UAL IMPLEMENTATION OF STRING SPACE CHARGE FORMALISM

Nikolay Malitsky, Brookhaven National Laboratory, and
Richard Talman, Laboratory of Elementary-Particle Physics,
Cornell University

Abstract

The "string space charge formalism" is a computational procedure that models the effect of space charge forces as direct intrabeam scattering, thereby avoiding the need for a particle-in-cell intermediate field determination. Individual particles are treated as points for their own dynamics but as uniformly charged longitudinal strings for purposes of calculating their influence on other particles. This procedure ameliorates the erratic behavior associated with close encounters in the presence of the Coulomb inverse square law dependence by reducing the singularity to be only logarithmic. This paper describes the implementation of this formalism within the Unified Accelerator Libraries (UAL) with emphasis on evaluating the emittance growth suffered by an intense bunch in a magnetic field.

1 FUNDAMENTAL SPACE CHARGE FORMALISM

While in a magnetic field a uniformly-charged longitudinally-aligned needle, or string, forms a circular arc of length $2L$ traveling along a radius R circular path, as shown in Fig. (1). The line charge density is $\lambda = q/(2L)$, the speed is $\beta = v/c$, and the corresponding current is $\beta c \lambda$, within the string, and zero otherwise. The length $2L$ can initially be regarded as arbitrary (though small compared to the bunch length of the charged bunch being analysed).

There are two major tasks in a fully-relativistic calculation of the force acting on a co-moving point charge q at point P, due to the charged string:

1. The electromagnetic field components at point P at, say, time $t = 0$, reflect not the instantaneous charge density at that time (which is indicated by an open curved box in Fig. (1) but rather the retarded time or or "effective" charge distribution, shown as a curved arc in the figure. It is necessary to find the angles $\overline{\alpha'}$ and $\underline{\alpha'}$ of the effective head and tail of the string. The equation determining the head is

$$\frac{R\overline{\alpha'} + s_t - L}{\beta} =$$
$$-\sqrt{2R^2\left(1 + \frac{x}{R}\right)(1 - \cos\overline{\alpha'}) + x^2 + y^2}, \quad (1)$$

where the symbols are defined in the figure. In particular, s_t is the londitudinal displacement of the "test point" P from the bunch center. The tail equation is similar.

2. In the string formalism both electric and magnetic fields are subsumed into a fundamental charged-string/point-charge force using the Lorentz force formula. The magnetic field is given by the "Jefimenko equation":

$$\mathbf{B}(P, t) = \frac{\mu_0}{4\pi} \int \left[\frac{\mathbf{I}(z', t_r)}{r^2} + \frac{\dot{\mathbf{I}}(z', t_r)}{cr}\right] \times \hat{\mathbf{r}}\, dz'. \quad (2)$$

The first term is just the Biot-Savart law, though applied to the retarded distribution. This will be referred to as the "body" term. The second term vanishes everwhere except at the string ends, where it gives δ-function contributions that will be referred to as "end" terms. There is a similar equation for the electric field and a similar division into Coulomb's law, body terms, and end effect terms. After finding the effective string ends, the second computational task is to evaluate all these terms. The extreme near-cancellation of electric and magnetic forces makes it necessary to handle the cancellation explicitly, which is why the formalism is restricted to give just the total force vector.

2 PROGRAM ARCHITECTURE

Assuming $\gamma >> 1$, the present report describes a restricted application that calculates the emittance growth of a dense electron bunch making a single pass around a more or less circular accelerator lattice. Space charge forces are neglected everywhere except within bending magnets, the only regions where coherent synchrotron radiation (CSR) and centrifugal space charge forces (CSCF) are important.

In a single particle simulation code a particle is deflected at each lattice element. In the intrabeam scattering code being described, a particle suffers a further deflection due to every other charge in the bunch, when, and only when, it is in a bending magnet. For a bunch containing N particles this requires N^2 calculations at each bending magnet. Computation time imposes a practical upper limit, perhaps $N < 10^4$. By introducing a grid of deflections this could be reduced to scaling as N. This would be appropriate for multiturn simulation, but seems unnecessary for calculating the emittance growth in one turn. As well as depending on the relative particle positions, the deflections depend on the bend radius and are proportional to the magnet length.
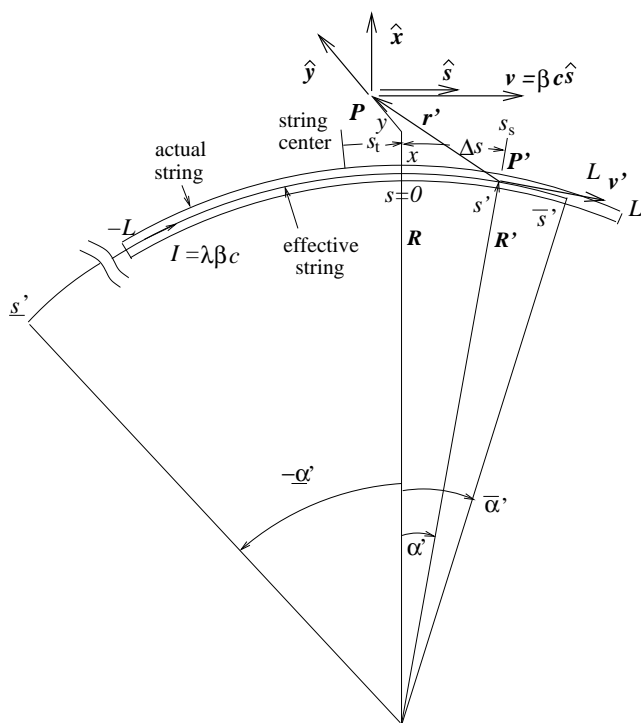
Figure 1: Definition of coordinates. A snapshot of the present, actual, charge distribution is shown and, superimposed on it is an electro/magneto-static configuration "equivalent" to the "true" electrodynamic system of a moving charged string. There are moving charges in just those locations of charges in the true system that, at some earlier time $t < 0$ contribute to fields at P at $t = 0$.

With the sum of all space charge forces on a particle ascribed to the bending magnet, particle evolution proceeds as in a single particle tracking code.

The structure of UAL has been described elsewhere.[2] Suffice it to say, here, that UAL provides a modularized framework, based on C++, for merging bunch processing methods encapsulated in various libraries. For the current task the basic libraries being used are TEAPOT[3] for conventional tracking, TIBETAN[4] for RF cavities, ACCSIM[5] for bunch generation and analysis and GSL[7] (the GNU Scientific Library), for equation solving and special function evaluation. The only new class necessitated by space charge is StringSCSolver whose methods include (numerical) retarded time equation solving and (analytical) definite integral evaluation.

The general code architecture can be inferred from the APDF (Accelerator Propagator Description Format) file shown next:

```
<apdf>
 <propagator id="stringsc" accelerator="ring">
  <create>
   <link algorithm="TEAPOT::DriftTracker"
                    types="Default" />
   <link algorithm="TEAPOT::DriftTracker"
                 types="Marker|Drift" />
```

```
   <link algorithm="TEAPOT::DipoleTracker"
                          types="Sbend" />
   <link algorithm="TEAPOT::MltTracker"
    types="Quadrupole|Sextupole|[VH]kicker"/>
   <link algorithm="TIBETAN::RfCavityTracker"
       types="RfCavity"/>
   <link algorithm="TEAPOT::StringSCKick"
       types="Kicker"/>
  </create>
 </propagator>
</apdf>
```

This XML file associates propagation algorithms with accelerator elements. Since this file is largely self-explanatory it should need only partial explanation. Most lattice elements are treated as they would be in the absence of space charge. Space charge kicks are associated with thin *kicker* elements located at the centers of every bending magnet. These artificial elements have to have been inserted into the original MAD[8] lattice description file. (Eventually an entirely new string space charge element type should be introduced to avoid this cannibalization of element type.) For elements of this type the APDF line containing TEAPOT::StringSCKick delegates all string calculations to StringSCSolver methods. Though the APDF file does not show it, the ACCSIM library is used for preprocessing (bunch generation) and postprocessing (bunch analysis).

All code mentioned so far is C++. But UAL also provides a "user friendly" interface to provide survey, lattice function evaluation, maps, tune and chromaticity tuning. The language of this interface is PERL. It has proved to a good productivity match to perform complicated (but routine) lattice calculations using the PERL interface while performing "new physics" calculations and debugging using C++.

3 NUMERICAL PROCEDURES

As previously mentioned, the numerical algorithms needed are available in the GNU Scientific Library.[7].

To find the effective (i.e. retarded) angle of the head of a source string from the position of a test particle, it is necessary to solve Eq. (1) for $\overline{\alpha'}$. Though this equation looks fairly simple, its coefficients are so close to irregular points (i.e. $\beta \approx 1$, $\alpha' \approx 0$, and $x, y << R$) that solution is not simple. A procedure that has been found to be robust in all cases tried is to start by finding a coarse solution using the root-bracketing procedure gsl_root_fsolver_brent which uses the so-called *Brent-Decker* method. From the physics there is certain to be just one solution of the equation and this method, though slow, is guaranteed to find it (approximately). It has not been investigated whether it is economical to iterate this method to a sufficiently accurate result. Rather, iteration is stopped when the relative change in $\overline{\alpha'}$ is less than 10^{-4}. Then the method gsl_root_fdfsolver_steffenson, employs the *Steffenson* method to "polish" the result to arbitrarily high accuracy. This method, which uses derivatives,

is not guaranteed to converge to the correct root, but we have observed no anomalous behavior. The same sequence of methods works for finding the effective tail angle $\underline{\alpha}'$.

The components of force on the test particle due to a source particles are known analytic functions of the head and tail angles. For example, the (fully-relativistic) horizontal force component F_{0x} is given by

$$\frac{R\lambda}{4\pi\epsilon_0} \int_{\underline{\alpha}'}^{\overline{\alpha}'} d\alpha' \frac{(2R+x)(1-\cos\alpha')+x}{r'^3}, \qquad (3)$$

where r' is distance from source point to test point.

The various terms of this integral are expressible in terms of *elliptic integral* special functions, given in the GNU scientific library as `gsl_sf_ellint_E` and `gsl_sf_ellint_F`. All other integrals, for both transverse and longitudinal forces, are similarly expressible, even in the non-relativistic regime where γ is of order 1.

It is the near vanishing of denominators in integrals like (3) that make their evaluation delicate. In fact, if the range of integration in (3) includes the origin, especially for the longitudinal force component, it is necessary to split the range and to exclude an "infinitesimal" range centered on the origin. Though the integrand is singular and discontinuous at the origin, the contribution from the excluded range cancels by symmetry. The end contributions are relatively simple (though also nearly singular) trigonometric functions of the end angles.

4 LIMITATIONS

The formalism that has been described has various limitations. The restriction to $\gamma >> 1$ is (superficially) not very serious, as fully-relativistic formulas are available. But it is only in the fully-relativistic regime where it is legitimate to neglect space charge forces everywhere except in bending magnets. Here it has been assumed, when calculating the space charge force on a particle in a magnet, that the particle has *always* been inside the same magnet field. But, for a particle that has just entered a magnet, the retarded time calculation should segment the effective charge distribution into a straight line segment (just outside the edge) and a curved section (just inside). For a bending magnet of length, say, 1 m and a string length L much less than say, 1 mm, this concern may seem far-fetched but, in fact, the relativistic dilation factor makes the problem of magnet entrance (though not magnet exit) quite complicated. For trajectory segments inside quadrupoles the complications are even worse, at least in principle.

Fortunately the difficulties mentioned in the previous paragraph do not prevent the estimation of emittance growth for some configurations of current interest. The beam line required to bend a short and intense electron beam through an angle that is some substantial fraction of 2π is made up mainly of bending magnets. Neglecting the effect of interposed quadrupoles, the assumption that the particles are always in the same magnet field is reasonably

good, especially since only a coarse estimate of a coarse parameter (emittance) is required.

Another serious complication is the effective reduction of coherent synchrotron radiation due to the "shielding" effect of the conductive beam tube. This effect, first accurately calculated by Schwinger[9], is known to suppress the long wavelength components of CSR. This would tend to reduce emittance growth but, since the fractional energy content at long wavelengths is relatively small, the suppression may be insignificant.

In the string space charge model the importance of beam-wall effects can be estimated by accounting only for the presence of the inner wall of the vacuum chamber. After finding the tail angle $\underline{\alpha}'$, the line joining source point and field point can be checked to see whether it misses the inner chamber wall. If the line misses the chamber wall then the formulas derived so far apply. Otherwise the effective charge distribution is "cut off" at a point determined by a tangency condition, effectively bringing $\underline{\alpha}'$ closer to the origin. This (straightforward) calculation has not yet been attempted. An optimist entertains the hope that, as well as accounting for the leading vacuum chamber effect, much of the uncertainty associated with magnet entry will also be "cut off" by the inner chamber wall.

The various delicate issues that have been mentioned require investigations not yet completed. Also investigations of dependencies on (artificial) string half-length L are required. For these reasons the present paper includes no numerical results.

5 REFERENCES

[1] R. Talman, *String Formulation of Space Charge Forces in a Deflecting Bunch.* Tentatively accepted for publication in PRSTAB.

[2] N. Malitsky and R. Talman, *The Framework of Unified Accelerator Libraries,* ICAP98, Monterey (1998), and *UAL User Guide*, BNL-71010-2003, and http://www.ual.bnl.gov

[3] L. Schachinger and R. Talman. *Teapot: A Thin-Element Accelerator Program for Optics and Tracking,* Particle Accelerators, **22**, 35 (1987)

[4] J. Wei, *Longitudinal Dynamics of the Non-Adiabatic Regime on Alternating-Gradient Synchrotrons,* SUNY PhD thesis (1990)

[5] F. Jones, *User's Guide to ACCSIM, 1990* and http://www.triumf.ca/compserv/accsim.html

[6] N. Malitsky, T. Satogata and R. Talman, *Configurable, UAL-based, Modeling Engine for Online Simulation*

[7] M. Galassi et al, *GNU Scientific Library Reference Manual,* Network Theory Limited, 15 Royal Park, Clifton, Bristol BS8 3AL, England

[8] D.C.Carey and F.C.Iselin. *Standard Input Language for Particle Beam and Accelerator Computer Programs,* Snowmass, Colorado, 1984

[9] J. Schwinger, *On Radiation by Electrons in a Betatron,* 1945, transcribed by MA Furman, July 29, 1996 (LBNL-39088/CBP Note-179)