MULTI-PASS BEAM-BREAKUP: THEORY AND CALCULATION*

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Abstract

Multi-pass, multi-bunch beam-breakup has been long known to be a potential limiting factor for the current in linac-based recirculating accelerators. New understanding of theoretical and computational aspects of the phenomenon are presented here. We also describe a detailed simulation study of BBU in the proposed 5 GeV Energy Recovery Linac light source at Cornell University.

INTRODUCTION

Several laboratories worldwide have proposed high power energy recovery linacs (ERLs) for different purposes [1]. One important limitation to the current that can be stored in such an accelerator is given by the beam-breakup (BBU) instability. In this kind of instability, the recirculating arc of the machine closes a feed-back loop, which translates a kick from a dipole-like higher order mode (HOM) in a RF cavity to a displacement of the bunch in the same cavity on the successive pass. When the energy deposited into the HOM by the offset beam exceeds the mode's damping, an exponential increase in beam induced voltage of the HOM develops, and beam loss occurs. This is particularly relevant for ERLs that use superconducting RF cavities because of the relatively high Q's of their HOMs. The size and cost of all these new accelerators certainly requires a very detailed understanding of this limitation.

Here we summarize some recent theoretical and computational advances made on this front. An extended theory of BBU which is applicable to ERLs has been presented elsewhere [2], and we present an overview that emphasizes its most salient findings. In particular, the theory treats time-dependence of HOM voltage in the cavities in explicit manner and, thus, can be used in case of arbitrary recirculation path length and corresponding bunch filling patterns. Excellent agreement of the theory with simulation results is demonstrated. Furthermore, a new and fast code capable of simulating both transverse and longitudinal BBU has been implemented. We have used this code to carry out detailed simulation studies of a 5 GeV ERL light source design, and the results of that study are summarized herein. We believe that our findings presented in this paper will be relevant to all those projects that aspire to achieve high average current ERLs.

THEORETICAL ASPECTS

A theory of BBU instability in recirculating linacs, where the energy is not recovered in the linac, but where energy is added to the beam when it returns after a recirculation time t_r in the linac, was presented in [3]. This original theory was additionally restricted to scenarios where the bunches of the different turns are in the same RF bucket, such as in the so-called continuous wave (CW) operation where every bucket is filled and all bunches are in the linac at about the same accelerating RF phase. Tracking simulations [4] compared well with this theory. In the following we therefore refer to it as the CW recirculator BBU theory. It determines the threshold current $I_{\rm th}$ above which the transverse bunch position x displays undamped oscillations in the presence of a HOM with frequency ω_λ. For $T_{12}\sin\omega_\lambda t_r < 0$, the following formula is obtained:

$$I_{\rm th} = -\frac{2c^2}{e(\frac{R}{Q})_\lambda Q_\lambda \omega_\lambda}\frac{1}{T_{12}\sin\omega_\lambda t_r}\,, \qquad (1)$$

where c is the speed of light, T_{12} is the element of the transport matrix that relates initial transverse momentum p_x before and x after the recirculation loop, e is the elementary charge, $(R/Q)_\lambda Q_\lambda$ is the impedance (here in units of Ω) of the HOM driving the instability, Q_λ is its quality factor.

To illustrate the approach taken in [2], consider the integral equation for the effective voltage $V(t)$ of a single dipole HOM in the case of one recirculation case:

$$V(t) = \int_{-\infty}^{t} W(t-t')I(t')T_{12}\frac{e}{c}V(t'-t_r)dt'\,. \qquad (2)$$

Here $W(\tau)$ is wake function of the mode. This equation relates the HOM's effective voltage to its value at previous times by using the mapping $x(t+t_r) = T_{12}p_x(t)$ for transverse offset $x(t)$ and momentum $p_x(t)$.

By treating the beam current at the injection point as a sum of delta functions representing individual bunches with interval t_b between them, so that the current on the second turn is given by $I(t) = I_0 t_b \sum_{m=-\infty}^{\infty}\delta_D(t-t_r-mt_b)$, the integral in Eq. (2) is reduced to a sum and the HOM voltage at a time $t = nt_b + t_r$ is given by

$$V(nt_b+t_r) = I_0 t_b T_{12}\frac{e}{c}\sum_{m=0}^{\infty}W(mt_b)V([n-m]t_b)\,. \qquad (3)$$

It is at this point that our treatment differs from previous works. Rather than writing voltage in the form $V(t) = V_0 e^{-i\omega t}$ for $t = nt_b$, an approach which is natural for CW recirculator configuration where bunches of different turns are in the same RF bucket, we proceed by writing $V(t)$ in terms of its Laplace transform, retaining all possible frequencies in HOM voltage, which automatically enables proper description of an arbitrary recirculating configuration. We refer the reader to [2] for more details on

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further development of the theory. We point out some of its features here:

1) Dispersion relation for arbitrary recirculation path length (c.f. [5]) for the simple case of one HOM, one recirculation, is given by

$$\begin{aligned} I_0 &= \frac{2}{\mathcal{K}T_{12}} e^{-i\omega n_r t_b} \\ &\times \frac{e^{\frac{\omega_\lambda}{2Q_\lambda}\delta t_b}[\cos(\omega^+ t_b) - \cos(\omega_\lambda t_b)]}{e^{-i\omega^+ t_b}\sin(\delta\omega_\lambda t_b) - \sin([\delta - 1]\omega_\lambda t_b)} , \end{aligned}$$

with $\mathcal{K} = t_b \frac{e}{c^2}(\frac{R}{Q})_\lambda \frac{\omega_\lambda^2}{2}$, an integer n_r and $\delta \in [0,1)$ are given by relation $t_r = (n_r - \delta)t_b$, and $\omega^+ = \omega + i\frac{\omega_\lambda}{2Q_\lambda}$, where ω, which appears in the Laplace transform, can be interpreted as the frequency of the instability. As usual, the threshold current $I_{\rm th}$ is the smallest real current I_0 for which there is a real ω that satisfies the dispersion relation. Fig. 1 shows a comparison between threshold current obtained by tracking and numerically solving this dispersion relation.

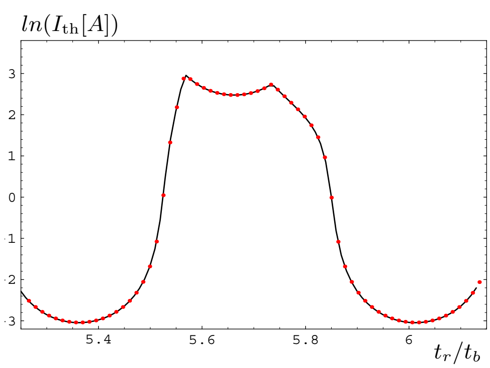

Figure 1: Threshold current obtained by tracking (dots) and by a numerical solution of the dispersion relation. Parameters: $n_r - \delta \in [6.135, 7.234]$, $(R/Q)_\lambda = 100\,\Omega$, $Q_\lambda = 10^4$, $T_{12} = -10^{-6}$ eV/c, $\omega_\lambda t_b = 9.67$.

2) The following approximate solution for the threshold current was obtained for the case when $\epsilon = \frac{\omega_\lambda}{2Q_\lambda}t_b$, $\epsilon \ll 1$, which describes a situation when HOM decay is negligible on the time scale of the bunch spacing t_b:

$$I_0 = -\frac{\epsilon}{\mathcal{K}}\frac{2}{T_{12}\sin\omega t_r} , \qquad (4)$$

with ω being a solution of equation $\Delta\omega t_b \sin\omega t_r = \epsilon\cos\omega t_r$ that is closest to $\omega_{\lambda,n_\pm} = \pm\omega_\lambda + n\frac{2\pi}{t_b}$ and at the same time produces a positive value of I_0 in Eq. (4). Here $\Delta\omega$ is given by $\Delta\omega = \omega - \omega_{\lambda,n_\pm}$.

3) Analysis of two HOMs and one recirculation demonstrated that for this case the threshold is given by the smaller value of the two modes as if the worse mode of the two was solely present. The interference of the two modes with the frequencies ω_1 and ω_2 occurs only when $\pm\omega_1 \bmod 2\pi/t_b$ and $\pm\omega_2 \bmod 2\pi/t_b$ are closer together than about $\Delta\omega_\lambda = \epsilon/t_b = \omega_\lambda/2Q_\lambda$.

Additional points covered in [2] include cavity misalignment treatment, BBU scaling in a multiple turn recirculator, estimates of rise time of the instability when beam current exceeds the threshold. It also contains generalization of BBU theory applicable to ERLs for arbitrary number of HOMs and recirculations, the result first obtained in [5].

CALCULATION ASPECTS

The tracking code `BI` (stands for beam instability) was developed to perform studies of beam breakup in recirculating linacs [6]. The algorithm models point charge bunch interactions with HOMs in linacs, taking into account proper time delays between the cavities, transfer maps, etc., allowing BBU simulations due to longitudinal, transverse and other higher order modes in a general linac configuration.

The basic algorithm can be summarized as following. The string of HOMs that a bunch sees in its lifetime between injection and ejection points is represented by a list of pointers to the actual cavities. The proper time delays between cavities is also stored for each pointer. E.g. for N HOMs and N_p passes, the list of pointers would be NN_p long pointing to N HOMs. This approach allows one to represent any recirculation configuration without limitations. As the train of bunches is injected into the structure, the next instance when any bunch sees any pointer is determined, and the HOM voltage in the corresponding cavity is updated. Then, this bunch is pushed to the next pointer where its coordinates are stored, waiting for its turn in time to be the next bunch going through a pointer. This way no bunches end up ahead of time precluding a situation when a bunch sees a cavity with incorrectly updated HOM fields. Furthermore, the algorithm is general enough to allow modeling of the longitudinal instability where timing between different bunches is no longer fixed. The practical realization of this algorithm is relatively fast, allowing the tracking of 0.1 ms beam duration in a complete 5 GeV ERL with 248 HOMs in less than a minute on an average personal computer. This duration is sufficient to determine the onset of transverse BBU instability in most practical cases.

The output of the code contains amplitudes of HOM voltages as a function of time, which is used to determine the growth rate of the instability by fitting an exponential. Several successive calls are made to the tracking unit to determine the threshold.

SIMULATION EXAMPLE

We have performed computational studies of BBU in a large scale, 5 GeV ERL. Details for this particular layout can be found elsewhere[7]. The linac consists of 248 SRF cavities, and 14 HOMs with highest $(R/Q)_\lambda Q_\lambda$ from the TESLA TDR [8] were used in these simulations.

Fig. 2 shows the BBU threshold for various HOMs assuming that all cavities have only one HOM with a) ex-

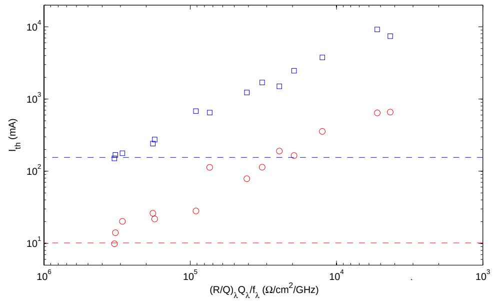

Figure 2: Threshold current for 14 worst HOMs for identical (red circles) and randomly (blue squares) distributed around nominal HOM frequencies (10 MHz interval).

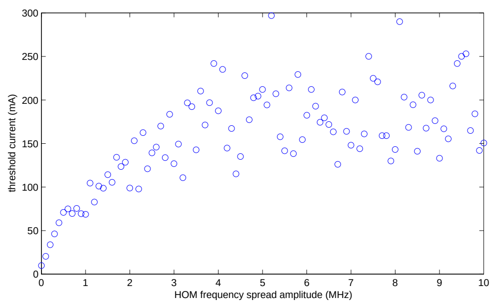

Figure 3: Threshold vs. HOM frequency spread amplitude. HOM parameters: $(R/Q)_\lambda = 11.21\,\Omega/\mathrm{cm}^2$, $Q_\lambda = 5 \times 10^4$, $f_\lambda = 1699.1$ MHz.

act same frequency (circles), and b) randomly distributed frequencies with a spread of 10 MHz around the nominal value for each HOM (squares). The magnitude of HOM frequency spread used is of the order that is actually being measured in existing SRF cavities. The plots also show thresholds when all 14 modes are included in simulations (dashed line). Clearly, the case with zero HOM frequency spread indicates the most pessimistic scenario, while introducing the spread between various cavities acts as effective damping by upsetting coherent interaction between HOMs. However, the effect of the frequency spread is limited in its usability to $\omega_\lambda/2Q_\lambda$ for any two given modes as was described earlier. Fig. 3 illustrates this point. Here, the threshold current is plotted as a function of rms frequency spread for a particular HOM. The same random number generator seed was used in all these cases, but the amplitude of the uniform spread was gradually increased. An estimate of frequency spread that would still produce lower threshold can be obtained by multiplying $\omega_\lambda/2Q_\lambda$ and the number of cavities, which gives 4.2 MHz spread or 1.2 MHz rms. This is in good agreement with simulation data shown in Fig. 3. Increasing frequency spread of HOMs beyond that does not increase the threshold current in general.

The somewhat chaotic behavior of the data plotted in Fig. 3 for larger frequency spread reflects the fact that the instability threshold can be quite sensitive to a particular

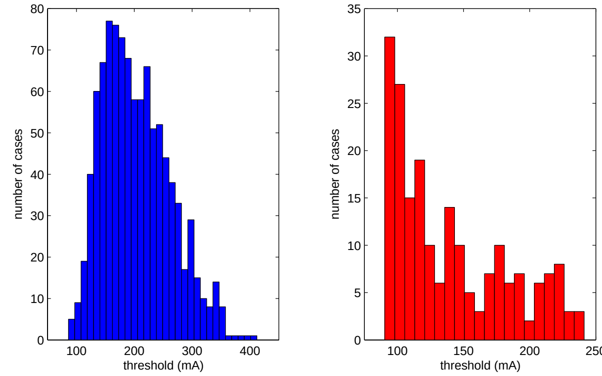

Figure 4: Statistics of BBU threshold. **Left**: 1000 cases with uniform HOM frequency spread distribution of 10 MHz maximal span (2.9 MHz rms). **Right**: 200 cases with uniformly distributed Q_λ's in the interval $(2.5, 7.5) \times 10^4$. Nominal parameters for the HOM are: $(R/Q)_\lambda = 11.21\,\Omega/\mathrm{cm}^2$, $Q_\lambda = 5 \times 10^4$, $f_\lambda = 1699.1$ MHz.

set of values of HOM frequencies. We analyze this by simulating 1000 random seeds all of which have the same rms values for the frequency spread, see Fig. 4 (left). The average threshold current is 204 mA and the rms uncertainty for different random seeds is 58 mA.

In addition to manufacturing uncertainties that result in spread of HOM frequencies for different cavities, there is also a spread in Q's of the modes. We simulate this situation in Fig. 4 (right) by determining thresholds in 200 random seeds that have Q's randomly distributed around its nominal value. The result shows that the threshold in this case can be as low as all modes having the highest Q, and as high as all modes having the lowest Q. Since the cavities at low energy contribute most significantly to the instability, this uncertainty of BBU threshold can be largely mitigated by ensuring that low energy cavities have properly damped Q's.

REFERENCES

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