SIMULATION CALCULATIONS OF STOCHASTIC COOLING FOR EXISTING AND PLANNED GSI FACILITIES

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Abstract

The process of longitudinal stochastic cooling is simulated using a Fokker-Planck model. The model includes the sensitivities of pick-up and kicker electrodes as calculated from field theoretical models. The effect of feedback through the beam is taken into account. The calculations cover the existing system at the ESR storage ring as well as the cooling system for secondary heavy ion and antiproton beams at the planned Collector Ring of the FAIR project. The paper discusses the resulting cooling times and requirements on the system layout as amplification factors and electrical power.

INTRODUCTION

Fast cooling of ion beams and antiprotons are key tasks of the FAIR project planned at GSI [1]. A large acceptance Collector Ring (CR) is an important part of this project [2]. The main purpose of the CR is the fast reduction of the phase space volume occupied by secondary Rare Isotope Beams (RIBs) or by antiproton beams. The required phase space reduction is achieved by the operation of a fast stochastic cooling system.

In this paper a numerical algorithm for the stochastic cooling simulations both at the ESR and at the CR in longitudinal phase space is presented. The algorithm is based on the Fokker-Planck equation. The simulations were performed to optimize the system parameters for maximum cooling rates.

FOKKER-PLANCK MODEL

The evolution of the momentum distribution function $f = f(x, t)$ (with $x=(p-p_0)/p_0$ the momentum deviation from the reference momentum p_0 and t the time) can be described by a Fokker-Planck equation (FPE) [3]:

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial x}\left(Ff - \frac{1}{2}D\frac{\partial f}{\partial x}\right). \qquad (1)$$

The drift coefficient F corresponds to the coherent component of the cooling force and becomes

$$F = \frac{(Qe\omega_0)^2}{4\pi^2\beta^2 E}\sum_m \frac{G_m}{\varepsilon_m} e^{-im\omega_0 \Delta T_{PK}}. \qquad (2)$$

Here, Qe is the ion charge ω_0 is the revolution frequency of the reference particle, $\beta=v/c$ is relativistic velocity factor and E is the kinetic energy. The summation is over the effective frequency range of the cooling system. The factor $exp[-im\omega_0\Delta T_{PK}]$ $\exp[-im\omega_0\Delta T_{PK}]$ is the effect of undesired mixing. G is a gain function including sensitivities of pick-up and kicker electrodes. ε_m is the signal suppression factor given by:

$$\varepsilon_m = 1 + \frac{N(Qe)^2}{\beta^2 E}\int dx_1 \frac{G_m\, \partial f/\partial x_1}{i(\omega(x) - \omega(x_1))}. \qquad (3)$$

The diffusion D consists of the two components, due to amplifier noise and Schottky noise. The Schottky coefficient D_S is

$$D_S = \frac{(Qe)^4\omega_0^3}{16\pi^4\beta^4 E^2\eta}\sum_m \frac{1}{m}\left|\frac{G_m}{\varepsilon_m}\right|^2 f, \qquad (4)$$

where η is the frequency slip factor. The thermal coefficient D_t is expressed in terms of the thermal temperature T_{eff} at the pick-up,

$$D_t = \frac{(Qe\omega_0)^2 k_B T_{eff}}{8\pi^3\beta^4 E^2}\sum_m \left|\frac{G_t}{\varepsilon_m}\right|^2, \qquad (5)$$

where G_t is the gain excluding the sensitivity of pick-up.

NUMERICAL METHOD

The drift and diffusion coefficients of stochastic cooling are non-linear and it is not possible to perform analytical calculations. Hence numerical simulation must be used. The following finite difference equation (compare with Eq.(1)) is used:

$$\begin{aligned} &f_i^{t+\Delta t} - f_i^t = \\ &\frac{\Delta t}{2h_i}\left(F_{i+1}f_{i+1}^{t+\Delta t} - F_{i-1}f_{i-1}^{t+\Delta t}\right) + \\ &\frac{\Delta t}{2\xi_i}\left(D_{i+\frac{1}{2}}\frac{f_{i+1}^{t+\Delta t} - f_i^{t+\Delta t}}{h_i} - D_{i-\frac{1}{2}}\frac{f_i^{t+\Delta t} - f_{i-1}^{t+\Delta t}}{h_{i-1}}\right) \end{aligned} \qquad (6)$$

The variables in this equation are $h_i=x_{i+1}-x_i$, $\xi_i=1/2(h_i+h_{i-1})$, and $D_{i+1/2}=1/2(D_{i+1}+D_i)$, and, Δt and h_i are the steps in time and momentum, i=1,2···I, enumerates mesh nodes for momentum. F_{i+1} and D_{i+1} are the drift and diffusion coefficients at point $(i+1)$ and are calculated by Eq.(2,4,5). The choice of the size-step depends on the desired accuracy. The linear systems obtained from Eq.6 are tridiagonal and are solved by the Gauss elimination algorithm.

COOLING SIMULATIONS

Stochastic cooling at the ESR

The calculations were performed for the ESR parameters for various ion species at 400 MeV/u, including an electrostatic model of the pick-up and kicker electrode sensitivities. The gain factors were determined in such a way that the optimal cooling rates at the beginning of the cooling process were reached. Fig. 1 shows the evolution

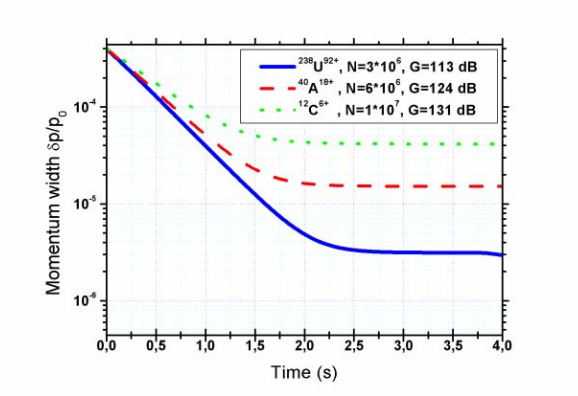

Figure 1. Evolution of the momentum width for different ion species.

of the 1σ width of the momentum distribution for uranium, argon and carbon ions in the course of the cooling process. *N* is the particle number, *G* the gain. Obviously the final equilibrium values are different for the given ion species. The reason for this behavior lies in the different signal to noise ratio of the Schottky signal at the pick-ups. For lighter ions a larger gain is needed in order to reach the same cooling rate as for highly charged ions. That increases the noise level and, as a consequence, the diffusion at the end of the cooling process. The observed higher equilibrium momentum width of the lighter ions with low charge is therefore mainly caused by noise. The calculations show also the well-known dependence of the cooling rates on the number of particles. Fig. 2 displays

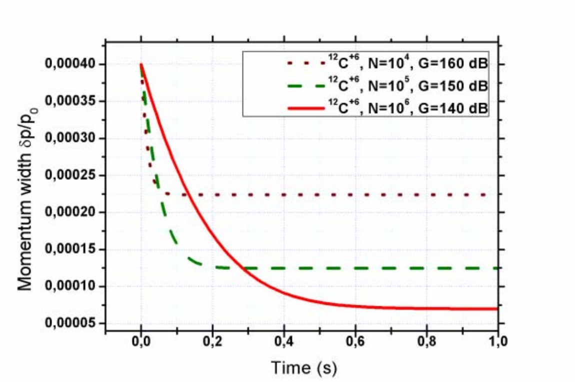

Figure 2. Evolution of the momentum width as a function of time for different numbers of carbon ions.

the effect for different numbers of carbon ions, where the effect of thermal noise is largest. It is well known that the cooling time is proportional to the number of particles. On the other hand, the diffusion term has two components: one is proportional to the Schottky noise density; the second is proportional to the thermal noise density. For the smallest number of particles the initial cooling rate is highest. However, the smallest equilibrium width is reached only for high particle numbers. This can be traced back to the large gain factor needed for small N, which enhances the diffusion rate due to thermal noise.

The results are compared to experimental results at the ESR [4] (see Tab.1). The good agreement for uranium supports the argument that the ESR stochastic cooling system is operated close to optimum gain, even though, in practice, there is a power limitation, as well. The discrepancy for argon is not very clear; it may be due to limited amplification.

Table 1: Comparison of simulations and experiment for the ESR.

Ion species		N	$t_{\|\|}$ *[s]*
$^{40}Ar^{18+}$	Experiment	$6 \cdot 10^6$	0.86
	Simulations	$6 \cdot 10^6$	0.52
$^{238}U^{92+}$	Experiment	$3 \cdot 10^6$	0.40
	Simulations	$3 \cdot 10^6$	0.42

Stochastic cooling at the CR

In the framework of the FAIR project we investigate the stochastic cooling process at the CR. We discuss Palmer cooling of RIBs at 740 MeV/u and filter cooling of antiprotons at 3 GeV. Since the momentum distribution at the exit of the superconducting fragment separator or of the antiproton separator is cut at the edges prior to injection, we consider a flat initial particle distribution with sharp drops at the edges. After bunch rotation and adiabatic debunching the momentum width for RIBs is $2.5{\times}10^{-3}$ and for antiprotons is $3.5{\times}10^{-3}$. The thermal temperature T_{eff} at the CR-pick-ups is by a factor of 10 smaller than at the ESR and therefore the cooling of RIBs is not only faster than at the ESR (see Fig.1 and Fig.3) but it also leads to a lower equilibrium momentum width.

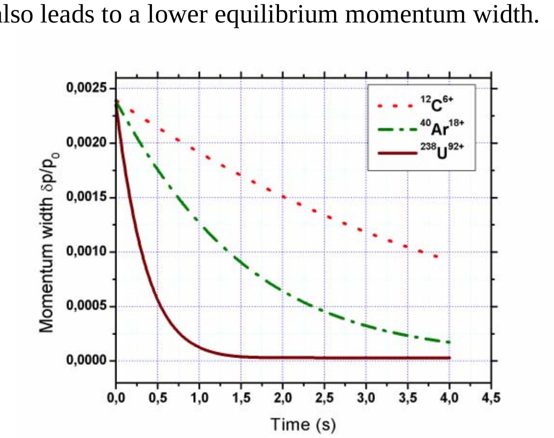

Figure 3. Evolution of the momentum width for different ion species at the CR.

A diagram of the cooling process of antiprotons is shown in Fig.4.The number of particles in this case is 10^8 and the cooling band is 1-2 GHz.

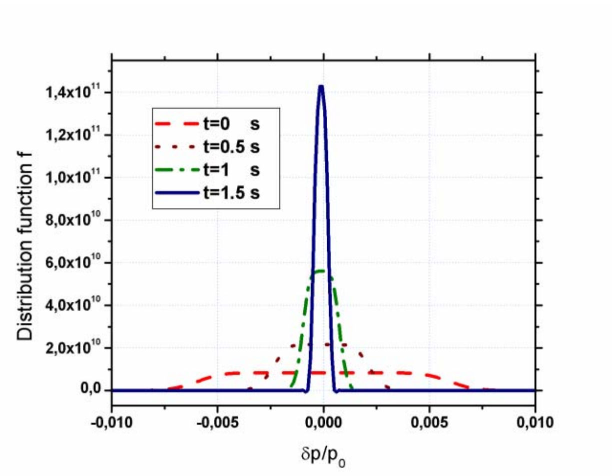

Figure 4. Evolution of the longitudinal distribution function of antiprotons at the CR.

The evolution of the momentum width for antiproton cooling for different settings of the initial electrical power is shown in Fig.5. Such simulations are useful to estimate the costs of the future power amplifiers. Note that the actual power to be purchased is by a factor 3 to 5 larger than in Fig.5 because of the random character of the signal.

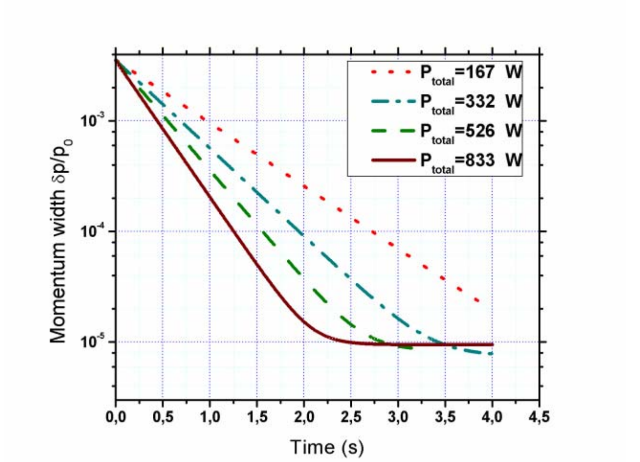

Figure 5. Evolution of the momentum width for a beam of 10^8 antiprotons at the CR for different electrical power.

Signal Suppression

Signal suppression is caused by collective effects inside the beam which effectively screen the signal seen by the pick-up. The effect can be analyzed by means of a Vlasov equation formalism. The screening may be described by a factor ε_m (see Eq.(3)) which modifies the system gain G and increases with the density of distribution function. As an example we present in Fig.6 and 7 drift and the diffusion coefficients at the end of antiproton cooling process. The number of particles is 10^{10} and the momentum width is 3.5×10^{-3}.

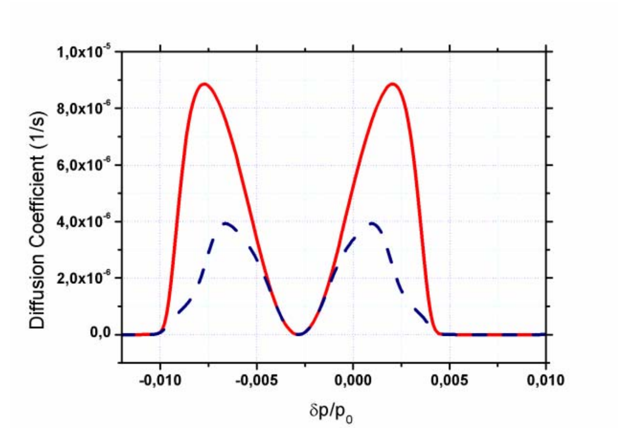

Figure 6. Calculated diffusion coefficient with signal suppression (dashed) and without (solid).

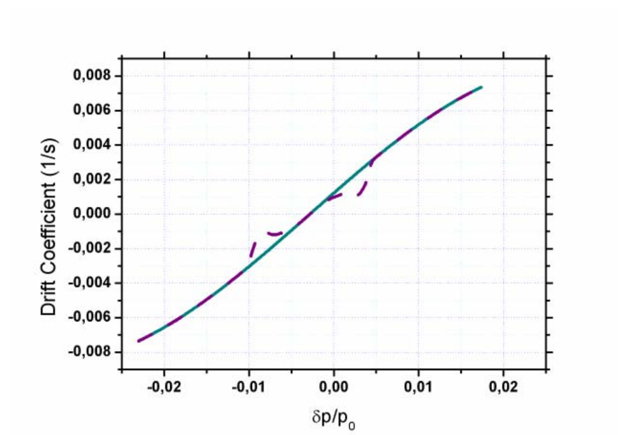

Figure 7. Calculated drift coefficient with signal suppression (dashed) and without (solid).

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