INJECTION SCHEMES FOR SELF-CONSISTENT SPACE CHARGE DISTRIBUTIONS

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Abstract

This paper is based on recently found sets of self-consistent 2D and 3D time-dependent space charge distributions. A subset of these distributions can be injection-painted into an accumulator ring, such as Spallation Neutron Source Ring, to produce periodic space charge conditions. The periodic condition guarantees zero space-charge-induced halo growth and beam loss during injection. In addition, the distributions are uniform in density, which is a requirement for neutron production targets. Practical aspects of such schemes are discussed, and simulations of a few specific cases are presented.

INTRODUCTION

Limiting halo and rms size growth in rings is mostly matter of getting self-consistent distributions during injection. The charge-exchange injection of H^- ions offers great possibility to create compact distributions of a predetermined shape. This can be done by varying the closed orbit in the vertical and horizontal directions according to a special law during injection. If both the horizontal and vertical closed orbit displacements go proportionally from their maximum to zero, the injection painting is called correlated; another painting, with one (e.g. vertical) displacement decreasing and another (e.g. horizontal) increasing is called anti-correlated. The latter can be used to create a self-consistent (in our case, preserving linear force under any linear transformations) Kapchinsky-Vladimirsky (KV) distribution [1], but it requires special fast injection kickers to keep the distribution self-consistent at all stages of injection. If the anti-correlated painting is slow, it initially creates a narrow distribution that has a tendency to blow up for high intensity beams. Therefore, the gaol is to keep the distribution self-consistent at all moments of injection. Recently, new sets of 2D (and 3D) self-consistent distributions were found [2]. Some of them are perfect for painting – with relatively small modifications of the injection scheme we can create them, and, moreover, they stay self-consistent at all moments of time. We start here with the 2D case. The simplest 2D self-consistent distribution and associated painting are shown in Figure 1. The left subplot shows the conventional correlated painting. The particles stripped by the foil (depicted by the red right-corner spot), don't have angles while merging with the circulating beam. By executing betatron oscillations, they fill the square if space charge forces are not included. In the process of injection the closed orbit moves away from the foil (the direction shown by the black arrow), increasing the size of the square. If the space charge force is strong, the square dilutes, producing tails and rms size growth. Now, instead of this situation, imagine we have equal betatron tunes and circular modes. We inject particles with the angle and coordinate offset from the closed orbit such that its trace exactly follows the circular mode motion (Fig. 1, right). If, in addition, the speed of closed orbit motion is arranged in such a way that the particles fill the rotating disk uniformly, the force is linear and the distribution is time-independent (except for growing monotonically in size). Moreover, it is valid at all moments of injection.

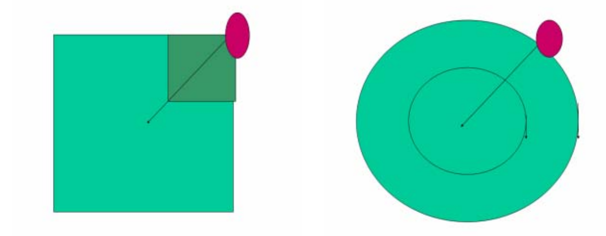

Figure 1: Conventional injection (left) and injection with an angle (right) to a circular mode.

If now we replace the circular modes by arbitrary elliptical modes in the presence of x-y coupling, we get a general relation of coordinates and angles of the linac beam at the foil relative to the closed orbit. Uniformly filled elliptical modes also represent a self-consistent case [2]. In general, we consider the distributions are well-suited for painting (with the above circular mode distribution as one of them), when they satisfy the following condition: **they are self-consistent and periodic (taking into account space charge force) at every moment of injection.** **(1)**

In the next section we present the realistic SNS ring structure with x-y coupling and a solution for kicker strengths for the present SNS configuration for this type of injection. We simulate the injection, including the influence of nonlinear perturbations on the distributions. In Section III we present new results and ideas on 3D self-consistent distributions for ring beams, the ultimate goal of this investigation. Section IV summarizes the results.

SNS RING IMPLEMENTATION

The SNS ring has 4-fold symmetry [2]. Each period has a bending arc and a straight section. The design tunes are 6.23 and 6.20 for horizontal and vertical motion, respectively. Since the tunes are close, we utilize the difference resonance to get circular modes by simply equalizing the tunes by a small adjustment of the quads and introducing solenoids in one of the straight sections. The straight section consists of two doublets, one drift for the RF cavities, and two empty drifts between doublets and arcs, each 6.85 meters long. We place 0.5 meter long, 1.41 T solenoids symmetrically in these two empty drifts at the point where vertical and horizontal beta functions are equal. The choice of the field is related to the tune split – our intent is to split the tunes of the circular modes to avoid nonlinearities influencing the motion. Figure 2 shows the simulation, performed by ORBIT [3] code, with the real SNS linear lattice with and without solenoids (to determine their importance), and including fringe field nonlinearities, and sextupoles needed to eliminate chromaticity of the betatron tunes. The particles are injected straight into one of the elliptical modes – the coordinates are determined by the foil position, and the angles are taken from the real parts eigenvectors of the revolution matrix at the foil. The upper plots in Fig.2 show the distribution at the end of injection without (left) and with (right) solenoids for low intensity beam. One can see that with solenoids the modes are almost unperturbed by nonlinearities and without solenoid at the difference resonance the motion becomes essentially nonlinear and the distribution is less controllable. The lower plots in Fig.2 show the same cases, but with an SNS coasting beam intensity of $3*10^{14}$ protons. One can see that with and without solenoid the distributions look elliptic and without much halo. The space charge introduces a stabilizing effect on the difference resonance.

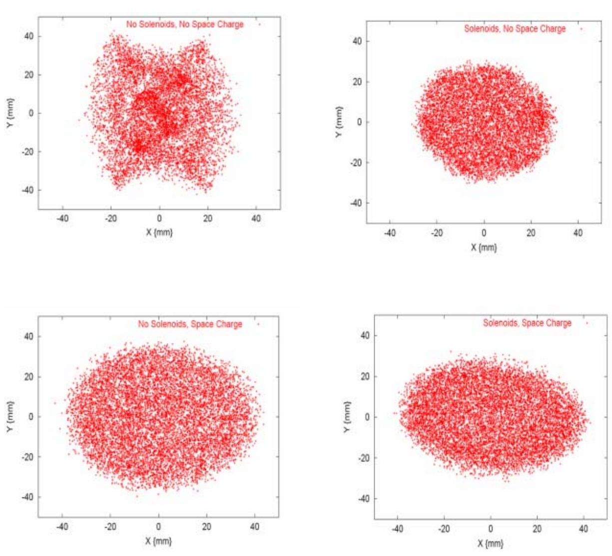

Figure 2: Results of painting into elliptical eigenmode with linearly increasing emittance. Top figures are without space charge and bottom figures include space charge. Left hand plots omit solenoids and right hand plots include solenoids in lattice.

We also found the injection kicker configurations for circular mode injection with angles. It can be performed with the present configuration of eight SNS kickers. All kicker strengths are within specified limits (about 12 mrad deflection per kicker). Figure 3 shows the closed orbit at the beginning (top plot), and at the end (lower plot) of injection. The foil is positioned approximately at the centre of the closed orbit excursion (12.5 meters from the start). One can see that at the beginning, when the closed orbit is near the foil, its angle is zero. At the end, both horizontal and vertical angles are large and have opposite signs. In the normal injection the orbit always has zero injection angle.

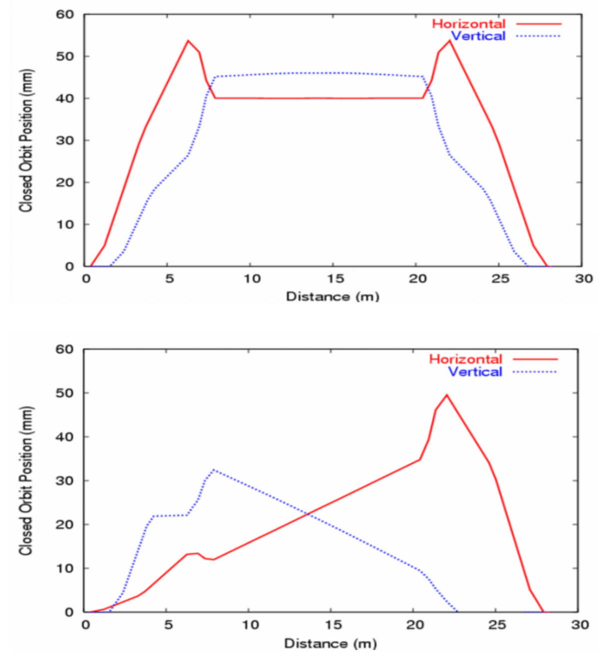

Figure 3: Closed orbit behavior during injection. Upper plots show the beginning and lower- the end of injection.

3D SELF-CONSISTENT RING DISTRIBUTIONS

In [4], three types of 3D self-consistent distributions which preserve the linearity of space charge forces (no vacuum chamber shielding is taken into account) under all linear transformations of phase space, were found:

a) $$f = \rho\delta(X' - e_{xx}X - e_{xy}Y - e_{xz}Z)\times \delta(Y' - e_{yx}X - e_{yy}Y - e_{yz}Z)\delta(Z' - e_{zx}X - e_{zy}Y - e_{zz}Z),$$

b) $$f = \frac{\rho\sqrt{d'_x + d'_y e'^2_{yx} + d'_z e'^2_{zx}}}{\pi\sqrt{2(H_b - H)}}\times \delta(Y' - e'_{yx}X' - e_{yx}X - e_{yy}Y - e_{yz}Z)\times \delta(Z' - e'_{zx}X' - e_{zx}X - e_{zy}Y - e_{zz}Z) \quad (2)$$

c) $$f = \frac{\rho\sqrt{d'_x d'_y + d'_z(d'_x e'^2_{zy} + d'_y e'^2_{zx})}}{\pi}\delta(2(H_b - H))\times \delta(Z' - e'_{zx}X' - e'_{zy}Y' - e_{zx}X - e_{zy}Y - e_{zz}Z),$$

where the e_{ij} and e'_{ij} are the arbitrary constants, H_b is positive number which determines the elliptical boundary of the beam and $H = \frac{1}{2}(d'_x X'^2 + d_x X^2 + d'_y Y'^2 + d_y Y^2 + d'_z Z'^2 + d_z Z^2) \le H_b$, (with arbitrary positive constants d'_i) is the positive quadratic form of the beam initial coordinates *X, Y, Z* and angles *X',Y',Z'*.

The distributions (2) are valid for linacs, where the bunch length is shorter than the vacuum chamber radius and the shielding effect is small. For rings, there are 3 basic effects, which make the space charge physics different from linacs:

1) the vacuum chamber shielding significantly modifies the longitudinal space charge force;

2) the closed orbits of particles depends on the energy offset;

3) the chromatic tune spread can be larger (and/or more important) than the space charge tuneshift.

The third item in the list, namely, combining chromatic and space charge effects, is not a resolved issue, therefore we assume below that the chromaticity of tunes is absent. But the first two issues can be taken care of!

First, if we have an elliptical 3D beam with a length much longer than the vacuum chamber diameter, the longitudinal force is proportional to the linear density derivative with respect to longitudinal coordinate. For a uniformly charged long ellipsoid, the linear density is proportional to the transverse cross-section *S* of the beam. For the ellipsoid with axes which coincide with *x,y,z* we have $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{S}{\pi ab} + \frac{z^2}{c^2} = 1$ or $S = \pi ab(1 - \frac{z^2}{c^2})$. One can see that the transverse area and the linear density of the beam is a quadratic function of the longitudinal coordinate *z*, and after differentiation it yields a linear longitudinal force. We have (for a beam radius much smaller than the vacuum chamber) the transverse force as the 2D linear force of a uniformly charged ellipse and **a linear longitudinal force.**

Second, the dispersion function may significantly change the space charge dynamics if the transverse size, associated with the energy spread, is comparable to the betatron beam size. This issue has been raised and equations for the transverse envelope in presence of dispersion were obtained in [5]. For our application, it is important to know if the distributions (2) are still self-consistent, when the (e.g.) horizontal *x* coordinate is determined by:

$$x = x_b + \eta \frac{\delta p}{p}, \tag{3}$$

where x_b is the coordinate of the betatron oscillation, η is the dispersion function (in principle, arbitrary function of time), and $\delta p/p$ is the relative longitudinal momentum offset from the reference momentum. Because $\delta p/p$ and *x* are variables of a unified 6D phase space, and the transformation (3) is linear, all distributions (2) will preserve the linear force if the beam enters a region with nonzero dispersion. Therefore, **for zero chromaticity**, **distributions (2) remain self-consistent even for rings!**

As for the injection into distributions (2), this is not a finished subject. The problem is to find methods of injection that satisfy the conditions in (1). We are currently working on effective methods to create 3D self-consistent distributions.

CONCLUSION

This paper demonstrates that a simple scheme exists to injection-paint the beam into two- dimensional self-consistent distributions with linear space-charge forces. It is shown that it is possible in the present SNS configuration. 3D ring distributions were also discussed.

ACKNOWLEDGEMENT

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