy in such a trap by equating the rotation radius to approximately 10% of the wavelength. For moderate 1 Tesla 10 GHz magnetic field this yields:

$$\mathrm{E}(eV) = 0.1\mathrm{B}c\lambda \approx 10^6 eV = 1MeV \ . \qquad (12)$$

This corresponds to an electron temperature of about 10 billion K. For an ultimate field of the order of 20 T, the electrons with density $n_e \approx 10^{14}$cm^{-3} have ultra long confinement times, comparable to accelerator lifetimes of particles (minutes or hours).

When the oscillating frequency of the field is much smaller than the particle oscillation frequency, the motion is approximately that of a time independent Hamiltonian. For this case there exist self consistent equilibrium distributions for fully or partially nonneutral plasmas for specific values of total charge, total angular momentum and total energy (see, e.g., [2-3]). This, in turn, suggests enhanced stability and long confinement for fusion purposes. If we get long living electron cloud with big density, it will trap positive ions in its potential with ultrahigh temperature and densities, comparable to the electron ones. This is a topic for a separate treatment and is beyond subject of this paper.

Now we present a quick estimate for the **laser trap**. The fields in Eqs. (7-8), except for the static magnetic component, can be again generated in open systems. Without a static magnetic field the motion can still be stable, but the region of stability in parameter space is smaller (see next paragraph after (1)). In laser fields the laser beam should be squeezed to obtain an electron rotation frequency, ν_e larger than the laser frequency, ν_l: $\nu_e = e\mathrm{B}/2\pi \mathrm{M}_e \approx 2.86\cdot 10^{10}\mathrm{B(T)} \geq \nu_l$. As an example, for an NdYAG laser, we have $\nu_l \approx 310^{14} c^{-1}$, and this gives $\mathrm{B} \geq 10^4 T$. Assuming the laser beam spot size near the trap center is about the size of the laser wavelength, we get a laser instant power of $\mathrm{P} \geq 23\ GW$. Then Eq. (11) gives n ≈ $1.4\cdot 10^{22}$ cm^{-3} in this micro spheroid.

CONCLUSION

This paper presents a new concept of traps for high temperature, high intensity plasma and charge confinement. It is shown that with achievable RF fields it is possible to practically confine electron plasma with temperature 10 Billion K and density $n \approx 10^{14}$cm^{-3} with ultra long particles lifetimes.

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